Architecture and Geometry

by Michael Rubin

Structural Topology (1) 1979

Abstract

The structural system of a building must be consistent with its appearance, and together they must reflect the function of that building. From an understanding of the geometry of 3-dimensional space arises the possibility of realizing these ideals as relationships within finished structures. We look back in time to see how the alliance between architecture and geometry has weathered the main periods of classical and European history. In particular we show convincing examples of the harmonious development of geometry and architecture in the Renaissance, and of a serious rift which has developed between them since the 18th century.

Introduction

The object of this essay is to show that architecture and geometry have been linked throughout history. Perhaps it would be more accurate to claim that good architecture has always reflected an understanding of geometry. Let us take architecture in its broadest sense, as Le Corbusier or Frank Lloyd Wright might conceive of it: **architecture** is the manipulation of space for human use. (Notice how we are lumping architectural design, structural engineering, environmental planning, etc. all under one roof). We will understand **geometry** to be the study of the properties and relationships of magnitudes in space. Our claim then, which is inherent in these definitions, is that geometry is an indispensable tool to the architect. Since he works in forms and since geometry is the language of form, a creative architect must have a broad base of geometric concepts.

Our claim is based on the more general conviction that there is an underlying unity between mathematics and art. This unity derives from the fact that both mathematicians and artists, according to their skills, are looking for an order in the universe. The ability to observe and represent the patterns that underlie the form and structure of nature is the mark of achievement in both fields. The ornamentation on his pottery, weaving and basketry, shows that as far back as ten thousand years ago, Neolithic man had a grasp of geometric patterns. In fact, considering that primitive ornamentation displayed the concepts of congruence, similarity and symmetry, we would be hard pressed to determine if such work should be properly regarded as mathematics or art. The aesthetic appeal of patterns led artists and decorators to exhaust all the symmetry groups in the plane, long before Federov (1891) proved that only seventeen exist. Indeed, all 17 patterns appear in one building, the Alhambra (1230) **(Figure 1).** It also seems apparent that mathematical and artistic creativity elicit the same sense of satisfaction in the creator and the same sense of "aesthetic appeal" in the eyes of the beholder. Possibly this is what led Birkhoff to formulate his quantitative "aesthetic measure" for works of art. (Birkhoff 1933).

The appeal of symmetry in art and nature led Hermann Weyl (in agreement with Plato) to theorize

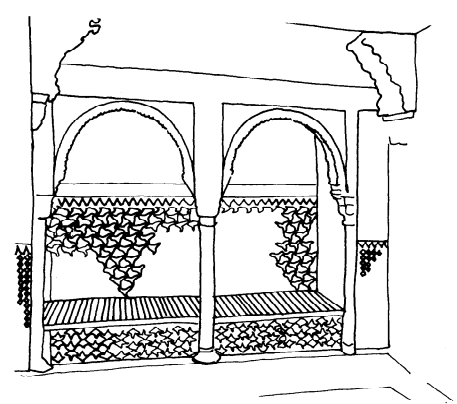

Figure 1.

that mathematics is the common origin of both. Mathematical laws are the origin of symmetry in nature, and the artists intuitive understanding of these laws is the source of symmetry in art (Weyl 1952, p6). That is, the artist is part mathematician. Conversely, G.H. Hardy claims "a mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas" (Hardy 1967). That is, the mathematician is part artist.

We shall not try to show all the links between mathematics and art but shall instead focus our attention on the links between geometry and architecture. This interrelationship is established by historical evidence. There have been entire architectural periods that were dominated by a particular geometric outlook. At times creative individuals have been involved in both architecture and geometry. The proportions of the Greek orders determined their architecture; the mason's geometric expertise determined the form of the Gothic Cathedral; projective geometry influenced Renaissance man's conception of space and hence, his architecture. The careers of Brunelleschi, Alberti, Leonardo da Vinci indicate the breadth of the Humanist's talents. It was an architect, Desargues, whose work on projective geometry was "the most original mathematical creation of the 17th century". Christopher Wren, architect of St. Paul's Cathedral, was labelled the most able geometer of his time by Newton. Lobachevsky, one of the founders of non-Euclidean geometry, learned architecture to supervise the building of an addition to the University of Kazan, where he was Rector. In our times Buckminster Fuller's domes are architectural constructions based on his geometric and topological theories.

Geometry performs a dual role for the architect: it helps him in a formal and in a technical sense. That is, geometry influences the visual and the structural aspects of design. Vitruvius understood this when he presented the three requirements of architecture: firmitas (structure), utilitas (function), and venustas (beauty). Thus a building must accomodate certain functions, stand securely and be visually stimulating. Our point of view is that the structural system of a building must be consistent with its appearance, and together they must reflect the function of that building. In our historical survey we notice that geometry has always determined the proportions of a building, hence its appearance. Structure was arrived at by empirical means, but sometimes, as in the Gothic Cathedral, structural system and geometric proportions combined to produce works of powerful visual impact. From the Renaissance on, more theoretical work was done in statics and mechanics and by the 17th century schools of engineering had been founded in France. Still, builders relied on their intuition, on experience, and on what Walter Whiteley calls "residual geometry", using geometric systems that had been worked out long before, instead of calling upon even the available structural theory of their day. Up to the middle of the 19th century this schism was not too critical as the empirical approach often sufficed for the stone and timber construction then common. However, from the industrial revolution on, man has made enormous advances in knowledge and use of materials and in structural theory. Furthermore architecture has had to respond to the complex demands of an industrialized society. Paradoxically, it is just at the time when architecture needs all the geometric and structural help it can get, it has divorced itself from contemporary geometric research into structure. "For as contradictory as it may seem, modern architecture is ignorant of modern geometry. Architecture has thus placed itself outside itself, outside of its own science" (Emmerich 1970, p2).

We would like to take you on a whirlwind tour of the architectural-geometric world, dwelling briefly on some periods and on some personalities. We should like to point out two major constraints on this review. First, we concentrate on western development and thus ignore the extraordinary geometric richness of Eastern architecture. Indeed, from the visual aspect, we consider the 2-dimensional and 3-dimensional patterns of Persian Moslem architecture to be the most striking example of geometry applied to architecture **(Figure 2)**. Second, the only way to appreciate architecture is to see it personally. Although we include some sketches and some photographs, they only hint at the desired effect.

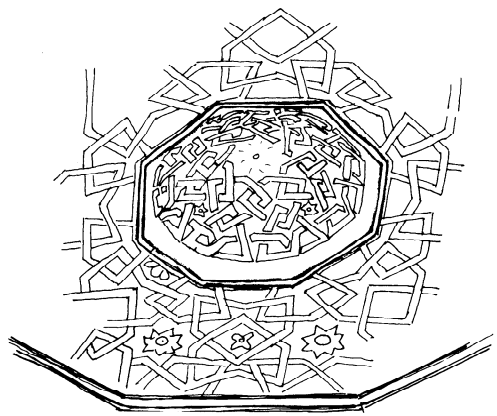

Figure 2.

Ancient Cultures

In the introduction, we referred to the fact that primitive man's use of ornamentation signified both geometric and artistic skills. Here we consider how his building reflected a merging of pattern and structure.

Ancient man built to accommodate his spatial needs. His dwellings possessed a natural geometry — based in part on the structural characteristics of his available materials. As an example, we take the Sumer reed house around 4000 B.C.E. The strong tall reeds of the Euphrates delta were used as the standard structural elements. These were bunched into bundles and bent to form either a circular or pointed arch. Reed matting was used as filler and the whole house was covered with mud. The house in its geometric simplicity contains all the structural elements of the Romanesque or Gothic cathedrals (Sandström, 1970, p7).

At about the same time, the Egyptians were harnessing the Nile and consequently urban developments, based on aristocracy, proliferated alongside it. Egyptian geometry developed through problems of mensuration after the floodings of the Nile. In fact, Herodotus used the word "geometry" to mean the measurement of the earth. The Egyptian's familiarity with geometric figures is evident in the sepulchers of the pharaohs — the pyramids **(Figure 3)**.

The pyramids and other ancient construction reflect man's attempt to model his human world on a "cosmic order" in order to symbolize his stability. Geometry was deeply related to astronomy and

religion, and was used to represent this cosmic order. Tons Brunes in his 2-volume work "The Secrets of Ancient Geometry" suggests that ancient architecture was based on an occult geometric system which he calls "ancient geometry". This geometry was a system of measurements, dimensions and proportions which were considered sacred and magic by religious leaders and which were used as a source of power by them. Brunes claims that this system was conserved right through the Middle Ages and is apparent in the Cologne Cathedral (15th century). Whether or not the concept of "ancient geometry" is correct, Brunes succeeds in showing that ancient temple construction is based on certain simple geometric patterns.

Greek Period

It is not our intent here to analyze the social, economic and geographical forces that resulted in the rise of Greek culture in the first millenium, reaching its highest development under Pericles in the 5th century B.C.E. We need only consider that the Greeks, with their emphasis on clarity of form, made contributions to mathematics and to art that laid the basis of Western civilization. It was the Greek mind, based on Ionian rationalism that first asked the modern scientific question — "why"? The Greeks sought to find man's place in the universe via logical means. They believed that the logic of nature expressed itself in the arrangement of her geometric shapes. Thus they placed architecture and sculpture on a higher level than painting.

Greek architecture is exemplified by temple construction **(Figure 4)**. It consisted of a raised platform from which a series of posts supported a continuous architrave which in turn supported the roof. Although they had discovered vault and arch construction, they rarely applied it as the 15 foot span of their beams or lintels sufficed in their human-scaled buildings. Their fine masonry joining gave the temple a sculptural look and implied the use of exact geometric measuring techniques.

The structure at first reflected the old timber construction techniques, but, after a while, familiarity with the stone materials led to new aesthetic notions. Both the structural and ornamental components of the building became standardized. The regulation of their position and shape led to the development of the Greek orders — doric, ionic, and corinthian — each with its exact proportions. The tapering of the doric column demonstrates the geometric sophistication they used to enhance the appearance of their work **(Figure 5)**. Similarly they were able to imperceptibly distort the main lines of their buildings so as to counteract the effects of perspective and foreshortening. Their search for proportion led them to a "modular" approach to construction: a "module", equal to half the lower diameter of the column, was the basis of the dimensions of component parts of the structure. Modern architecture, too, is looking for a modular system, albeit on a different scale and for different reasons, to solve some of our contemporary architectural problems.

It would appear that Greek geometry manifested itself more in the use of proportion and visual effects than in the development of structural systems. This is consistent with the Sophists' accent on understanding rather than utility in their mathematics and the Pythagorians use of mystical numbers and geometric patterns. Plato believed craftsmen were inferior to philosophers. He believed that geometry draws the soul towards "truth" whereas art represented falsity. Greek temples, which were sanctuaries of the Gods, depended upon geometric propositions to represent that ideal. From this point of view it is easy to understand why Greek architects concentrated upon exact proportions rather than on structural systems to make better use of their stone and marble building materials.

Ivins makes an interesting criticism on both Greek art and geometry (Ivins, 1964). He claims that in their geometry they worked out many relationships between measurements of lines, angles, areas and volumes but they missed the underlying structural qualities of lines and consequently never reached the general notions of duality and geometric continuity. Similarly in their art they never studied spatial organization or movement. We may extend his argument by claiming that they never considered the underlying structure of their materials in their architecture.

Greek geometry developed each theorem as a separate case, just as in architecture Greek temples

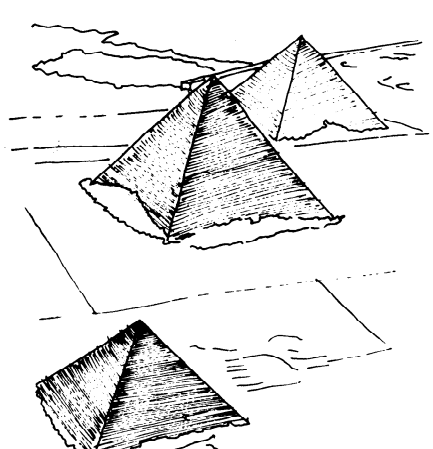

Figure 3.

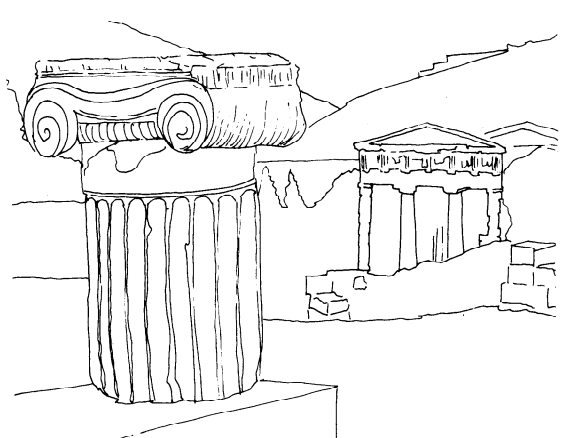

Figure 4.

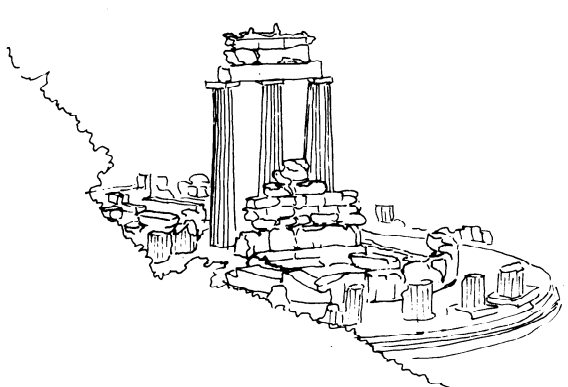

Figure 5.

Gothic Period

Building construction was developing rapidly in England and France in the 12th and 13th centuries. Technical achievement and visual impact of the Gothic cathedrals showed such a richness of shape and pattern that these constructions have been called "the architecture of geometry". It was an architecture that combined the technical experimentation and innovation of the master craftsmen with the knowledge of Greek mathematics that had come to Europe via the Arabs. Technical skills were perfected in the course of the vast amount of religious building at this time: between 1150 and 1280 about 80 cathedrals were built or rebuilt in France. Arab influence and their transmission of Greek mathematics came about in two ways: the crusades, and the recapture of Spain by Europeans in the 12th century. Is it coincidence that new Gothic architecture was taking hold in France at the same time that the Latin translation of Euclid arrived there?

The technical achievements of the Gothic period were, in brief: I. Differentiating between bearing columns and non-bearing walls; 2. utilizing the pointed arch; 3. use of vault-supporting ribs; 4. development of flying buttresses; 5. large use of glass and window tracery. The 13th century mason did not solve his structural problems by analysis as a modern engineer would, but by insightful trial-and-error, aided by experience and geometric rules of design. The design depended more on getting the shape, that is the geometry, of the structural members than on calculating the magnitude of forces. Their formulas and geometric rules were concerned with form and composition, based on understanding their materials, rather than structural theory. Their results can be appreciated in the cathedrals of Chartres, Amiens and Rheims **(Figure 9).**

Platonism, with its emphasis on the universe as being mathematical, was interpreted by St. Augustine to give theological meaning to simple numerical relationships; for example, the numeral "3" represented the Trinity and, thus, God. Augustine's numerology had strong influence on building; St. Bernard of Clairvaux suggested its use to determine the proportions of Cistercian churches.

Moreover Platonic thought encouraged interest in Euclidean geometry. Chartres became a centre of study of Euclid and was for a time the most important school of mathematics in the West. Master craftsmen and architects of the region realized the practical value of geometry in their work and were often enrolled in cathedral schools **(Figure 10).** "Such were the architects of the cathedrals — literate men who knew Latin and who were quite capable of keeping in touch with academic geometry" (Pacey 1976, p77).

We learn much about Gothic art and geometry from the sketchbook of Villard de Honnecourt, a wandering artist craftsman. He wrote that "the art of geometry commands and teaches" and his sketches show the geometric basis to many types of design.

In design of buildings, a geometric technique called the "ad quadratum design" was widely used. It was a system of construction lines based on the square, by which the height of a cathedral and other dimensions were worked out. It is no accident that the Gothic Cathedral arouses in the observer a keen awareness of geometric design.

Renaissance

Although anticipated earlier, especially by the work of Petrach, Dante, and Giotto, the full blossoming of the Renaissance began in the 15th century. The movement began on a scholarly level but was linked with technological, ecclesiastical and economic changes. Growing wealth in the northern cities of Italy (Florence, Genoa, Pisa, Milan, etc.) gave powerful stimulus for the development of fine art and engineering. This was coupled with a concern to seek unity with the whole classical world of Greece and Rome. Humanism, the study of classical culture, shifted man's interest from religion to himself. Humanism, like its Greek predecessor, sought to find the order of the universe.

Renaissance architecture exhibits the renunciation of Gothic style and the revival of the classic style.

Figure 9.

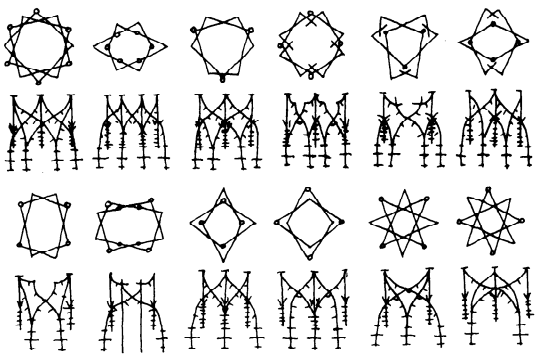

Figure 10.

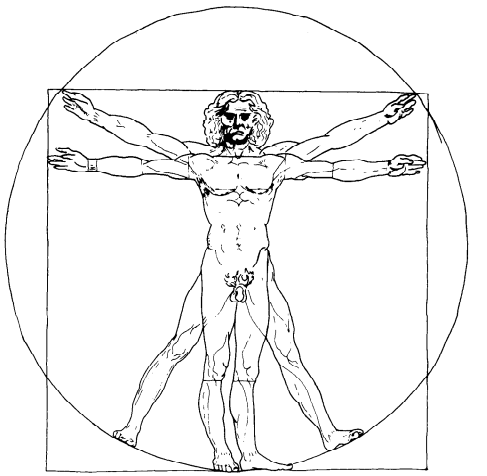

Figure 11.

This reaction of one period to its predecessor is a common theme in architectural history and from the Renaissance on occurs at more frequent intervals. The Renaissance rejected the "incommensurability, infinitudes, and dispension of Gothic spaces" (Zevi 1957, p112) and sought a unifying order, a discipline. Furthermore, what separates the Renaissance from the middle ages is the consideration of man as creator — man at the centre of the universe **(Figure 11).** In Gothic construction, a cathedral sometimes took two centuries to build; in the Renaissance one man is the designer. In Gothic architecture we know the name of the building; in Renaissance construction we recognize the work of known architects.

Thus it was an architect-engineer, Filippo Brunelleschi (1377-1446) who was associated with the start of Renaissance architecture. His practical skill is demonstrated through his many technical inventions. For example, he devised special techniques for constructing the dome of the Florence cathedral. He studied Roman ruins and incorporated Roman construction techniques in his building. His search for order led him to the design of the first Renaissance buildings: The Foundling Hospital (1419) and the Church of San Lorenzo (1420) in Florence. It is noteworthy that Brunelleschi should also be the discoverer of theoretical geometry behind central perspective. The development of **projective geometry** (modern terminology for perspective) was an event that was of both geometric and architectural importance.

The development of perspective is an instance of technical skill leading to intellectual achievement. In the 14th century, painters, especially around the Po Valley, and not geometers, greatly advanced the skill of drawing. The work of Altichiero and Avango in the Oratory of S. Giorgio show how refined their perspectives were. Yet it remained for Brunelleschi, working within the renewed interest in classic texts on optics and catoptrics, to discover the theory and construction techniques of central projection. In the 15th century many of the painters were good geometers. Vasari in his "Lives of the Painters" stresses the involvement of many quattrocento artists with solid geometry.

Leone Battista Alberti

Leone Battista Albert (1424-1472) was an example of the Renaissance "artist engineer" who contributed greatly to the theory of perspective. Alberti came from a patrician Florentine family and, as unlike his cathedral-building predecessors, he was weaned on scholarship rather than craftsmanship. "...Despising everything but books, (he) gave himself up entirely to the improvement of his mind, and made so great a progress in the sciences, that he outstript all the great men of that age who were most famous for their learning." (Alberti 1955, pxii).

Alberti's "Ten Books on Architecture" (printed in 1485) showed the range of the architect's skill. They include discussions of city planning, sewers, bridges, shipbuilding, hoists, and water supply. With respect to mechanics and surveying, he writes: "My design is to speak of these things not like a mathematician, but like a workman (VI,7)." This is consistent with the practice of the masons of the Gothic era who also used empirical knowledge of structure in construction. Again, not unlike the Gothic craftsman, it is in his treatment of architectural proportion that he is very concerned with mathematics. Mathematics unlocks the secrets of nature. "The Ancients knowing from the Nature of things... did in their works propose to themselves chiefly the Imitation of Nature, as the greatest Artist of all Manner of Compositions; and for this purpose they laboured, as far as the Industry of Man could reach, to discover the laws upon which she herself acted in the Production of her Works, in order to transfer them to the Business of Architecture" (IX,5).

Echoing (or fashioning) Renaissance neo-platonism, Alberti advised the study of symmetry and the proportions of the human figure in order to find proper ratios for architecture. It should be noted, however, that in addition to the Greek concept of mathematical beauty, the Renaissance architect added the Roman interest in engineering. Whereas to Plato matter was a debasement of the original mathematical idea of the world, and art was the "image of an image of an image", to the Humanist matter was a reflection of natural law and a constant source of study. If geometry ruled the Greeks, Nature ruled the Renaissance-man. Perspective was the geometric construct to represent nature. In his "Trattato della pittura" (1443), Alberti presented the first coherent theory of "pictorial science" — **perspective**. Although his "costruzione legittima" technique was cumbersome (based on a double projection), he succeeded in presenting a geometrical scheme for depicting objects in a unified space. He defined a painting as the intersection by a plane with the "visual pyramid": the artist's eye is at the vertex and the object is at the base. The intersecting plane is the artists' tableau.

Alberti's architecture shows how he put his theory to work. His search for order and his knowledge of Roman architecture resulted in very classic architecture, such as the Palazzo Rucellai (1446) in Florence shows **(Figure 12).** It was an instance of what Bruno Zevi calls "man's intellectual control over architectural space." (Zevi 1957, p114)

The laws seen to govern space in the Renaissance arose from the theory of perspective. Perspective objectively fixed the three-dimensional building, putting the observer in command. It was consistent with the Humanist philosophy of individualism and immanence. Leonardo da Vinci wrote that: "Perspective which shows how linear rays differ according to demonstrable conditions, should therefore be placed first among all the sciences and disciplines of man, for it crowns not mathematics so much as the natural sciences" (quoted in Reti 1974, p295).

Figure 12.

It is not surprising then that both artists and geometers contributed to the development of perspective. Between 1450-1550 it seemed that artists — Durer, Pelerin, Piero della Francesca — did most of the work, while between 1550-1600 the focus shifted to geometry, as evidenced by the treatises of Commandino, Danti, Ubaldi.

Post-Renaissance

After the Renaissance different architectural styles developed, each challenging its predecessor, each commanding for a period, then yielding to the challenge of the next style. So Renaissance gave way to Mannerism, Mannerism gave way to Baroque, Baroque to Rococo, Rococo to Neo-classicism. Neo-classicism to Eclecticism, Eclecticism to the Modern movement and the Modern movement to "post modernism" if we can so label presently—expressed longings for Baroque vitality.

It would be too tedious to go through all these styles and trace their relationship to geometry. Instead we isolate a few people who were active in both architecture and geometry and use them to exhibit the links between these disciplines.

It was in the Renaissance - Mannerist - Baroque period that art and mathematics were still closely linked, so our examples are concentrated in early post-Renaissance. We have seen Brunelleschi and Alberti as exemplary Renaissance designers. We now turn to Vignola, a Mannerist architect.

Giacomo de Vignola (1507—1573) was an architect who studied perspective. He taught both the "construzioni legittima" and the "distance-point" method (originally discovered by Pelerin) and showed that both techniques yielded identical results. He was noteworthy as a theorist and his "Regola delle Cinque Ordini d'Architettura" was considered authoritative. Vignola's Castello Farnese (completed 1564) was a much copied example of Mannerism, a style so called because it aimed to display a "mannered" look. Vignola also contributed to St. Peters. However, his most important work was the Gésu in Rome (begun 1568) **(Figure 13),** the building that most completely marks the transition from Mannerism to Baroque. The Gésu influenced many of the Roman Catholic churches of the Baroque period. "That Vignola, one of the designers of St. Peter's and the architect of the Gésu in Rome, should have been one of the great masters of theoretical perspective throws much light upon the spatial origins and character of the Baroque" (Ivins 1964).

As Summerson points out "there is no such category as "pure Baroque" — just because there is a word, it does not mean there is a pure essence to match it" (Summerson 1971, p30). "Baroque" originally signified the odd, extravagant shapes that 17th century Italian architects built as distinct from the symmetric shapes of the classicists, and now is used to designate the works of art of that century.

We turn to Francesco Borromini (1599-1667) not because he was involved in theoretical geometry but because his work demonstrated how geometry determined the architecture of the Italian Baroque. Borromini began as a craftsman. He was a simple stonemason at St. Peter's and later apprenticed himself to Carlo Maderna, whom he always called his master. He combined the practical experience of working in architectural trades with the theory he gained from much reading and with the artistic talent he developed through constant sketching to train himself as an architect. His first important work, the church of S. Carlo alle Quattro Fontane (begun 1633), shows his talent at spatial composition **(Figure 14).** The oval theme, which he uses to communicate a feeling of movement in space, is a form often used in Baroque architecture. (We wonder, in passing, if Kepler's development of his elliptical theory of planetery motion at the beginning of the 17th century had any bearing on the use of ovals in design.) We suggest the reader thumb through Portoghesi's book on Borromini to see his use of geometry in design and to get a visual understanding of the Italian Baroque (Portoghesi 1967).

A disciple of Borromini working mostly in Turin, Guarino Guarini (1624-1683), brought extreme plasticity and use of ovals to his work. Guarini was an architect, dramatist, Theatrine monk, philosopher

Figure 13.

Figure 14.

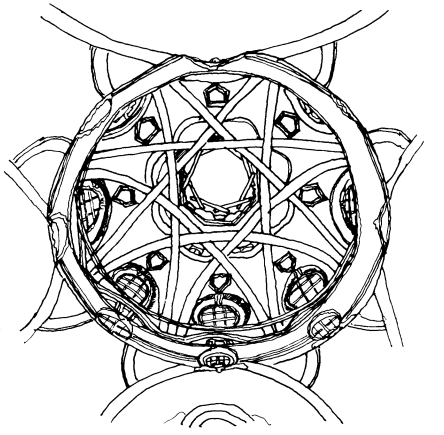

Figure 15.

and mathematician. His churches, S. Lorenzo at Turin (1666) for example, show an imaginative interplay of convex and concave parts **(Figure 15).** "It is not easy to understand them solely with the help of one's eyes and Guarini was probably no less interested in them as a mathematician than as an artist." (Pevsner 1972, p262)

One architect who had more influence on the mathematical world than on the architectural world was Gerard Desargues (1591-1661). He was a self-taught geometer who wanted to put into compact form a large number of disparate geometric theorems so that they would be available to artists, engineers and stone cutters. His coining of terms "stump", "knot" and the like in his scientific writing was presumably intended to attract an audience of scientific laymen, but may have instead had the effect of offending mathematicians. Whatever the reason his work was lost until the 19th century when Chasle accidently found in a bookshop a manuscript copy made by La Hire.

Desargue's mathematical contribution was the development of projective geometry as the study of qualitative or positional properties instead of metric ones. Besides "Desargues' Theorem" on perspective triangles, he did work on points at infinity, involutions and polarities. What concerns us in particular is that geometry was so fundamental to architecture that an architect could lay the groundwork for non-metrical geometry.

The Rift Between Engineering and Architecture

Desargues was active in Paris when that city was the centre of mathematical research in an era so given to scientific enquiry and experimental research that we call it the "Age of Reason". At the centre of the centre was the Franciscan P. Mersenne, about whom gathered scholars, mathematicians and artists. As an active correspondent and intermediary he linked such men as Fermat, Descartes, Desargues, Pascal, Huygens, Roberval, Dubreuil and Niceron (Flocon 1963, p57).

It is ironic that at the same time that Baroque architecture was displaying a richness of geometric shapes and that Desargues, Christopher Wren and others were working in both fields, the philosophical and practical emphasis on pure science was planting the seeds of separation between engineering and architecture. For up to this point, as we have seen, these two fields were highly interconnected. It was the Baroque architect Fontana who was responsible for the most amazing technical feat of the 16th century: the removal of the Egyptian obelisk from Circus Maximus and its transport and erection in front of St. Peter's. (Sandström 1970, P190). But the 17th century brought the Descartian - Newtonian emphasis on empirical rationalism, leading engineering towards a more theoretical approach and causing engineering to be considered as distinct from architecture. Their separation is evidenced by the educational academies instituted by Colbert in France. In 1663, he established the Academy of Sciences, in 1671, the Academy of Architecture and in 1675, the Corps de Génie for military engineers. The training of architectural students was the responsibility of a single professor, François Blondel, an architect and a mathematician. He tried to combine two antithetical approaches: the science of proportions as used in antiquity based on Vitruvius and "mathematical science" the geometry of space as applied to the structural solidity of buildings. "Mathematical science" was not yet sufficiently developed to provide an adequate base for architectural training and remained outside the realm of the architect's competence. The science of proportions, on the hand, was his domain, but it was a reactionary system clinging to the traditional symbolism of the contemporary social order (Pelpel 1978, p8).

The establishment of the Ecole des Ponts & Chaussées in 1747 further consolidated the rupture between architecture and engineering. Architecture retreated into revivalism and historicism, while theoretical advances were being made in structural engineering. In 1776, Augustus Coulomb summarized the static behaviour of building materials into a coherent system thereby founding structural analysis. By the beginning of the 19th century, the Ecole Polytechnique under Gaspard Monge was the centre for the development of structural theory. Monge had developed descriptive theory as a branch of geometry and his pupil Poncelet later continued the work of Desargues and founded projective geometry. It was due to Monge's influence that geometry began to flourish at the school. What is fundamental to our survey is to point out that when engineering and architecture went their separate ways, geometric research followed engineering and not architecture. This research led to the development of a geometric method of analyzing structures, called **graphic statics,** by Culmann, Cremona and Mohr in Europe and by Rankine and Maxwell in England. Culmann's pupil Wilhelm Ritter published his teacher's classic four volume work on static graphics (1888-1906), which had the effect of eliminating the lengthy calculations of the older analytic approach.

Parallel to the theoretical development in the 19th century were great advances in the use of materials. The research work done especially in bridge construction, resulted in the use of cast-iron, then wrought-iron and finally, by the 1850's of steel in construction. The monuments of the 19th century are primarily engineering works and not traditional architectural construction. The Crystal Palace, built outside London for the 1851 International exhibition by the gardener-engineer Sir Joseph Paxton, showed the vast spanning possibilities of glass-iron construction. Built in 16 weeks and demountable, the structure foreshadowed the advantages of modern prefabricated, integrated building systems. Yet, by conventional architectural standards it was an aberration; a "glass monster" cried Pugin. Gustave Eiffel's tower, 1000 feet high and constructed of wrought-iron, was the 1889 International exhibit's tribute to technology. John and Washington Roebling's Brooklyn Bridge (1883) demonstrated the intrinsic artistic beauty of the functional use of steel.

Use of steel did have a positive influence on American architecture, in particular on the development of the steel-framed skyscraper in Chicago in the 1880's and 1890's. The creative force in Chicago was Louis H. Sullivan, whose work shows the understanding of the new materials and the new technology. But Sullivan was a tragic figure his creativity crushed by the regressive use of classical architecture by lesser minds, as displayed by the Chicago Exhibition in 1893. The next generation of American architecture clung to the classicism of the Exhibition, and Sullivan was ignored.

By the 1890's the building material which was to have the most far-reaching affects in 20th century architecture came into popularity: reinforced concrete. But as yet its vast potential in compression, tension, elasticity and plasticity remained unharnessed. Eliel Saarinen who would later exploit its characteristics criticized architecture at that time. "Certainly in those days architecture did not inspire one's fancy. Architecture was a dead art form and it had gradually become the mere crowing of obsolete and meaningless stylistic decoration on the building surface. And so long had this state of things already lasted that a break would have been considered almost as much of a sacrilege as the breaking of the most essential principles of religion. So was architecture understood."(Saarinen 1948, pXI).

Architecture summed up a basic contradiction of the 19th century: the pursuit of technological progress as opposed to the retreat to the icons of the past (Huxtable 1960). Machine-produced objects were overlayed with styling imitative of the work of a craftsman. In building, the steel-skeleton was hidden behind an excess of Victorian ornamentation. The divorce between architecture and engineering had ultimately caused the divorce between structure and appearance. Geometry was not eliminated from architecture, but it was relegated to 2-dimensional stylization and was dislodged from its natural position as a determinant of spatial design.

Modern Movement

The Modern Movement was a reaction against the excessive ornamentation of Eclecticism. Seeing that it represents a plurality of activity that is too vast to analyze here, we refer the reader to Jencks' "Modern Movement in Architecture" for such analysis. If there was one unifying theme to Modern architecture it was in the striving for "functionalism". It attempted to make use of Modern materials — concrete, steel and glass — to create an architecture that clearly expressed the function of the building. Pevsner compares the creativity of the pioneers of the Modern Movement to the creators of Renaissance architecture, who reacted against the highly ornate design of High Gothic. He even prefers the creative spirit of the moderns who forged their own style while the Humanist fell back on the language of classic architecture (Pevsner 1972, p 424)

The Modern architectural movement called for nothing less than a revolution in geometric design. The honesty it demanded was that spatial form and not applied ornamentation be the expression of design. We see in the works of its masters a successful accomplishment of this task. Frank Lloyd Wright, in his earlier work, building with the "natural" materials of wood and stone, freed residential building from its cubic confinement. His use of irregular plans, his interpretation of interior and exterior space, his organic use of materials reveals a mind of extraordinary spatial imagination. Le Corbusier believed that simple geometric forms produced "primary sensations" in human beings. His understanding of reinforced concrete allowed him to create buildings which are at once both controlled and lyrical **(Figure 16).** Mies Van Der Rohe, whose aphorism "less is more" became the slogan of the International Style (as Modern architecture came to be known), designed buildings with a geometric simplicity and attention to detail reminiscent of the Doric temple.

Le Corbusier, Mies Van Der Rohe and Walter Gropius were the major forces behind the International Style. Yet this movement has ultimately ended in failure. Modern architecture failed to continue the 3-dimensional searching of its pioneers. Instead it copied their work with minor variations until our cities became saturated with skyscrapers which are essentially 2-dimensional conceptions. As happened when the Renaissance evolved into Baroque architecture, critics now demand more visual complexity in design than the International Style provided. They inveigh against the cold, severe style of functionalism, even praising the once damned "wedding-cake" decoration of Victorian design. For the two-dimensional patterns of ornamentation presents a more exciting geometry than do the monolithic slabs which dominate our cities. Nor we do need critics to tell us what our eyes plainly see: the monotony of high-rise design. Our criticism is not merely stylistic — it cuts right into the heart of functionalism. Did not the Modern Movement preach that the design should express the function of the building? Yet who today can differentiate between high-rise apartments, public buildings, office towers and other structures?

Conclusion

We believe that we are now in a position to realize the dreams of the Modern Movement. Whereas formerly masonry provided the standard building module and ornamentation was determined by those dimensions, we now have the capacity to construct space-enclosing polyhedra as the basic module. Polyhedra will at once determine the structure and ornamentation of the design. The unity of form and structure through the use of reinforced concrete has already been demonstrated by engineers such as Candella, Morandi and Nervi **(Figure 17).** In housing projects, formalists like Zvi Hecker and functionalists like Moishe Safdie have demonstrated the rich geometric potential of modular design. We believe that architecture must follow the exploratory work of such men.

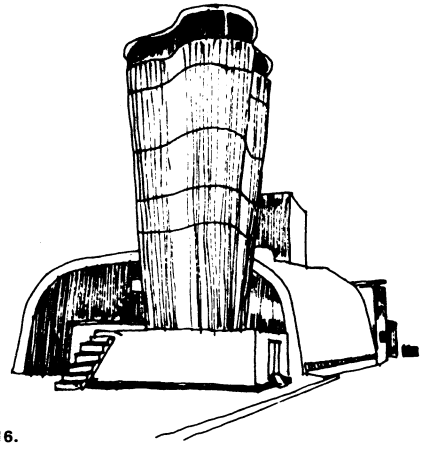

Figure 16.

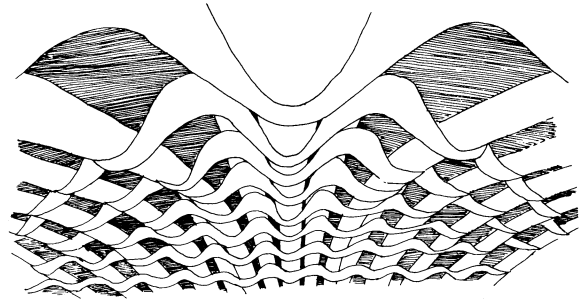

Figure 17.

We are also aware of the limitation of natural resources with its subsequent demand for economy in design. Industrialization, which has been so successfully harnessed in the automotive and aerodynamic industries, provides a powerful tool for architecture to reduce costs and to design with spatial modules. Modularization, mass production and prefabrication all impose constraints on design, so the architect must be trained to understand them. Further, the juxtaposition of modules and joining techniques will demand 3-dimensional insight in the minds of designers. Each of these features of architectural design will make exacting demands on geometry, and on the successful development of collaboration between geometry and architecture.

In our historical survey we have seen different examples of architectural styles. Usually they have been churches, villas, monumental architecture — but never buildings that affected the day-to-day lives of ordinary people. Today architecture presents a far more public face. Masses of people are dependent upon it to create the environment in which they live and in which they work. Inasmuch as man's environment has a determining influence on his physical and emotional state, architecture must now respond to deep social responsibilities. We believe that the need of sensual stimulation is so intrinsic to man that a varied 3-dimensional built environment is a necessity. In short, formalism in design takes on functional significance. Through the imaginative use of spatial design, form and structure can be united.

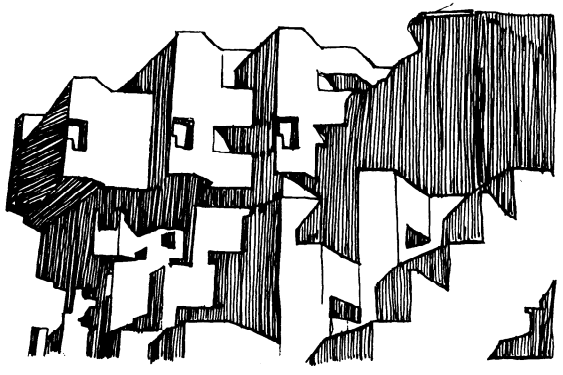

Figure 18

References

For a general history of western architecture we suggest the reader see Pevsner's **An Outline of European Architecture**. The articles on different periods and personalities in the **Encyclopedia of World Art** are thorough, as are the articles on "Architecture" and on "Perspective". Summerson's **The Classical Language of Architecture** is an excellent account of classicism from the 15th to 19th centuries. For a summary view of mathematics the reader can consult Struik's **A Concise History of Mathematics** or Coolidge's **A History of Geometrical Methods**. Sandstrom's **Man the Builder** is an easily-read history of building techniques. **The Maze of Ingenuity** by Arnold Pacey has good chapters on Gothic construction and Renaissance mathematics and art. Albert Flocon's **La Perspective** traces the evolution of perspective geometry. Finally, an intriguing enquiry into spatial conceptions is presented by William Ivins in **Art and Geometry**. Here for once is an author who criticizes Greek art and mathematics.

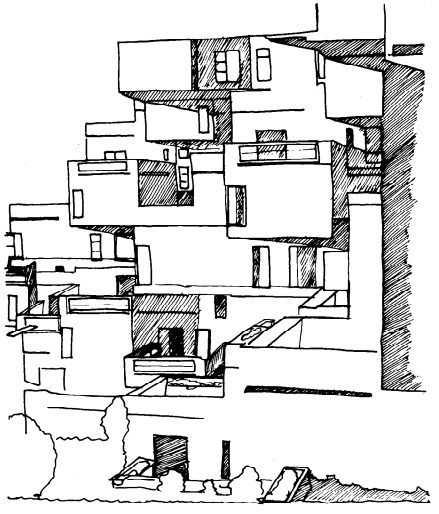

Figure 19.

List of Illustrations

Figure 1. Baths of the harem, Alhambra, Granada, 13th - 14th centuries.

Figure 2. Masjid i Jami, Isfahan. Braided Kufic inscription, circa 1310.

Figure 3. The pyramids at Gizeh, 3rd millenium B.C.E. Simple form suggests stability.

Figure 4. Treasury of the Athenians, Delphi, 5th century B.C.E.

Figure 5. Marble tholos at Delphi, 4th century B.C.E. An example of Greek elegance.

Figure 6. Interior of the Pantheon, Rome, 2nd century. Roman vaulting techniques allowed them to span great inner spaces.

Figure 7. The Pont-du-Gard, Nîmes, 1st century. The arch was the dominant theme in Roman engineering.

Figure 8. St. Sophia, Constantinople, 6th century. Basilia and domed square were typical of Byzantine design.

Figure 9. Rhiems Cathedral, 12th century. The culmination of French Gothic.

Figure 10. Medieval mason's working drawing. The craftsmen had great skill in geometry.

Figure 11. Vitruvian Man, by Leonardo da Vinci. Man at the centre of the universe.

Figure 12. Palazzo Rucellai, Florence, 1446, Alberti. Pursuing geometrical order in architecture.

Figule 13. Church of the Gesù, Rome, begun 1568, Vignola. The building that foreshadowed the Baroque.

Figure 14. San Carlo alle quatro fontane, Rome, begun 1638, Borromini. Undulating façade as example of Baroque vitality.

Figure 15. San Lorenzo, Turin, begun 1668, Guarini. The dome shows geometrical sophistication.

Figure 16. Unité d'habitation, Marseilles, begun 1947, Le Corbusier. This roof detail shows his sculptural use of concrete.

Figure 17. Exhibition Halls, Turin, 1949, Nervi. This roof detail shows structure as ornament.

Figure 18. The Dubliner Apartment House, Ramat Gan, begun 1960, Neumann Hecker & Sharon. An example of polyhedric architecture.

Figure 19. Habitat 67, Montréal, Safdie. 1967, Safdie. The module here is cubic.

The architectural sketches for this article were drawn by **Katia Montillet**.

They are visual impressions of photographs obtained from various sources, including **World Architecture** by H.R. Hitchcock, et al, McGraw-Hill Book Co. Ltd. (1963), A.U. Pope's **Persian Architecture**, George Braz iller Inc. (1965), and D. Sharp's **A Visual History of Twentieth Century Architecture**, New York Graphic Society Ltd. (1972).

Bibliography

The code in the first block of each bibliographic item consists of three parts, separated by dashes. The first letter indicates whether the item is a

B ook
A rticle
P reprint, or
C ourse notes.

The middle letter(s) indicates whether the piece was intended primarily for an audience of

M athematicians,
A rchitects, or
E ngineers.

The final letter(s) indicates if the piece touches on one or more of the principal themes of our work:

P olyhedra,
J uxtaposition' or
R igidity.

The key words or other annotations in the third column are intended to show the relevance of the work to research in structural topology, and do not necessarily reflect its overall contents, or the intent of the author.

Alberti 1955 Leone Battista Alberti B—AE	**Ten Books on Architecture.** Edited by Joseph Rytavert. Alec Tiranti Ltd., London, 1955.	Classic treatise dealing with all aspects of Renaissance architecture and engineering.
Brunes, 1967 Tons Brunes B—	**The Secrets of Ancient Geometry.** Rhodes, Copenhagen, 1967.	
Coolidge 1963 Julian Lowell Coolidge B—	**A History of Geometrical Methods.** Dover Publications Inc., New York, 1963.	
Emmerich 1970 David Georges Emmerich B—A—PJ	**Constructive Geometry.** University of Washington, Seattle, USA, 1970.	
Flocon 1963 Albert Flocon B—MA—	**La Perspective.** Presses Universitaires de France, Paris, 1963.	
Hardy 1967 G.H. Hardy P—M—	**A Mathematician's Apology.** Cambridge University Press, London, 1967.	
Hitchcock 1963 H.R. Hitchcock, S. Lloyd, D.T. Rice, N. Lynton A. Boyd, A. Carden, P. Rawson, J. Jacobus B—A—	**World Architecture.** McGraw-Hill, London, 1963.	Comprehensive and illustrated history of architecture.

Huxtable 1960 Ada Louise Huxtable B—AE—	**Pier Luigi Nervi** George Braziller Inc., New York, 1960.	Good summary of split between architecture and engineering in 19th and 20th centuries. Analysis and photo of Nervi's work.
Ivins 1964 William N. Ivins Jr. B—MA—	**Art and Geometry.** Dover Publications Inc., New York, 1964.	Good criticism of Greek art. Relationships between geometric and artistic concepts.
May 1967 Kenneth O. May A—MA—	**Mathematics and Art.** The Mathematics Teacher, October 1967, Vol. 60.	Links between mathematics and art. Historical examples.
Pacey 1976 Arnold Pacey B—AE—	**The Maze of Ingenuity.** M.I.T. Press, Cambridge, 1976.	Idealism in development of technology. Good section on cathedrals and on 16th century art and math.
Pelpel 1978 Pelpel A—A—	**La Formation Architecturale au XVIIIème Siècle en France.** Les Cahiers de la recherche architecturale, No.2, Mars, 1978.	
Pevsner 1972 Pevsner B—A—	**An Outline of European Architecture.** Penguin Books, Scotland, 1972.	European architecture an expression of western civilization illustrated.
Portoghesi 1967 Paolo Portoghesi B—A—	**The Rome of Borromini.** George Braziller Inc., New York, 1967.	Borromini as Baroque architect. Excellent illustrations.
Reti 1974 Ladislav Reti, editor B—AE—	**The Unknown Leonardi.** McGraw-Hill Book Co., Switzerland, 1974.	Leonardi as the Renaissance man.
Saarinen 1948 Eliel Saarinen B—AE—	**Search for Form.** Rheinhold Publishing Corporation, New York, 1948.	
Sandström 1970 Gösta E. Sandström B—AE—	**Man the Builder.** McGraw-Hill, Stockholm, 1970.	

Struik 1967 Dirk J. Struik B—M—	**A Concise History of Mathematics.** Dover Publications, New York, 1967.	
Summerson 1971 John Summerson B—A—	**The Classical Language of Architecture.** M.I.T. Press, Cambridge, 1971.	Essentials of Classicism. The orders, historical analysis, illustrations.
Vitruvius 1960 Vitruvius B—AE—	**The Ten Books on Architecture.** Dover Publications, New York, 1960.	Major source of information on classic architecture.
Weyl 1952 Hermann Weyl B—MA—	**Symmetry.** Princeton University Press, Princeton, USA, 1952.	Symmetry, applications in art and nature.
Zevi 1957 Bruno Zevi B—A—	**Architecture in Space.** Horizon Press, New York, 1957.	
Encyclopedia 1959 A—	**Encyclopedia of World Art.** McGraw-Hill, Toronto, c1959-1968.	See articles on different periods of architecture, on "architecture" and on "perspective".