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Singularities of Robot Mechanisms

Numerical Computation and
Avoidance Path Planning



Springer

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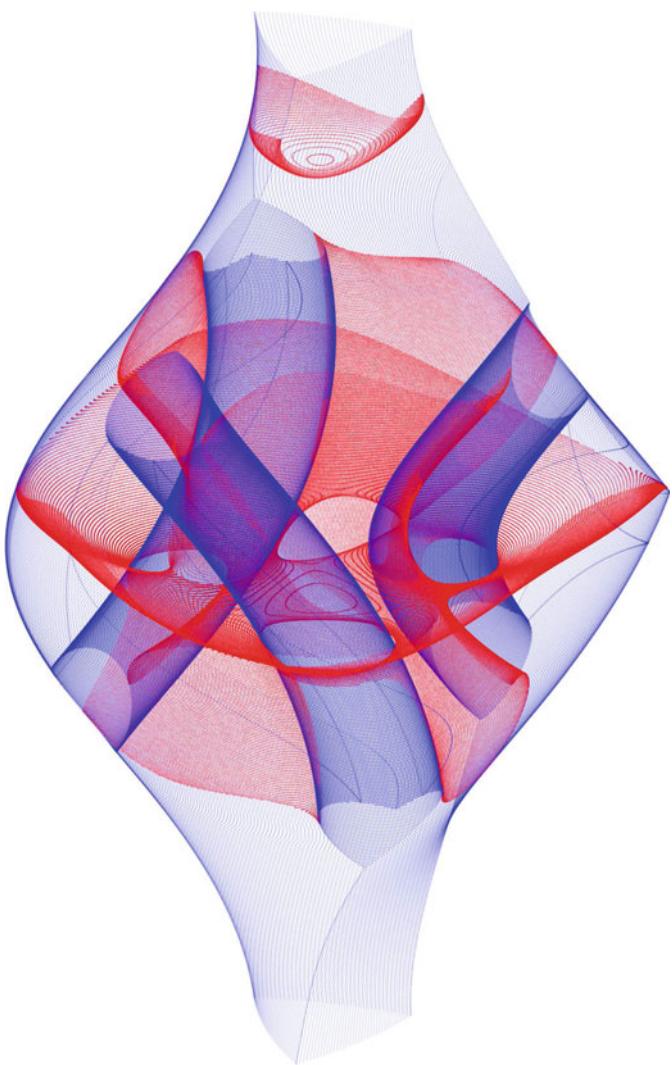
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To our families



Preface

Motivation and Purpose

Robotics is flourishing. Innovative robot mechanisms constantly see the light of day, and their use may increase dramatically in the near future. Whether on Earth or in Space, from research labs, to medicine, or industry, we see parallel and walking robots, force-feedback devices, flying manipulators, anthropomorphic hands and arms, humanoids, and other sophisticated machines in action. The capacity to perform complex motions in a precise and reliable way is essential for such devices, but the analysis and planning of such motions is by no means trivial. It requires a deep understanding and treatment of so-called *singular* configurations, the special postures of the mechanism in which kinetostatic behavior degrades to a huge extent.

The purpose of this book is to provide a consistent presentation of all singularity types that can be encountered in a mechanism, together with general robust methods for their interpretation, computation, and avoidance path planning. Obtaining such methods is crucial, because singularities generally pose problems for the normal operation of a robot, and thus they must be taken into account before the actual construction of a prototype. The book also shows how the computation of the singularity set provides detailed information on the local and global motion capabilities of a mechanism: its projections onto the task and joint spaces determine the working regions in such spaces, may inform about the existence of different assembly configurations, and highlight areas where control or dexterity losses arise. These projections also supply a fair view of the feasible movements of the mechanism, but do not reveal all possible singularity-free motions. In order to also tackle this issue, the book develops general path planning methods that avoid problematic singularities, and extends such methods to consider wrench-feasibility constraints in rigid-limbed or cable-driven hexapods.

Although the key role played by singular configurations has been known for years, methods for singularity set computation or avoidance have only been designed for specific mechanisms to date. A distinguishing feature of the book is that it proposes, for the first time, a toolbox of methods applicable to nonredundant

mechanisms of large generality, thus facilitating the development of new or more complex robot designs. Emphasis is put on mechanisms with nonredundant actuation because these allow an easier, more symmetric presentation of results, but the techniques, duly extended, could be used in redundant cases as well. In sum, the work seeks to eliminate barriers in design creativity, and to contribute to the general understanding on how the motions of complex multibody systems can be predicted, planned, and controlled in an efficient and reliable way.

Highlights

Overall, the book:

- Makes an effort to provide a clear definition and interpretation of all singularity types, whose descriptions have often been confusing, and very much dispersed in paper publications over the years.
- Provides solutions to two open problems of robot kinematics: the exhaustive computation of the singularity set, and the planning of singularity-free paths, on nonredundant mechanisms of general architecture.
- Shows how the ability to compute the singularity set yields a general method for workspace determination.
- Disseminates powerful branch-and-prune and higher-dimensional continuation methods in the fields of robotics and mechanism science.
- Develops an innovative method for planning wrench-feasible paths for hexapodal parallel kinematic machines.
- Illustrates the performance of the methods in robotic devices of practical interest, including planar, spherical, and spatial manipulators with closed-kinematic chains, and in challenging cable-driven robots.
- Makes intensive use of drawings and pictures to facilitate the reader's understanding of the subject.

The algorithms in the book are distributed within the CUIK Suite, an open-source software package developed by the Kinematics and Robot Design Group at IRI (CSIC-UPC), Barcelona [1]. This package can be downloaded from <http://www.iri.upc.edu/cuik>, and it can be used to replicate any of the test cases analyzed. The equation files needed to run such test cases can be found in the companion web page of the book <http://www.iri.upc.edu/srm>, which provides further complementary material, including videos, animated versions of several figures, and an introductory tutorial.

The results of the book were presented by the first author in the Advanced School on “Singular Configurations of Mechanisms and Manipulators,” held on September 22–26, 2014 in the International Center for Mechanical Sciences in Udine, Italy [2]. The presentation included a hands-on session in which the attendees could experiment with the algorithms of the book, applying them to simple and illustrative mechanisms. The material of that session is available

in <http://www.iri.upc.edu/srm>, and it can be used as an additional source of information.

Intended Audience

The book is the outcome of several years of collaborative research by the authors, primarily in the context of the Ph.D. work by the first author, supervised by the second and third authors. The style is that of a research monograph, but care has been taken to make the book accessible to a wide audience, from graduate students to professional engineers, mathematicians, or researchers working in robotics, mechanism design, or related fields. The book is also a source of supplementary material for robotics courses at graduate level, especially those devoted to robot kinematics. Altogether, the results may have an impact in a variety of domains, including the design of new parallel robots, haptic devices, 3D printers, mechatronic prostheses, or bio-inspired robots. They may also find application in emerging fields such as programmable surfaces for Earth and Space applications.

Acknowledgments

Several people have contributed to successfully complete this project. First of all, we owe a large debt to Josep M. Porta for his careful and rigorous work on the CUIK Suite over the years. The algorithms in the suite provide solid foundations on which this work is based. Without them it would have been impossible to come up with the solutions proposed. Also, Dr. Dimiter Zlatanov from University of Genoa provided invaluable help on the analysis of mechanism singularities presented in Chaps. 2 and 3, and Dr. Michael E. Henderson, from the IBM T.J. Watson Research Center, introduced the authors to the powerful continuation methods employed in Chaps. 5 and 6. The work of Prof. Illian Bonev, from ÉTS Montréal has likewise been particularly inspiring, and we appreciate his useful feedback on an earlier version of the manuscript. On the hardware side, Alejandro Rajoy and Patrick Grosch did the excellent job of designing and constructing some of the prototypes shown. They worked hard on the parallel 3-RRR robot of Chap. 5, and Patrick also developed the cable-driven robot of Chap. 6. Alex, moreover, produced beautiful illustrations for the book, including the one in this preface.

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Notation

Scalars, Vectors, and Tuples

x	A scalar variable, typically a coordinate
t	The time parameter
θ	An angle
v	A linear velocity
ω	An angular velocity
\mathbf{x}	A tuple or vector (understood by context). When \mathbf{x} is a vector appearing in an operation, it should be thought of as a column vector
\mathbf{x}^T	The transpose of vector \mathbf{x}
$\mathbf{x}(t)$	A time-dependent vector function
$\dot{\mathbf{x}}(t)$	The time derivative of $\mathbf{x}(t)$. Sometimes the dependency on t is omitted
$\mathbf{0}$	A column vector of zeros

Vector Components and Compound Vectors

Whenever the components of a vector need to be made explicit in a displayed formula, they will be written in a column with square brackets, as in

$$\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}.$$

Inline with the text, however, we shall prefer the notation $\mathbf{v} = (v_1, \dots, v_n)$ to refer to such components. In a similar way, we shall use $\mathbf{u} = (\mathbf{u}_1, \dots, \mathbf{u}_n)$ to refer to the column vector \mathbf{u} that results from the linear concatenation of the vectors $\mathbf{u}_1, \dots, \mathbf{u}_n$. Such a notation is meant to be equivalent to the expression $\mathbf{u} = [\mathbf{u}_1^T, \dots, \mathbf{u}_n^T]^T$, in which the \mathbf{u}_i are thought of as column vectors.

Screws

- $\hat{\mathbf{S}}$ A unit screw in axis coordinates, i.e., in (moment, vector) form
- $\hat{\mathbf{w}}$ A wrench in ray coordinates, i.e., in (vector, moment) form
- $\hat{\mathbf{T}}$ The twist of the end effector, in axis coordinates

Matrices

- \mathbf{X} A matrix
- \mathbf{X}^T The transpose of \mathbf{X}
- \mathbf{X}^{-1} The inverse of \mathbf{X}
- \mathbf{X}^i The submatrix of \mathbf{X} obtained by removing its i th row
- $\mathbf{0}$ A vector or a matrix of zeros, understood by context
- $\mathbf{0}_{m \times n}$ The $m \times n$ matrix of zeros
- \mathbf{I} An identity matrix
- $\mathbf{I}_{n \times n}$ The $n \times n$ identity matrix
- \mathbf{D} A diagonal matrix
- \mathbf{R} A 2×2 or 3×3 rotation matrix

Sets

- \mathcal{A} A set. In case of being a manifold, it is explicitly noted
- $\partial\mathcal{A}$ The boundary of \mathcal{A}
- $\mathcal{A} \setminus \mathcal{G}$ Set subtraction: the points of \mathcal{A} except those of \mathcal{G}
- $\mathcal{A} \times \mathcal{G}$ The Cartesian product of \mathcal{A} and \mathcal{G}
- \mathcal{C} The configuration space, or C-space, of a mechanism
- $T_{\mathbf{q}}\mathcal{C}$ The tangent space of \mathcal{C} at configuration \mathbf{q}
- \mathcal{G} The set of points \mathbf{q} of \mathcal{C} in which $\Phi_{\mathbf{q}}(\mathbf{q})$ is rank-deficient
- \mathcal{S} The singularity set of a mechanism
- \mathcal{S}_f The set of forward singularities of a mechanism
- \mathcal{C}_{sf} The singularity-free C-space, i.e., $\mathcal{C} \setminus \mathcal{S}$
- $[\underline{d}, \bar{d}]$ The closed real interval of values x such that $\underline{d} \leq x \leq \bar{d}$
- (\underline{d}, \bar{d}) The open real interval of values x such that $\underline{d} < x < \bar{d}$
- \mathcal{B} A box, i.e., a Cartesian product of closed real intervals
- $B^{\mathcal{W}}$ A box approximation of set \mathcal{W}
- \mathcal{P} A polytope
- \mathbb{R}^n The n -dimensional vector space over the reals

\mathbb{Z}	The set of integer numbers
$SO(m)$	The special orthogonal group in dimension m
$SE(m)$	The special Euclidean group in dimension m

Mechanism Symbols

L_j	The j th link of a mechanism
\mathcal{F}_j	The reference frame attached to the j th link
\mathcal{F}_1	The reference frame of L_1 , which usually acts as the absolute frame
\mathbf{r}_j	The position vector of the origin of \mathcal{F}_j in the absolute frame
\mathbf{R}_j	The rotation matrix providing the orientation of \mathcal{F}_j relative to \mathcal{F}_1
J_i	The i th joint of a mechanism
ω_i	The relative velocity at the i th joint, which can be linear or angular
P	A point on a link
$\mathbf{p}^{\mathcal{F}_j}$	The position vector of point P in reference frame \mathcal{F}_j
\mathbf{p}	The position vector of P in the absolute frame, usually \mathcal{F}_1
\mathbf{q}	The configuration tuple of a mechanism
\mathbf{v}	The tuple of input actuated degrees of freedom of a mechanism
\mathbf{u}	The tuple of output coordinates defining the mechanism functionality
\mathcal{Q}	The manifold of all possible \mathbf{q} values
\mathcal{V}	The manifold of all possible \mathbf{v} values
\mathcal{U}	The manifold of all possible \mathbf{u} values
n	The dimension of the configuration space \mathcal{C}
\mathbf{m}	The velocity vector of a mechanism $\mathbf{m} = (\mathbf{m}_u, \mathbf{m}_v, \mathbf{m}_p)$
\mathbf{m}_u	The output velocity components (those of the end effector usually)
\mathbf{m}_v	The input velocity components (those of the actuators)
\mathbf{m}_p	The passive velocity components
\mathbf{L}	The coefficients matrix of the velocity equation of the mechanism
\mathbf{f}	A vector of actuator forces in a mechanism

Maps

φ	A scalar-valued map
φ^{-1}	The inverse map of φ
$\boldsymbol{\varphi}$	A vector-valued map
$\boldsymbol{\varphi}^{-1}$	The inverse map of $\boldsymbol{\varphi}$
$\Phi(\mathbf{q})$	The differentiable nonlinear map defining the C-space
$\Phi_{\mathbf{q}}$	The Jacobian matrix of $\Phi(\mathbf{q})$, whose (i, j) entry is $\partial\Phi_i/\partial q_j$
$\Phi_{\mathbf{y}}$	The Jacobian matrix of $\Phi(\mathbf{q})$ with respect to the components of \mathbf{y} , a subvector of \mathbf{q}

$\Phi _{\mathcal{A}}$	The map Φ with domain restricted to the set \mathcal{A}
$\pi_{\mathbf{u}}$	The projection map on the \mathbf{u} space

Other Symbols

$\ \mathbf{x}\ $	The norm of vector \mathbf{x}
$\mathbf{x} \cdot \mathbf{y}$	The dot product of \mathbf{x} and \mathbf{y}
$\mathbf{x} \times \mathbf{y}$	The cross product of \mathbf{x} and \mathbf{y}
$\ker(\mathbf{X})$	The kernel of the linear transformation represented by matrix \mathbf{X}
$\det(\mathbf{X})$	The determinant of the square matrix \mathbf{X}

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