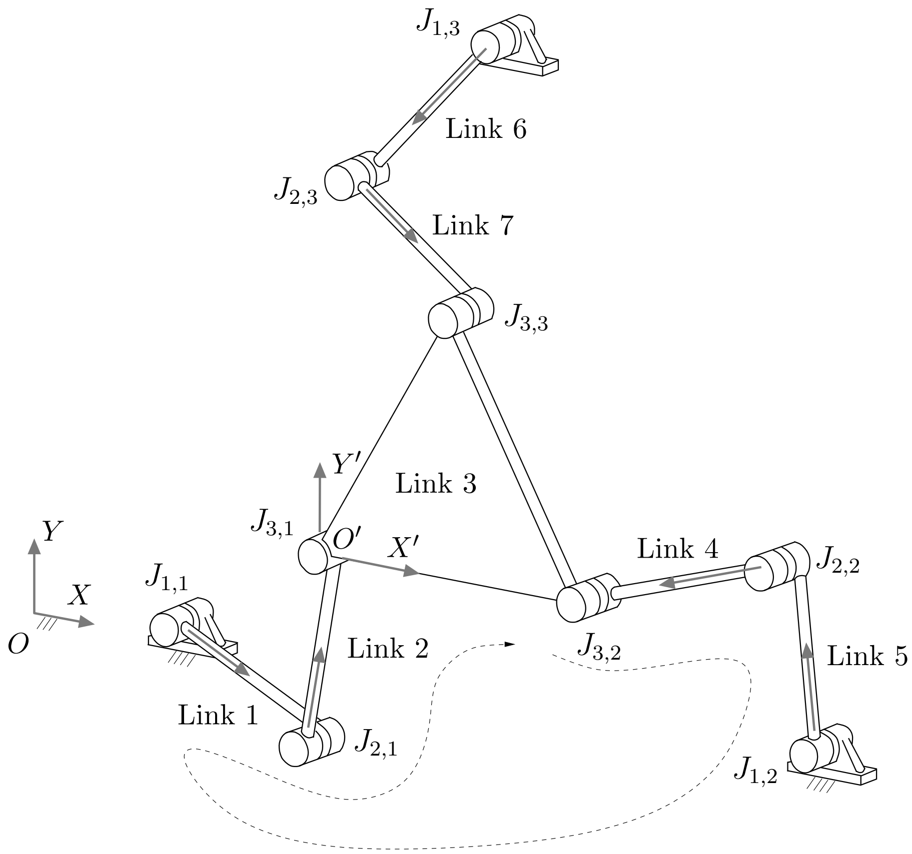
GFR, final exam

24 January, 2014

Test questions (10/20 points)

Mark the answer in the attached answer sheet. Each question has only one correct answer. Failed answers do not subtract points.

1. **(1 point)** Consider the 3-<u>R</u>RR manipulator of the figure, where the lengths of the leg links are all a, the distance from $J_{3,1}$ to $J_{3,2}$ is b, and the distance from $J_{1,1}$ to $J_{1,2}$ is c. We attach a local reference frame to every link i (indicated in grey in the figure) and define θ_i as the angle between such frame and the OXY frame. For the frames of the leg links, only the X axis is indicated, the Y axis being at 90° counterclockwise. Joints $J_{1,1}$ and $J_{1,2}$ are on the X axis.



In frame OXY, the loop-closure equation of the indicated loop is:

(a) $\begin{cases} a\,\cos\theta_1 + a\,\cos\theta_2 + b\,\cos\theta_3 = a\,\cos\theta_4 + a\,\cos\theta_5 \\ a\,\sin\theta_1 + a\,\sin\theta_2 + b\,\sin\theta_3 = a\,\sin\theta_4 + a\,\sin\theta_5 \end{cases}$

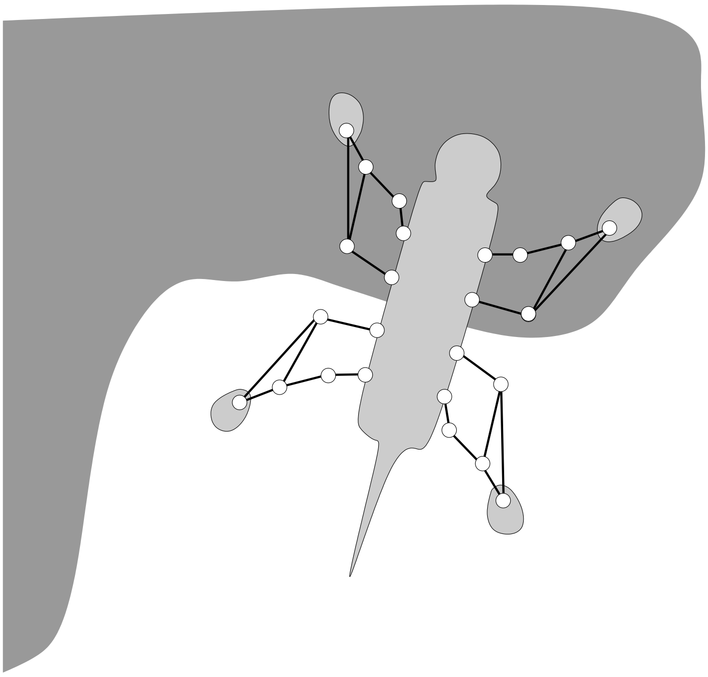
(b) $\begin{cases} a\,\cos\theta_1 + a\,\cos\theta_2 + b\,\cos\theta_3 = -a\,\cos\theta_4 - a\,\cos\theta_5 \\ a\,\sin\theta_1 + a\,\sin\theta_2 + b\,\sin\theta_3 = -a\,\sin\theta_4 - a\,\sin\theta_5 \end{cases}$

(c) $\begin{cases} a\,\cos\theta_1 + a\,\cos\theta_2 + b\,\cos\theta_3 - a\,\cos\theta_4 - a\,\cos\theta_5 - c = 0 \\ a\,\sin\theta_1 + a\,\sin\theta_2 + b\,\sin\theta_3 - a\,\sin\theta_4 - a\,\sin\theta_5 = 0 \end{cases}$

(d) $\begin{cases} a\,\cos\theta_1 + a\,\cos\theta_2 + b\,\cos\theta_3 - a\,\cos\theta_4 - a\,\cos\theta_5 = 0 \\ a\,\sin\theta_1 + a\,\sin\theta_2 + b\,\sin\theta_3 - a\,\sin\theta_4 - a\,\sin\theta_5 - c = 0 \end{cases}$

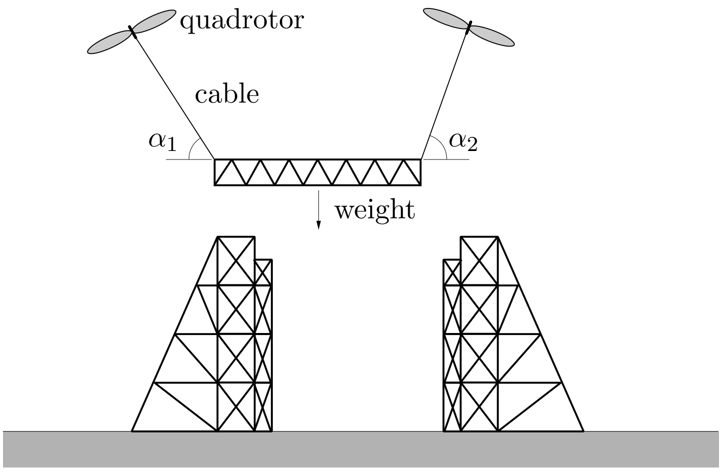
(e) None of the previous

2. **(1 point)** The lower feet of Stickybot have lost contact with the wall in an overhang. How many actuators are needed to fix the whole robot configuration? Assume that the pose of the upper adhesive feet is fixed to the wall, and that all circles correspond to movable revolute joints.



(a) 3
(b) 4
(c) 5
(d) 9
(e) 11

3. **(1 point)** Two quadrotors are transporting a structure in order to mount it on a larger construction, as shown in the figure. Assume that the load is always kept horizontal, that the center of gravity is in the middle of the load, and that the quadrotors and cables can resist any forces considered. The angles can vary in the range $0 \leq \alpha_i \leq \pi$.



Mark the false statement about such a system.

(a) Vertical forces can always be equilibrated, in all possible configurations of the system.

(b) If the weight is in equilibrium with the cable tensions, in abscence of other forces and couples, then $\alpha_1 = \alpha_2$.

(c) If the action lines of the cables intersect the center of gravity, then the system is unable to equilibrate additional couples applied to the carried structure.

(d) If $0 < \alpha_1 = \alpha_2 < \pi/2$, the system can equilibrate a small perturbation force in any direction, as long as its action line is concurrent with the cable lines.

(e) There is at least one configuration for which the system is unable to equilibrate the effect of horizontal wind.

4. **(1 point)** The configuration of the 3-RRR manipulator of the figure is mirror-symmetric with respect to line ADG, and three of the joints are actuated. Mark the false statement:

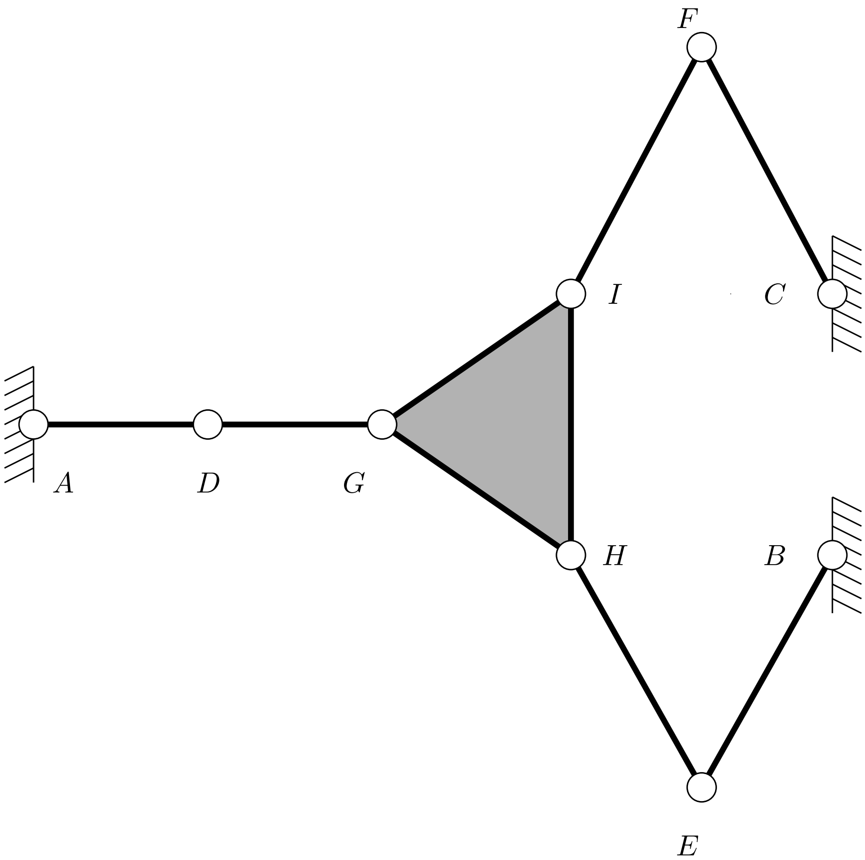
   (a) If D, E, and F are actuated, then the configuration is a forward kinematic singularity.

   (b) If A, B, and C are actuated, then the configuration is a forward kinematic singularity.

   (c) The configuration is a forward kinematic singularity regardless of the location of the actuators.

   (d) The configuration is an inverse kinematic singularity regardless of the location of the actuators.

   (e) There is a loss of platform dexterity. The platform cannot perform an arbitrary infinitesimal displacement $\delta\hat{D}$, and therefore not all small position errors can be corrected.



5. **(1 point)** Consider the identity matrix $\mathbf{I}$ and the triangular matrix $\mathbf{T}$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

   Mark the correct answer:

   (a) No serial robot exists whose kinematic Jacobian could be $\mathbf{J} = \mathbf{I}$ in a given configuration.

   (b) No serial robot exists whose static Jacobian could be $\mathbf{J}^\mathsf{T} = \mathbf{I}$ in a given configuration.

   (c) A 3-R$\underline{\text{P}}$R robot exists whose static Jacobian is $\mathbf{j} = \mathbf{T}$ in a given configuration.

   (d) A serial robot exists whose kinematic Jacobian is $\mathbf{J} = \mathbf{T}$ in a given configuration.

   (e) A serial robot could only have a kinematic Jacobian $\mathbf{J} = \mathbf{T}^\mathsf{T}$ in a singular configuration.

6. **(1 point)** A lamina is instantly rotating under a given twist $\hat{t} = \{\boldsymbol{\omega}; \boldsymbol{v}_o\}$, while receiving the action of a wrench $\hat{w} = \{\boldsymbol{f}; \boldsymbol{c}_o\}$. Mark the incorrect statement:

   (a) The wrench $\hat{w}$ can be interpreted as a force $\boldsymbol{f}$ acting on a given line, or as a force $\boldsymbol{f}$ applied to the origin point of the lamina, together with a couple $\boldsymbol{c}_o$.

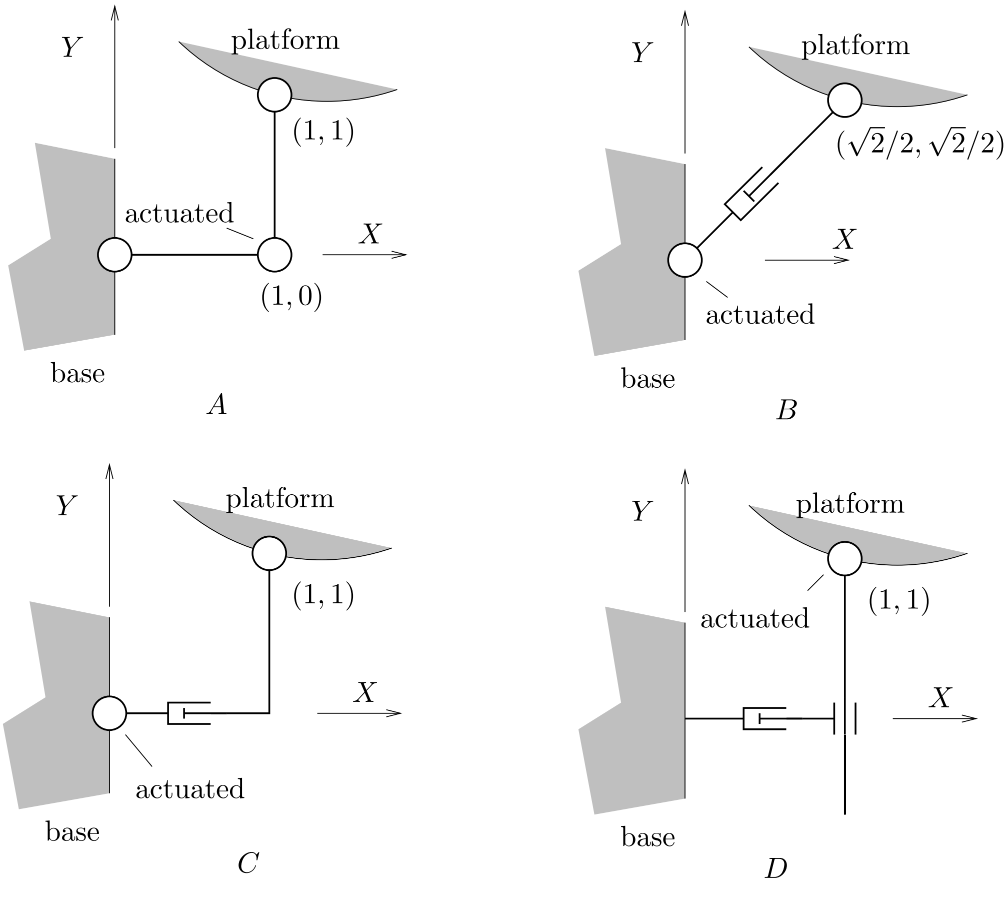
   (b) It is possible to determine the Plücker coordinates of the action line of $\hat{w}$ irrespectively of whether it is given in ray or axis coordinates.

   (c) If $\boldsymbol{\omega} = \mathbf{0}$, it is not possible to determine the projective coordinates of the instant center.

   (d) The twist $\hat{t}$ can be viewed as encoding the velocity field induced by an angular velocity $\boldsymbol{\omega}$ around a specific line perpendicular to the plane, or as the one induced by an angular velocity $\boldsymbol{\omega}$ around the origin point of the lamina, plus a linear velocity $\boldsymbol{v}_o$.

   (e) The action line of $\hat{w}$ intersects the instant center of the lamina if, and only if, $\boldsymbol{f}^{\mathsf{T}} \cdot \boldsymbol{v}_o + \boldsymbol{c}_o^{\mathsf{T}} \cdot \boldsymbol{\omega} = 0$.

7. **(1 point)** Consider the parallel robots whose leg architectures are displayed in the figure. Relate each one of these architectures with the unit line coordinates of the reciprocal wrench $\hat{s}_i$ intervening in matrix $\mathbf{A}$ of the kinematic equation $\mathbf{A}\hat{T} = \mathbf{B}\boldsymbol{\gamma}$.



$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ (1) \qquad \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix} \ (2) \qquad \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \\ 1 \end{bmatrix} \ (3) \qquad \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \ (4)$$

   (a) A-1, B-2, C-3, D-4.

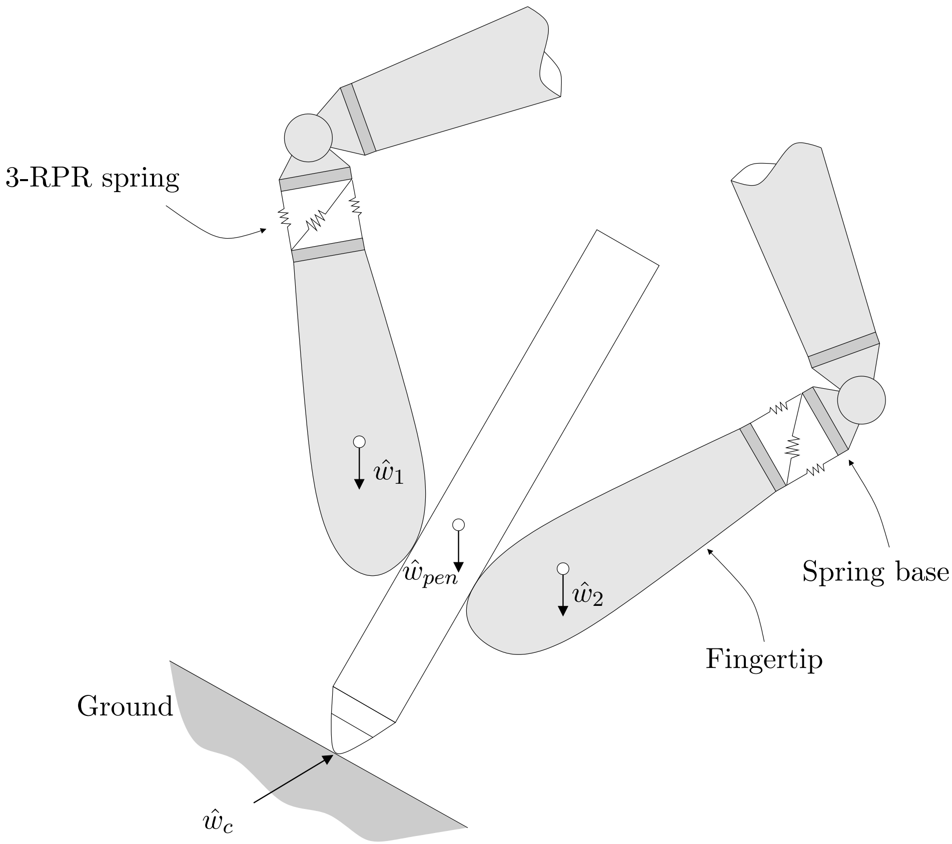
   (b) A-2, B-3, C-4, D-1.

   (c) A-3, B-2, C-4, D-1.

   (d) A-2, B-3, C-1, D-4.

   (e) A-4, B-3, C-2, D-1.

8. **(1 point)** In the position of the figure, two fingers of a robot hand are grasping a pen, just before making contact with the ground. From such position, the fingertips move slightly, pressing the pen against the ground. We wish to estimate the contact wrench $\hat{w}_c$ that the ground is applying to the pen, assuming that for each finger i we know the rigidity matrix $\mathbf{K}_i$ of its 3-RPR spring, the infinitesimal displacement $\delta\hat{D}_i$ that arises (of the fingertip relative to the spring base), and the wrench $\hat{w}_i$ of the finger weight. The wrench $\hat{w}_{pen}$ of the pen weight is also known.



The correct expression for $\hat{w}_c$ is:

(a) $\hat{w}_c = \mathbf{K}_1\,\delta\hat{D}_1 + \mathbf{K}_2\,\delta\hat{D}_2 - \hat{w}_1 - \hat{w}_2 - \hat{w}_{pen}$

(b) $\hat{w}_c = -\mathbf{K}_1\,\delta\hat{D}_1 - \mathbf{K}_2\,\delta\hat{D}_2 - \hat{w}_1 - \hat{w}_2 - \hat{w}_{pen}$

(c) $\hat{w}_c = -\mathbf{K}_1\ - \delta\hat{D}_1 - \mathbf{K}_2\,\delta\hat{D}_2 + \hat{w}_1 + \hat{w}_2 + \hat{w}_{pen}$

(d) $\hat{w}_c = \mathbf{K}_1\,\delta\hat{D}_1 + \mathbf{K}_2\,\delta\hat{D}_2 - \hat{w}_{pen}$

(e) $\hat{w}_c = \mathbf{K}_1\,\delta\hat{D}_1 + \mathbf{K}_2\,\delta\hat{D}_2$.

9. **(1 point)** The end effector of a spatial robot is moving under a twist $\hat{t} = [\boldsymbol{\omega}^\mathsf{T}, \boldsymbol{v}_o^\mathsf{T}]^\mathsf{T} = [0, 0, 1, 0, -1, 0]^\mathsf{T}$ in a given frame $OXYZ$. Mark the correct statement:

(a) The velocities of all points of the effector are parallel to the XY plane.

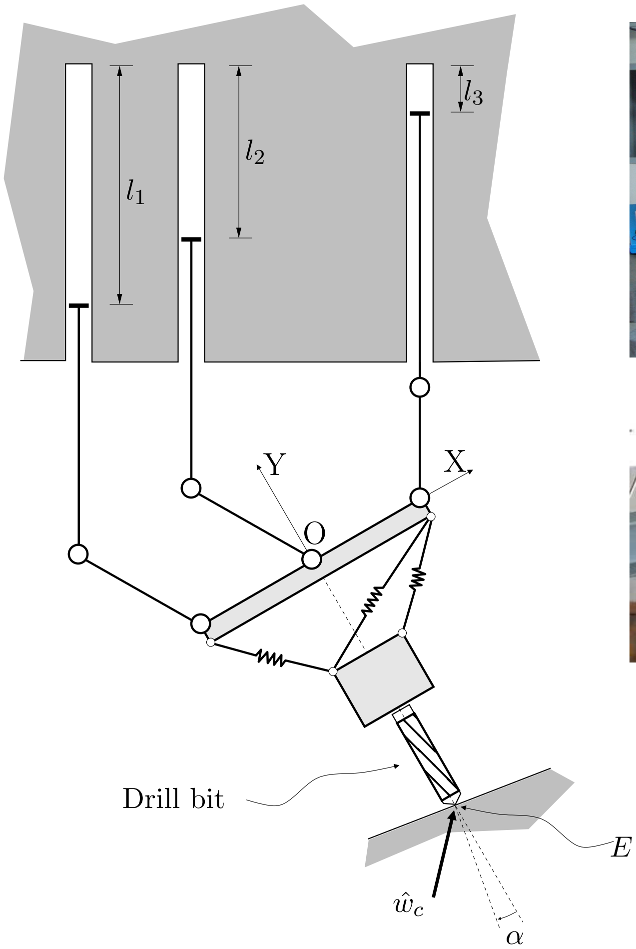
(b) The velocities of all points of the effector are parallel to the X axis.

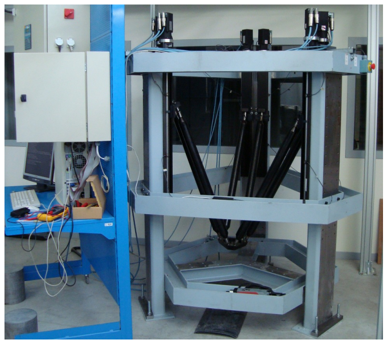
(c) The velocities of all points of the effector are parallel to the Y axis.

(d) The velocities of all points of the effector are parallel to the Z axis.

(e) No point of the end effector has a null velocity.

10. **(1 point)** The following 3-PRR drilling machine has a compliant 3-RPR wrist mounted between the drill and the moving platform (the pictures show spatial analogues of the machine). The drill bit-surface contact is punctual with friction. At the shown configuration we wish to eliminate the small contact wrench $\hat{w}_c$ applied by the surface to the drill bit, and simultaneously perform a small clockwise rotation $\delta\hat{D}_E$ of the drill bit about point E, of angle α.





Mark the correct statement:

(a) Since the right leg is fully extended, the manipulator is in an inverse kinematic singularity. There is a loss of dexterity, and the machine is unable to perform arbitrary correcting displacements $\delta\hat{D}_E$. The configuration is not suitable to hybrid control purposes.

(b) The contact type constrains the wrench $\hat{w}_c$ to act on a line orthogonal to the surface to be drilled.

(c) The vector of small actuator corrections $\boldsymbol{\delta l} = [\delta l_1, \delta l_2, \delta l_3]^\mathsf{T}$ needed to remove $\hat{w}_c$ while simultaneously performing $\delta\hat{D}_E$ is

$$\boldsymbol{\delta l} = \mathbf{B}^{-1}\mathbf{A}[-\mathbf{K}^{-1}\delta\hat{w}_a + \delta\hat{D}_E]$$

where $\delta w_a = -\hat{w}_c$, $\mathbf{A}$ and $\mathbf{B}$ are the standard matrices of the velocity equation of the machine, and $\mathbf{K}$ is the rigidity matrix of the compliant wrist.

(d) There is no problem if instead of actuating the P joints at the ground we actuate the revolute joints of the moving platform.

(e) If the distance from O to E is d, then we have

$$\delta\hat{D}_E = \alpha \begin{bmatrix} 0 \\ d \\ 1 \end{bmatrix} \qquad \text{(in reference frame } OXY\text{).}$$