Geometric Fundamentals for Robot Design

Final exam, 27 January, 2015

Test questions (10/30 points)

Mark your answers in the attached answer sheet. Each question has only one correct answer. Failed answers do not subtract points.

1. **(1.25 points)** Consider a 3R serial robot equipped with a welding tool in its end effector. Let (x, y) denote the coordinates of the tip point of the tool. The robot has to weld along a rectilinear segment, with the tip point starting at $P = (2, 0)$ and finishing at $Q = (2, 2)$. We denote by l_1, l_2, and l_3 the distances between joints 1 and 2, joints 2 and 3, and joint 3 and the tip point, respectively.

   Mark the false statement about the following `cuik` file

```
[CONSTANTS]
    Px := 2
    Py := 0
    Qx := 2
    Qy := 2
    l1 := 1
    l2 := 1
    l3 := 1
    ct3 = 1
    st3 = 0

[SYSTEM VARS]
    x:[-l1-l2-l3,l1+l2+l3]
    y:[-l1-l2-l3,l1+l2+l3]
    ct1:[-1,1]
    st1:[-1,1]
    ct2:[-1,1]
    st2:[-1,1]
    t:[0,1]

[SYSTEM EQS]
    ct1^2 + st1^2 = 1;
    ct2^2 + st2^2 = 1;
    x = l1 * ct1 + l2 * ct2 + l3 * ct3;
    y = l1 * st1 + l2 * st2 + l3 * st3;
    x = Px + (Qx-Px) * t;
    y = Py + (Qy-Py) * t;
%   st1 * ct2 - st2 * ct1 = 0;
```

   (a) The file can be used to determine whether the tip point can reach all points of the PQ segment.

   (b) Counting the number of variables and constraints we can anticipate that, in general, the file will yield a one-dimensional solution set.

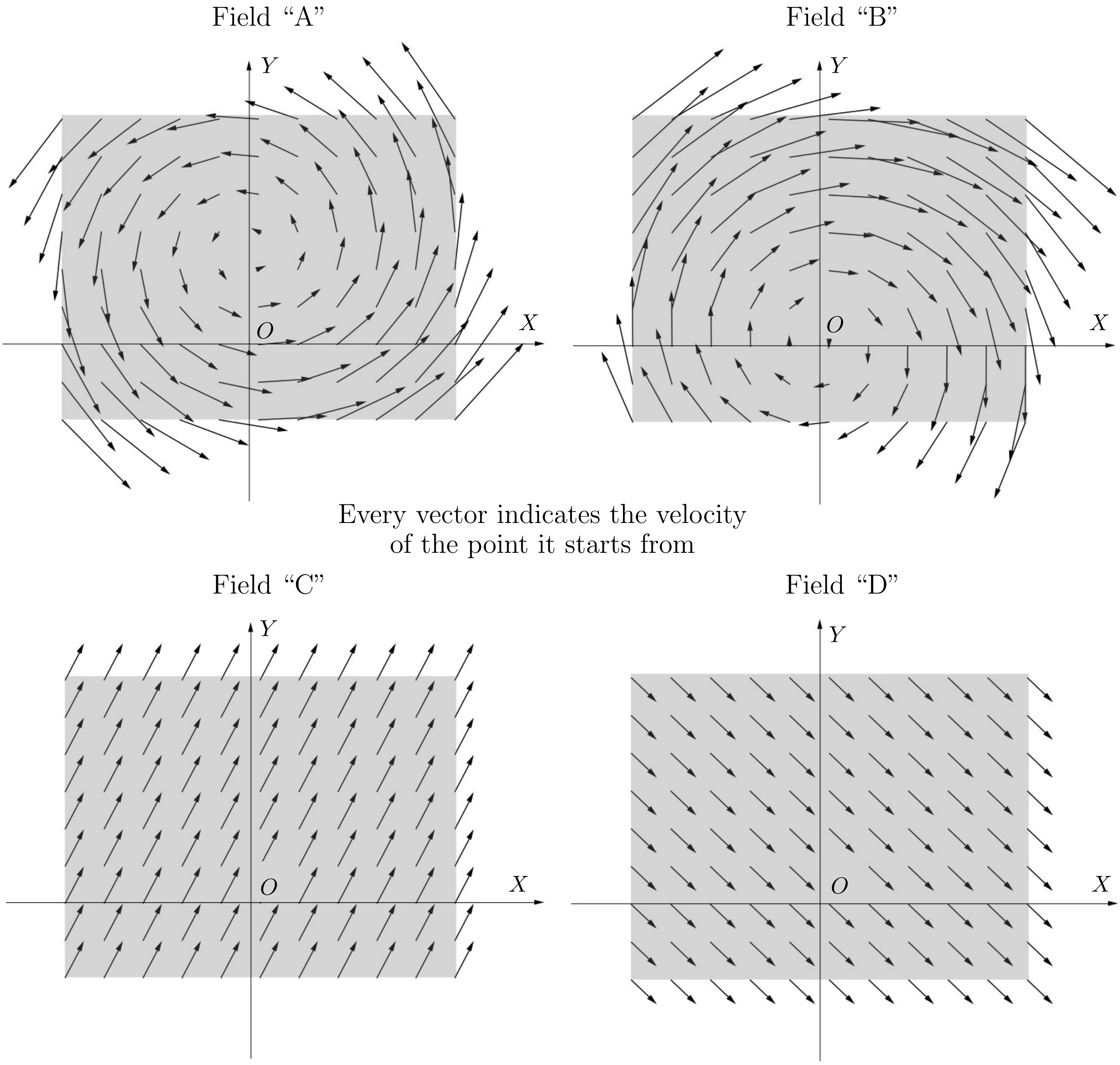
   (c) If we uncomment the last line, then the file allows us to detect the presence of singularities of the robot during the welding task. These singularities reveal dexterity losses of the welding tool.

   (d) The variables `st2` and `ct2` are the sine and cosine of the angle between the link of length l_2 and the link of length l_1.

   (e) In non-singular configurations, for each feasible value of t we will find two configurations of the arm.

2. **(1.25 points)** The following twists encode the velocity field of a rectangular lamina moving relative to the background plane, in each of four situations. All twists are written in axis coordinates in the OXY coordinate system indicated, and SI units are used throughout. Match each twist with the velocity field it encodes.

$$\hat{T}_1 = \begin{bmatrix} 0 \\ 0 \\ -0.5 \end{bmatrix} \qquad \hat{T}_2 = \begin{bmatrix} 1.5 \\ 3 \\ 0 \end{bmatrix} \qquad \hat{T}_3 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \qquad \hat{T}_4 = \begin{bmatrix} 3 \\ 0 \\ 0.5 \end{bmatrix}$$



(a) $T_1 \to$ A, $T_2 \to$ C, $T_3 \to$ D, $T_4 \to$ B

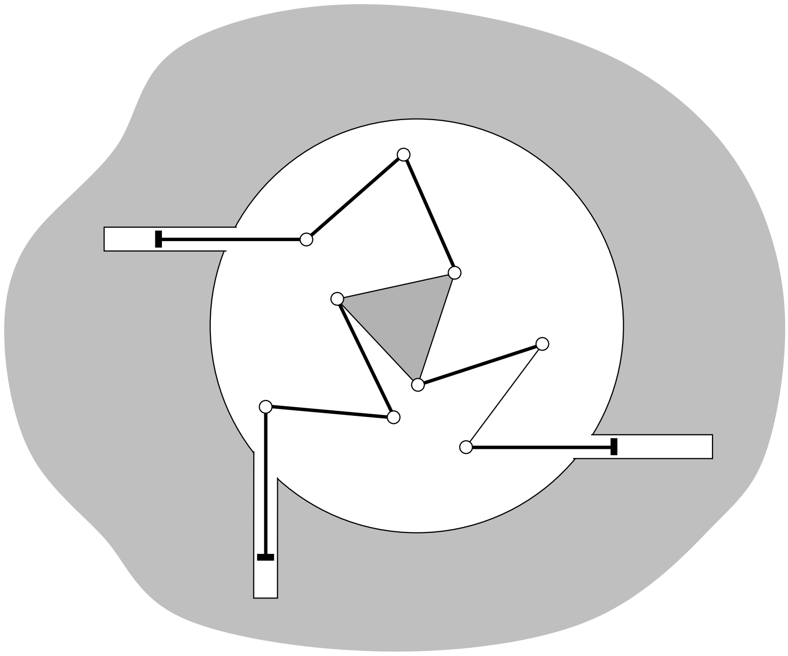
(b) $T_1 \to$ A, $T_2 \to$ D, $T_3 \to$ C, $T_4 \to$ B

(c) $T_1 \to$ B, $T_2 \to$ C, $T_3 \to$ D, $T_4 \to$ A

(d) $T_1 \to$ B, $T_2 \to$ D, $T_3 \to$ C, $T_4 \to$ A

(e) $T_1 \to$ D, $T_2 \to$ A, $T_3 \to$ B, $T_4 \to$ C

3. **(1.25 points)** Mark the correct statement about the mechanism of the figure.



(a) The mobility of the mechanism is 1.

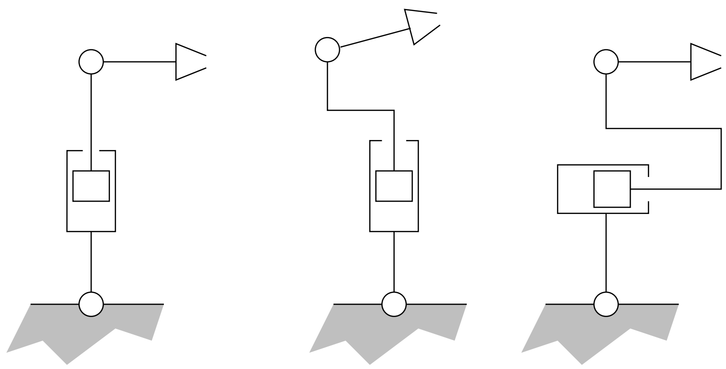
(b) The mobility of the mechanism is 6.

(c) The mobility of the mechanism is 3.

(d) The mobility of the mechanism is 5.

(e) After locking all joints of one leg, the mobility of the resulting mechanism is 3.

4. **(1.25 points)** Mark the correct statement about the following RPR manipulators and their feasible configurations:



(a) Robot A is now in a singular configuration but robot B isn't.

(b) Robot C is not in a singular configuration.

(c) Due to the shown limits of the prismatic joint, robot A can never be in a singular configuration.

(d) The three robots are in a singular configuration.

(e) All the previous statements are false.

5. **(1.25 points)** Mark the false statement, where $\mathbf{J}$ and $\mathbf{j}$ are the usual 3×3 Jacobians of the course:

(a) A 3R serial robot loses dexterity in a singularity. This can be seen mathematically on equation $\delta\hat{D} = \mathbf{J} \cdot \delta\boldsymbol{\theta}$, which becomes unsolvable for some values of $\delta\hat{D}$ when $\det \mathbf{J} = 0$.

(b) In a serial 3R robot, an internal velocity is a vector $\boldsymbol{\gamma} \neq \mathbf{0}$ satisfying $\mathbf{J}\,\boldsymbol{\gamma} = \mathbf{0}$.

(c) After locking its actuators, a 3-R<u>P</u>R robot is "shaky" at a singular configuration. This can be seen mathematically on equation $\mathbf{j}^{\mathsf{T}}\,\hat{T} = \mathbf{0}$, which has infinitely-many solutions when $\det \mathbf{j} = 0$.

(d) In a 3-R<u>P</u>R robot, an internal force (or self-stress) is a vector $\boldsymbol{\lambda} \neq \mathbf{0}$ satisfying $\mathbf{j}\,\boldsymbol{\lambda} = \mathbf{0}$.

(e) If $\operatorname{rank} \mathbf{j} = 2$, then $\operatorname{rank} \mathbf{j}^{\mathsf{T}} = 2$, $\operatorname{Null} \mathbf{j} = 2$, and $\operatorname{Null} \mathbf{j}^{\mathsf{T}} = 2$.

6. **(1.25 points)** Consider the following drilling machine based on a fully parallel 3-PRR robot, whose kinematic equation adopts the form $\mathbf{A} \cdot \hat{T} = \mathbf{B} \cdot \boldsymbol{\gamma}$ using the conventions of the course.

Mark the false statement regarding the shown configuration:

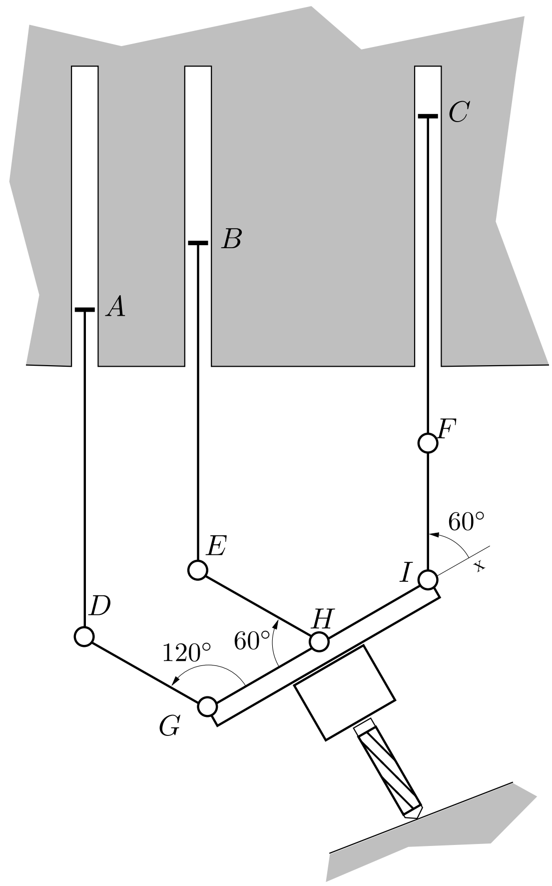
(a) If joints A, B, and C are actuated, $\det \mathbf{A} = 0$ and the manipulator is in a forward kinematic singularity.

(b) If joints D, E, and F are actuated, $\det \mathbf{A} = 0$ and the manipulator is in a forward kinematic singularity.

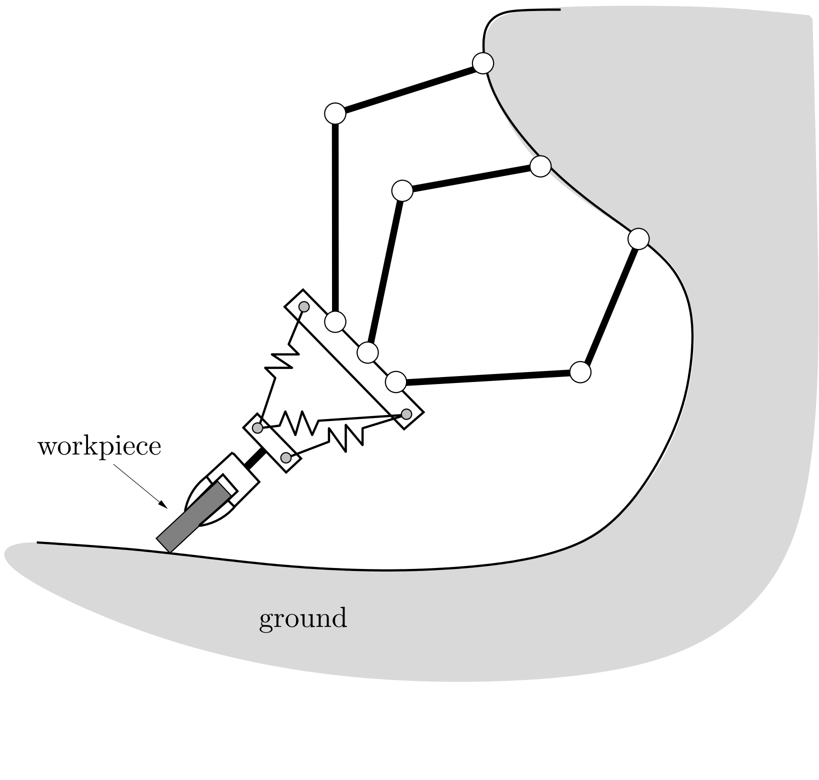
(c) If joints G, H, and I are actuated, $\det \mathbf{A} = 0$ and the manipulator is in a forward kinematic singularity.

(d) If joints A, B, and C are actuated, we have $\det \mathbf{B} \neq 0$ and the manipulator is not in an inverse kinematic singularity.

(e) In this configuration the manipulator can't be in an inverse kinematic singularity, irrespective of the choice of actuated joint in each leg.



7. **(1.25 points)** The figure shows a 3-RRR robot with a compliant 3-RPR wrist, and a gripper holding a workpiece which is being pressed against the ground. At the shown configuration, the Jacobian matrices of the 3-RRR robot are $\mathbf{A}$ and $\mathbf{B}$, in the usual notation of the course, and the rigidity matrix of the wrist is $\mathbf{K}$. From this configuration, we wish to achieve a small variation $\delta\hat{w}$ of the wrench that the ground is exerting on the workpiece, and a small displacement $\delta\hat{D}$ of the workpiece relative to the ground, both compatible with the contact model assumed.



Mark the correct expression for the vector $\delta\boldsymbol{\theta} = [\delta\theta_1, \delta\theta_2, \delta\theta_3]^\mathsf{T}$ of small angular increments that the actuators of the 3-RRR robot should undergo to achieve $\delta\hat{w}$ and $\delta\hat{D}$:

(a) $\delta\boldsymbol{\theta} = \mathbf{B}^{-1}\mathbf{A}\,(\delta\hat{D} - \mathbf{K}^{-1}\delta\hat{w})$

(b) $\delta\boldsymbol{\theta} = \mathbf{A}^{-1}\mathbf{B}\,(\delta\hat{D} - \mathbf{K}^{-1}\delta\hat{w})$

(c) $\delta\boldsymbol{\theta} = \mathbf{B}^{-1}\mathbf{A}\,(\delta\hat{D} - \mathbf{K}\delta\hat{w})$

(d) $\delta\boldsymbol{\theta} = \delta\hat{D} - \mathbf{K}^{-1}\delta\hat{w}$

(e) $\delta\boldsymbol{\theta} = -\delta\hat{D} + \mathbf{K}^{-1}\delta\hat{w}$

Note: $\mathbf{A}$, $\mathbf{B}$, $\mathbf{K}$, $\delta\hat{w}$, and $\delta\hat{D}$ are all given in the same reference frame.