

# Contents

|          |  |    |
|----------|--|----|
| <b>1</b> | <b>Introduction</b>                              | 1  |
| 1.1      | Historical Context                               | 3  |
| 1.2      | Assumptions and Scope                            | 5  |
| 1.3      | Reader's Guide                                   | 8  |
|          | References                                       | 9  |
| <b>2</b> | <b>Singularity Types</b>                         | 13 |
| 2.1      | Forward and Inverse Singularities                | 13 |
| 2.2      | A Geometric Interpretation of Singularities      | 17 |
| 2.2.1    | Singularities Yield Shaky Mechanisms             | 17 |
| 2.2.2    | C-Space, Input, and Output Singularities         | 20 |
| 2.2.3    | Singularities as Silhouette Points               | 22 |
| 2.2.4    | Indeterminacies in the Mechanism Trajectory      | 26 |
| 2.3      | Lower-Level Singularity Types                    | 27 |
| 2.4      | A Simple Mechanism with All Singularities        | 33 |
|          | References                                       | 35 |
| <b>3</b> | <b>Numerical Computation of Singularity Sets</b> | 37 |
| 3.1      | A Suitable Approach                              | 37 |
| 3.2      | Formulating the Equations of the Singularity Set | 39 |
| 3.2.1    | The Assembly Constraints                         | 39 |
| 3.2.2    | The Velocity Equation                            | 42 |
| 3.3      | Isolating the Singularity Set                    | 45 |
| 3.3.1    | Reduction to a Simple Quadratic Form             | 45 |
| 3.3.2    | Initial Bounding Box                             | 46 |
| 3.3.3    | A Branch-and-Prune Method                        | 46 |
| 3.3.4    | Computational Cost and Parallelization           | 50 |
| 3.4      | Visualising the Singularity Sets                 | 50 |

|   |            |
|---|------------|
| 3.5 Case Studies . . . . .  | 52         |
| 3.5.1 The 3-RRR Mechanism . . . . .                               | 52         |
| 3.5.2 The Gough-Stewart Platform . . . . .                        | 61         |
| 3.5.3 A Double-Loop Mechanism . . . . .                           | 66         |
| References . . . . .  | 72         |
| <b>4 Workspace Determination . . . . .</b>                        | <b>75</b>  |
| 4.1 The Need of a General Method . . . . .                        | 75         |
| 4.2 The Workspace and Its Boundaries . . . . .                    | 76         |
| 4.2.1 Jacobian Rank Deficiency Conditions . . . . .               | 77         |
| 4.2.2 Barrier Analysis . . . . .                                  | 79         |
| 4.3 Issues of Continuation Methods . . . . .                      | 81         |
| 4.4 Exploiting the Branch-and-Prune Machinery . . . . .           | 84         |
| 4.4.1 Joint Limit Constraints . . . . .                           | 84         |
| 4.4.2 Equations of the Generalised Singularity Set . . . . .      | 86         |
| 4.4.3 Numerical Solution and Boundary Identification . . . . .    | 89         |
| 4.5 Case Studies . . . . .  | 91         |
| 4.5.1 Multicomponent Workspaces . . . . .                         | 91         |
| 4.5.2 Hidden Barriers . . . . .                                   | 92         |
| 4.5.3 Degenerate Barriers . . . . .                               | 98         |
| 4.5.4 Very Complex Mechanisms . . . . .                           | 107        |
| References . . . . .  | 108        |
| <b>5 Singularity-Free Path Planning . . . . .</b>                 | <b>111</b> |
| 5.1 Related Work . . . . .  | 112        |
| 5.2 Formulating the Singularity-Free C-Space . . . . .            | 113        |
| 5.3 Exploring the Singularity-Free C-Space . . . . .              | 116        |
| 5.3.1 Constructing a Chart . . . . .                              | 116        |
| 5.3.2 Constructing an Atlas . . . . .                             | 117        |
| 5.3.3 Focusing on the Path to the Goal . . . . .                  | 121        |
| 5.3.4 The Planner Algorithm . . . . .                             | 122        |
| 5.4 Case Studies . . . . .  | 124        |
| 5.4.1 A Simple Example . . . . .                                  | 125        |
| 5.4.2 A 3- <u>RRR</u> Parallel Robot . . . . .                    | 126        |
| References . . . . .  | 134        |
| <b>6 Planning with Further Constraints . . . . .</b>              | <b>137</b> |
| 6.1 Wrench-Feasibility Constraints . . . . .                      | 138        |
| 6.2 The Planning Problem . . . . .                                | 142        |
| 6.2.1 A Characterisation of the Wrench-Feasible C-Space . . . . . | 142        |
| 6.2.2 Conversion into Equality Form . . . . .                     | 144        |
| 6.2.3 The Navigation Manifold . . . . .                           | 145        |
| 6.2.4 Addition of Pose Constraints . . . . .                      | 146        |

|                      |   |
|----------------------|---|
| Contents             | xiii  |
| 6.3                  | Proofs of the Properties . . . . . 146                |
| 6.3.1                | Nonsingularity of the Screw Jacobian . . . . . 147    |
| 6.3.2                | Smoothness of the Navigation Manifold . . . . . 147   |
| 6.3.3                | Smoothness of Lower-Dimensional Subsets . . . . . 148 |
| 6.4                  | Case Studies . . . . . 150                            |
| 6.4.1                | Planning in Illustrative Slices . . . . . 150         |
| 6.4.2                | Planning in the IRI Hexacrane . . . . . 155           |
| 6.4.3                | Planning in Rigid-Limbed Hexapods . . . . . 158       |
| 6.4.4                | Problem Sizes and Computation Times . . . . . 159     |
| 6.5                  | Details About the Wrench Ellipsoid . . . . . 160      |
| 6.6                  | Extensions . . . . . 163                              |
| References           | . . . . . 164   |
| 7                    | <b>Conclusions</b> . . . . . 167                      |
| 7.1                  | Summary of Results . . . . . 167                      |
| 7.2                  | Future Research Directions . . . . . 169              |
| References           | . . . . . 172   |
| <b>Author Index</b>  | . . . . . 175   |
| <b>Subject Index</b> | . . . . . 179   |