A Reconfigurable Asymmetric 3-U$\underline{\text{P}}$U Parallel Robot

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***Abstract*—Parallel robots with three U$\underline{\text{P}}$U legs have received a lot of attention due to the possibility of assembling these legs so that the robot performs either a pure translational or a pure rotational motion. Nevertheless, some arrangements, despite their theoretical interest, are of doubtful practical utility due to their sensitivity to errors and the presence in their workspaces of mixed-modes that involve both translations and rotations. The introduction of some sort of asymmetry has been revealed of relevance to come up with more robust designs. In this context, we present an asymmetric 3-U$\underline{\text{P}}$U robot, that can be reconfigured to work either as a translational or as a rotational robot by simply flipping upside down its moving platform.**

I. Introduction

A general-purpose parallel robot has six degrees of freedom (DOF) to manipulate an object freely in three-dimensional space. A parallel robot with limited-DOF has fewer than six DOF. Among all limited-DOF parallel robots, the three-DOF family has received significant attention from researchers. Some of them provide the platform with a pure translational motion [1]–[5] and are of interest in automated assembly, especially for pick-and-place operations, and in machine tools as alternative structure to the serial positioning devices. Others provide the moving platform with a pure relative rotation about a fixed point [6]–[9] and are used as wrists of manipulators or, in general, as pointing devices.

The investment cost to purchase a parallel robot for a particular task could be worth if there is the possibility to reconfigure it for another task. In this sense, it is interesting to observe that the 3-U$\underline{\text{P}}$U architecture, where the prismatic joint is underlined to denote that it is actuated, can be configured so that the resulting robot provides the platform with either translational or rotational motions. This paper focuses on the possibility of designing a robot with such an architecture that could be reconfigured to work in either of these two modes.

A static reconfiguration denotes a manual rebuilding of a robot which might lead to a robot with new kinematic characteristics and a new workspace [10]–[12]. In this paper, we present a 3-U$\underline{\text{P}}$U robot that can be statically reconfigured to work either as a translational or a rotational robot by simply flipping upside down its moving platform. Since this kind of robot consists of universal and prismatic joints only, it is very attractive from the manufacturing point of view. This operation can also be simplified by introducing rT joints as explained in [13].

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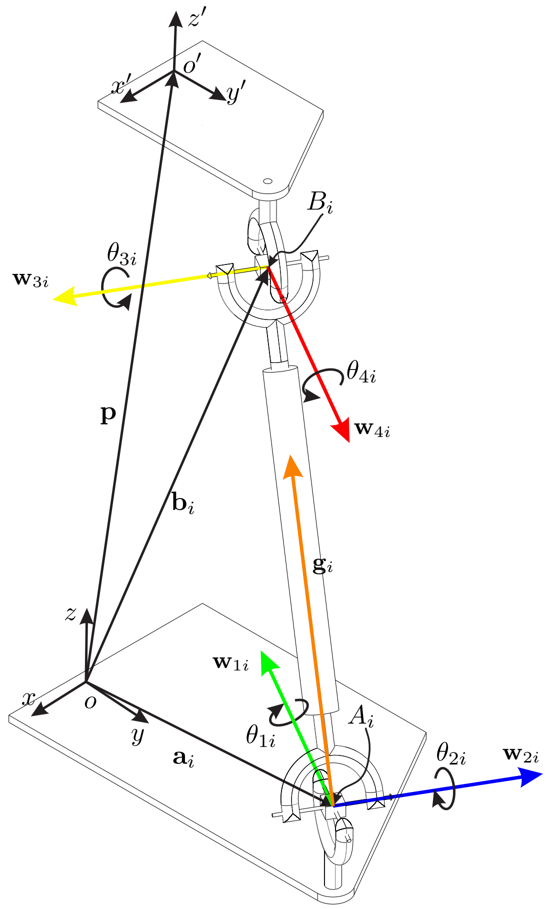


Fig. 1: Notation associated with the i^{th} leg of a general 3-U$\underline{\text{P}}$U robot.

This paper is organized as follows. In the next section, some basic notions are reminded and the notation used throughout the paper is presented. In Section III, the family of 3-U$\underline{\text{P}}$U robots is reviewed. The instantaneous kinematics of this family of robots is analyzed in Section IV, first for the general case and then particularized to the translational and rotational cases. In this analysis, the emphasis is put on singularities, more particularly on constraint singularities. In Section V, the forward kinematics of 3-U$\underline{\text{P}}$U robots with different arrangements for their universal joints is numerically solved using a branch-and-prune method. This provides an interesting insight into the effect of rearranging universal joints in 3-U$\underline{\text{P}}$U robots that permits to speculate about the ultimate reason for the better behavior of asymmetric designs. In Section VI, the reconfigurable robot is presented. Finally, Section VII provides the conclusions and the prospects for further research.

II. NO(TA)TIONS

A 3-UPU parallel robot consists of a fixed base and moving platform connected by three serial chains, or legs, each of them having a universal-prismatic-universal joint arranged in sequence. Fig. 1 shows one of these legs. The universal joints are passive. Only the prismatic joint are actuated.

With reference to Fig. 1, $\mathbf{w}_{1i}$ and $\mathbf{w}_{2i}$ are two mutually orthogonal unit vectors defined by the revolute axes of the universal joint centered at A_i. Likewise, $\mathbf{w}_{3i}$ and $\mathbf{w}_{4i}$ are the two mutually orthogonal unit vectors of the axes of the two revolute pairs constituting the universal joint centered at B_i. $\mathbf{a}_i$ and $\mathbf{b}_i$ are the position vectors of A_i and B_i, respectively, in a generic Cartesian reference fixed to the base, whereas $\mathbf{p}$ is the position vector of the origin, O', of the reference frame associated with the moving platform. θ_{ji}, $j = 1, \ldots, 4$, is a joint variable denoting a rotation angle around the joint-axis defined by $\mathbf{w}_{ji}$, $j = 1, \ldots, 4$, using the right-hand rule. The length of the i^{th} leg is equal to $\|\mathbf{b}_i - \mathbf{a}_i\|$, and it will be denoted l_i. Moreover, we define

$$\begin{aligned}
\mathbf{g}_i &= (\mathbf{b}_i - \mathbf{a}_i)/l_i,\\
\mathbf{h}_i &= \mathbf{w}_{3i} \times \mathbf{w}_{4i},\\
\mathbf{r}_i &= \mathbf{w}_{1i} \times \mathbf{w}_{2i},\\
\mathbf{s}_i &= \mathbf{h}_i \times \mathbf{r}_i - [\mathbf{g}_i \cdot (\mathbf{h}_i \times \mathbf{r}_i)]\mathbf{g}_i.
\end{aligned}$$

Observe that $\mathbf{s}_i$ is just the component of $\mathbf{h}_i \times \mathbf{r}_i$ perpendicular to $\mathbf{g}_i$.

III. THE REMARKABLE FAMILY OF 3-UPU ROBOTS

In 1996, Tsai proposed a 3-UPU parallel robot with three translational degrees of freedom in [4]. The axes of the universal joints of this particular robot, henceforth called Tsai manipulator, are arranged as follows (see Fig. 2a):

(a1) the axes of the three revolute joints embedded in the base/platform (shown in green/red Fig. 2a) form a triangle.
(a2) the two triangles are similar.
(a3) for each leg, the axes of the intermediate revolute pairs are parallel to each other and perpendicular to the axis of the prismatic pair.

The sensitivity of this robot to geometric parameter variations and manufacturing tolerances was analyzed in [14], where it was shown that small torsions in the legs generate large deviations in the position of the moving platform. Therefore, applications of the Tsai's robot are limited by this pseudo-singular behaviour. The sensitivity of this robot to other manufacturing errors is studied in [15], [16]. Di Gregorio studied its singularities in [17]. The same analysis was later preformed by Joshi and Tsai in [18] using screw calculus.

In 1998, Di Gregorio and Parenti-Castelli [19] studied the more general 3-RRPRR architecture and, from this analysis, they arrived at the important conclusion that the geometric conditions for a 3-UPU robot to have three translational DOFs can algebraically be expressed as:

(b1) $|\mathbf{w}_{1,1} \cdot \mathbf{w}_{1,2}| = |\mathbf{w}_{4,1} \cdot \mathbf{w}_{4,2}|$.
(b2) $|\mathbf{w}_{1,1} \cdot \mathbf{w}_{1,3}| = |\mathbf{w}_{4,1} \cdot \mathbf{w}_{4,3}|$.
(b3) $|\mathbf{w}_{1,2} \cdot \mathbf{w}_{1,3}| = |\mathbf{w}_{4,2} \cdot \mathbf{w}_{4,3}|$.
(b4) $\mathbf{w}_{2,i} = \pm\mathbf{w}_{3,i},\ i = 1, 2, 3$.
(b5) $\mathbf{w}_{1,i} = \pm\mathbf{w}_{4,i},\ i = 1, 2, 3$.

Another important conclusion in [19] is that the pure translation of the moving platform does not only depend on the leg topology, but also on specific mounting conditions. In this sense, while the above conditions (b1), (b2), (b3), and (b4) are *manufacturing conditions*, (b5) is a *mounting condition*.

As a result of this analysis, Tsai's robot can be seen as a particular case of a large family of 3-UPU translational robots. Another particular translational 3-UPU robot results if all the revolute-pair axes at the leg endings converge, while remaining coplanar, toward a single point and every leg has the two intermediate revolute-pair axes parallel to each other and perpendicular to the straight line through the universal joint centers (see Fig. 2b). This particular 3-UPU robot, which we will call *central robot*, was studied in [20] and [21]. In [20], Walter *et al.* showed that the translational motion of this robot is rather doubtful due to the presence of at least 16 different assembly modes including the pure translational one. Thus, it is important to highlight that for a given set of leg lengths a translational 3-UPU manipulator have, in general, different assembly modes and, only if the platform is properly assembled, it can have a pure translational motion.

In 2006, Lu and Hu proposed a family of asymmetrical 3-UPU robots [22]. This family of robots included a translational design (see Fig. 2c). Lu and Hu argued that, contrarily to what happens with the above two symmetrical designs, condition (b5) is easier to satisfy due to the peculiar joint disposal of their design, thus concluding that it provides a significant advantage with respect to Tsai's robot. In our opinion, as we explain in Section V, this is not the main reason for the better behavior of their design.

At this point, we have three 3-UPU robots with identical pure translational DOFs and an identical actuator arrangement. Nevertheless, they necessarily differ in terms of singularity configuration and stiffness due to the different arrangement of their universal joints. For example, as it is proved in [23], Tsai's robot has a singularity plane and a singularity cylindrical surface, while Lu-Hu's robot has two singularity planes.

Observe that condition (b5) means that the axes defined by the first and the fourth revolute axes in each leg should be parallel, and, if this condition is satisfied, (b1), (b2) and (b3) are also satisfied. Thus, the geometric conditions for a 3-UPU robot to have three translational DOFs can be simply expressed as the conjunction of (b4) and (b5). There is no need that the axes from different legs intersect in finite points. Actually, these unnecessary extra geometric constraints seem to be the ultimate reason for the poor behavior of Tsai's and the central 3-UPU robots.

In 2000, Karouia and Hervé showed that a 3-UPU robot, under some mounting and manufacturing conditions, can provide its moving platform with spherical motions [24]. These conditions are as follows (see Fig. 2d):

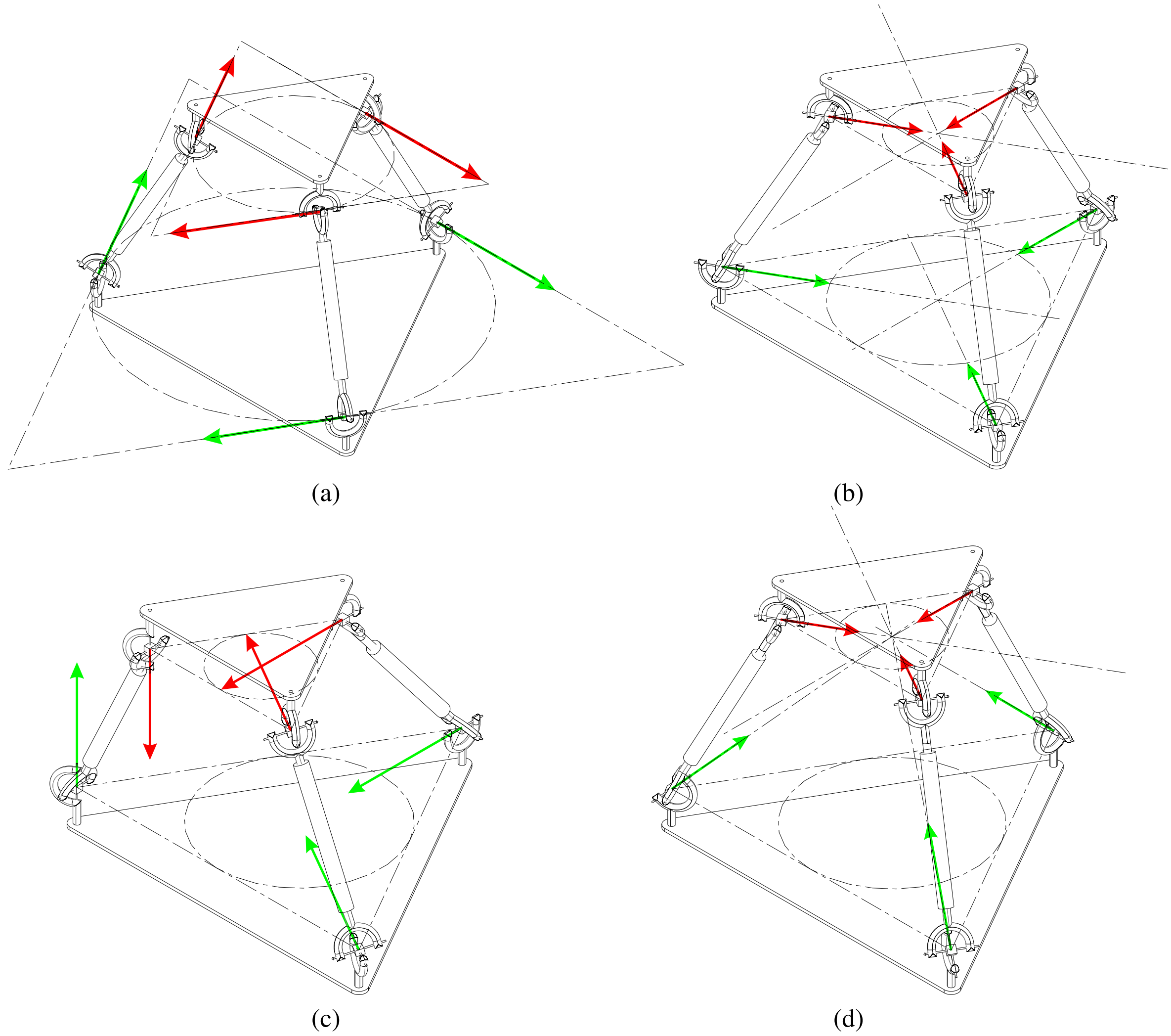


Fig. 2: Four 3-UPU robots: (a) Tsai's robot, (b) central robot, (c) Lu-Hu's robot, and (d) Hervé's robot.

(c1) The three revolute pairs axes fixed to the platform (base) must converge at a point fixed in the platform (base).
(c2) In each leg, the intermediate revolute pair axes must be parallel to each other and perpendicular to the leg axis which is the line through the universal joints' centers.
(c3) The point located at the intersection of the platform's revolute pair axes must coincide with the point located at the intersection of the base's revolute pair axes.

In this case, (c1) and (c2) are manufacturing conditions, and (c3) is a mounting condition. Different aspects of the kinematics of this robot were studied in [25]–[27].

IV. Instantaneous kinematics and singularities of 3-UPU robots

The derivation of the input-output velocity relationships for 6-DOF spatial parallel manipulators, in which the connectivity of each serial chain limb is equal to the mobility of the end effector, leads to a satisfactory formulations of their Jacobian matrices. Nevertheless, this approach is not valid, in general, for parallel manipulators with less than 6-DOF. In the case of 3-UPU robots, this approach leads to a 3×3 Jacobian matrix whose analysis cannot predict all possible singularities, as detailed in [28].

Then, it can be proved for a general 3-UPU robot that:

$$\begin{pmatrix} \mathbf{1}_{3\times3} \\ \mathbf{0}_{3\times3} \end{pmatrix} \dot{\mathbf{l}} = \begin{pmatrix} \mathbf{G}_{3\times3} & \mathbf{K}_{3\times3} \\ \mathbf{S}_{3\times3} & \mathbf{J}_{3\times3} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{p}} \\ \boldsymbol{\omega} \end{pmatrix} \tag{1}$$

where $\mathbf{1}_{3\times3}$ and $\mathbf{0}_{3\times3}$ are the 3×3 identity and zero matrix, respectively, $\dot{\mathbf{l}} = (\dot{l}_1, \dot{l}_2, \dot{l}_3)$ is the vector of velocities in the actuators, $\boldsymbol{\omega}$ is the angular velocity of the moving platform, and

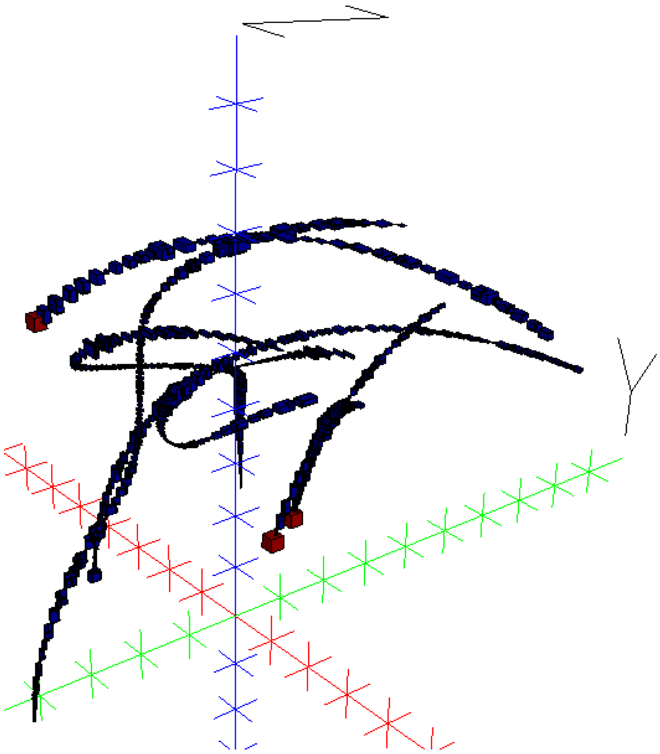
$$\mathbf{G}^T[i,:] = \mathbf{g}_i \tag{2}$$

$$\mathbf{K}^T[i,:] = (\mathbf{b}_i - \mathbf{p}) \times \mathbf{g}_i \tag{3}$$

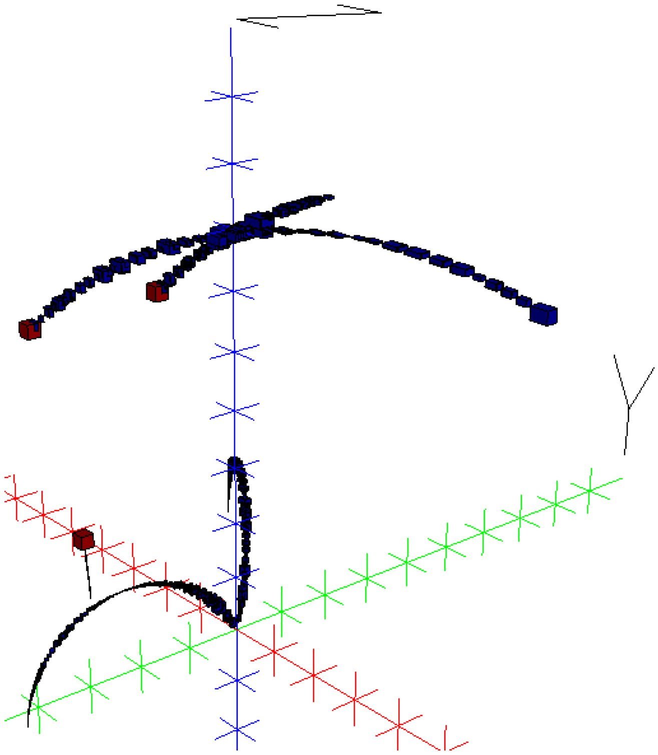
$$\mathbf{S}^T[i,:] = \mathbf{s}_i \tag{4}$$

$$\mathbf{J}^T[i,:] = (\mathbf{b}_i - \mathbf{p}) \times \mathbf{s}_i - l_i(\mathbf{r}_i \cdot \mathbf{g}_i)\mathbf{h}_i \tag{5}$$

where $\mathbf{A}[i,:]$ denotes the i-th row of $\mathbf{A}$.



(a)



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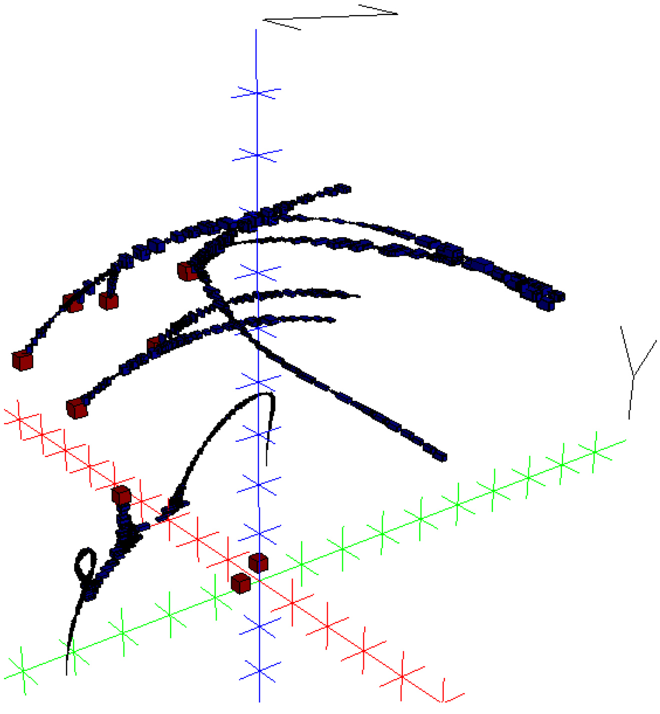
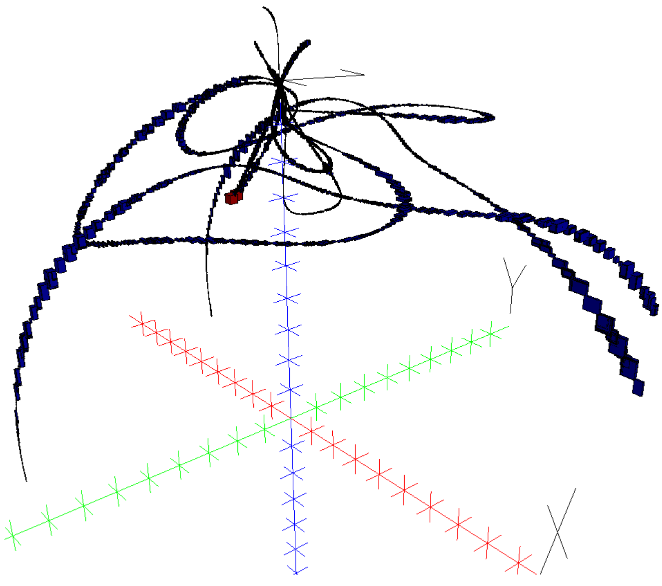


(c)



(d)

Fig. 3: Trajectories followed by the center of the moving platform, as one leg is extended, for all assembly modes of: (a) Tsai's robot, (b) central robot, (c) Lu-Hu's robot, and (d) Hervé's robot. The large boxes in dark red correspond to the initial configurations. Intersections between trajectories do not necessarily correspond to singularities because in these plots the orientation of the moving platform is not considered.

The derivation of (1) is a bit lengthy and for this reason it is not included here, but the interested reader can follow the steps detailed in [29].

If we compare equation (1) with its counterpart in [18], we realize that they do not coincide. The reason is that, while in [18] the analysis is particularized to the case in which the robot is of translational type, equation (1) is general for any 3-U$\underline{\text{P}}$U robot.

Equation (1) can be rewritten as follows:

$$\dot{\mathbf{l}} = \underbrace{\begin{pmatrix}\mathbf{G}_{3\times 3} & \mathbf{K}_{3\times 3}\end{pmatrix}}_{\mathbf{J}_x}\begin{pmatrix}\dot{\mathbf{p}}\\ \boldsymbol{\omega}\end{pmatrix}, \tag{6}$$

$$\mathbf{0}_{3\times 1} = \underbrace{\begin{pmatrix}\mathbf{S}_{3\times 3} & \mathbf{J}_{3\times 3}\end{pmatrix}}_{\mathbf{J}_c}\begin{pmatrix}\dot{\mathbf{p}}\\ \boldsymbol{\omega}\end{pmatrix}. \tag{7}$$

Now, equation (6) relates the twist of the moving platform with the linear velocities of the prismatic actuators. If $\mathbf{J}_x$ —sometimes called *Jacobian of actuations*— is rank-deficient for all possible locations of the moving platform, the robot is said to be *architecturally singular*. Observe how $\mathbf{J}_x$ is independent on how the universal joints are arranged; it only depends on the three leg lines.

Equation (7) models the internal constraints. If $\mathbf{J}_c$ —sometimes called *Jacobian of constraints*— is rank-deficient, there are non-null twist for the moving platform that satisfy (7). When this happens, the robot is said to be in a *constraint singularity*.

The Jacobian of constraints depends on how the universal joints are arranged. In a translational 3-U$\underline{\text{P}}$U robot, $\mathbf{w}_{1,i} = \mathbf{w}_{4,i}$ and $\mathbf{w}_{2,i} = \mathbf{w}_{3,i}$. In this case, $\mathbf{h}_i$ and $\mathbf{r}_i$ are aligned and, as a consequence, $\mathbf{s}_i = 0$. Therefore, it can be checked that

$$\mathbf{J}_c^{\text{translational}} = \begin{pmatrix}\mathbf{0}_{1\times 3} & (\mathbf{b}_1-\mathbf{a}_1)^T\\ \mathbf{0}_{1\times 3} & (\mathbf{b}_2-\mathbf{a}_2)^T\\ \mathbf{0}_{1\times 3} & (\mathbf{b}_3-\mathbf{a}_3)^T\end{pmatrix}, \tag{8}$$

which concurs with the result reported in [18]. Hence, if $\mathbf{g}_1$, $\mathbf{g}_2$ and $\mathbf{g}_3$ are linearly independent, the only feasible solution to (7) is $\boldsymbol{\omega} = (0\,0\,0)^T$, as expected.

In a rotational 3-U$\underline{\text{P}}$U we can make coincident O' with the center of rotation without loss of generality. Then, $(\mathbf{b}_i-\mathbf{p})$ is aligned with $\mathbf{w}_{4i}$ and $(\mathbf{a}_i-\mathbf{p})$ is aligned with $\mathbf{w}_{1i}$. Moreover, $\mathbf{h}_i$, $\mathbf{r}_i$, and $\mathbf{g}_i$ lie on the plane defined by O', A_i and B_i. As a consequence, the triple product $\mathbf{g}_i\cdot(\mathbf{h}_i\times\mathbf{r}_i)$ vanishes, and $\mathbf{J}_c$ can be simplified as follows:

$$\mathbf{J}_c^{\text{rotational}} = \begin{pmatrix}(\mathbf{h}_1\times\mathbf{r}_1)^T & \mathbf{0}_{1\times 3}\\ (\mathbf{h}_2\times\mathbf{r}_2)^T & \mathbf{0}_{1\times 3}\\ (\mathbf{h}_3\times\mathbf{r}_3)^T & \mathbf{0}_{1\times 3}\end{pmatrix}. \tag{9}$$

Hence, if $\mathbf{h}_1\times\mathbf{r}_1$, $\mathbf{h}_2\times\mathbf{r}_2$ and $\mathbf{h}_3\times\mathbf{r}_3$ are linearly independent, the only feasible solution to (7) is $\dot{\mathbf{p}} = (0\,0\,0)^T$, as expected.

When the three planes defined by O', A_i and B_i, $i = 1,2,3$, intersect in a single line, vectors $\mathbf{h}_1\times\mathbf{r}_1$, $\mathbf{h}_2\times\mathbf{r}_2$ and $\mathbf{h}_3\times\mathbf{r}_3$ are coplanar and, as a consequence, linearly dependent. In this case, the rotational robot is in a constraint singularity. Within this singularity it can also happen that the three leg lines intersect in a point. If this happens, not only $\mathbf{J}_c$ is rank-deficient but also $\mathbf{J}_x$ is. In this pathological situation, the rank of the full Jacobian is, at most, 4; that is, the robot locally gains two DOF.

V. The Forward Kinematics of 3-U$\underline{\text{P}}$U Robots Solved Numerically

The existence of important differences between the four robots represented in Fig. 2 becomes apparent by assembling their models using SolidWorks. Indeed, if we implement the different parts of the robot and assemble them by introducing the corresponding geometric constraints, we can drag the moving platform to observe how the extensible prismatic actuators evolve. While in the case of the Lu-Hu's robot the moving platform can only perform translational motions if properly assembled in the initial configuration, the Tsai's robot, starting from the same configuration, sometimes falls in a mixed-mode where the platform performs a combined translational-rotational motion, and the central robot simply leads to numerical errors that prevents any motion simulation. This experimental observation is consistent with the theoretical analyses performed to date that lead to think that the best option for a practical 3-U$\underline{\text{P}}$U robot is Lu-Hu's robot.

For a deeper analysis, we have numerically solved the forward kinematics of the four models in Fig. 2 using a branch-and-prune technique [30], [31]. First, we have computed all the assembly modes for the case in which two legs have the same lengths and the remaining one is shorter. The exact values are now irrelevant as we are just interested on a qualitative analysis of the assembly modes. We can represent the centers of the moving platform for each assembly mode in the robot workspace. They appear as dark red boxes in Fig. 3.

Regarding the translational robots, while the number of real assembly modes is three, both for Tsai's and the central robot, this number is nine for Lu-Hu's robot. Only one configuration in each set corresponds to the case in which the base and the moving platform are parallel and hence satisfies the mounting condition for the corresponding robot to work as a translational one. Now, if we progressively extend the shorter leg, we can see how these assembly modes evolve. The center of the moving platform follows different trajectories depending on the initial assembly mode. Only one of these trajectories keeps the moving platform parallel to the base. These trajectories are shown as sequences of blue boxes in Fig. 3. Observe that they intersect each other. Nevertheless, these intersections do not necessarily correspond to intersections in the configuration space of the moving platform because we are not considering the orientation of the moving platform in this representation. Actually, these trajectories can be seen as the projection of the trajectories in the configuration space ($\mathbb{R}^3 \times SO(3)$) onto the workspace ($\mathbb{R}^3$). While intersections in the configuration space correspond to singularities, the intersections in Fig. 3 are not necessarily harmful. Nevertheless, clusters of boxes around an intersection indicates that they indeed correspond to an intersection in the configuration space and hence to an

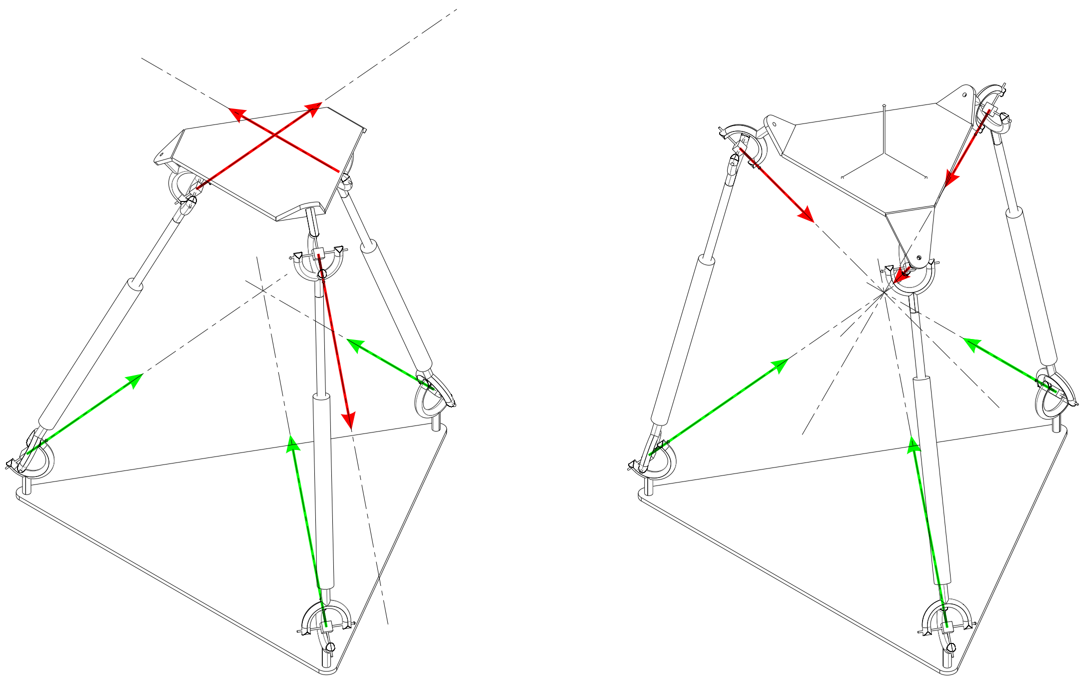


Fig. 4: The proposed reconfigurable 3-U$\underline{P}$U robot assembled as translational robot (left), or as rotational robot (right). They only differ in the way the moving platform is assembled.

actual singularity. These clusters are clearly present in Tsai's and in the central robot.

Regarding the Hervé's rotational robot, we have two assembly modes that follow a trajectories that bifurcates when all three legs have the same leg lengths. This bifurcation point corresponds to the constraint singularity already identified for this robot at the end of Section IV.

The above results allow us to speculate on why the asymmetric design works better for translational robots. An arbitrary 3-U$\underline{P}$U robot has, in general, multiple real assembly modes. As we introduce geometric constraints in the arrangement of their universal joint —such as coplanarities, axes intersections, etc.— the number of assembly modes is reduced because some of them coalesce. Nevertheless, since in practice none of the introduced geometric constraints can be exactly satisfied, we have clusters of assembly modes in which the parallel robot becomes shaky. To avoid this situation, the best solution is to minimize the number of extra geometric constraints to be satisfied.

VI. The proposed reconfigurable 3-U$\underline{P}$U robot

The proposed reconfigurable robot is easily understood by observing Fig. 4. On the left, we have a 3-U$\underline{P}$U robot that satisfies conditions (b1)-(b5), and, as a consequence, it is a translational robot. On the right, the robot satisfies conditions (c1)-(c3), and, as a consequence, it is a rotational robot. The interesting thing is that both robots only differ in the way the moving platform is assembled.

In the case in which the robot is assembled to work as a translational robot, we can repeat an analysis similar to that described in the previous section. The result is presented in Fig. 5. Obviously, the represented trajectories not only depend on the chosen dimensions for the different elements but also, due to the asymmetry of the robot, on which leg is extended. An optimization is required to dimension the elements leading to the largest possible workspace free from singularities. Nevertheless, this is left as a point for further research.

In the case in which the robot is assembled to work as a rotational robot, the result is similar to that presented in Section IV for Hervé's robot.

VII. Conclusion

We have presented a 3-U$\underline{P}$U robot that can be reconfigured to work either as a translational or a rotational robot by flipping upside down its moving platform. Although this reconfiguration is designed to be performed off-line, it can be potentially implemented dynamically by exchanging the role of the fixed base and the moving platform and introducing a binary actuator that turns over three re-orienting elements. This is certainly a possibility that deserves further attention.

Our current efforts are addressed to the dimensioning of the robot elements to optimize the volume of its workspace. This is probably the most important point to translate the presented conceptual design into a system definition that is suitable for detailed design and subsequent manufacture.

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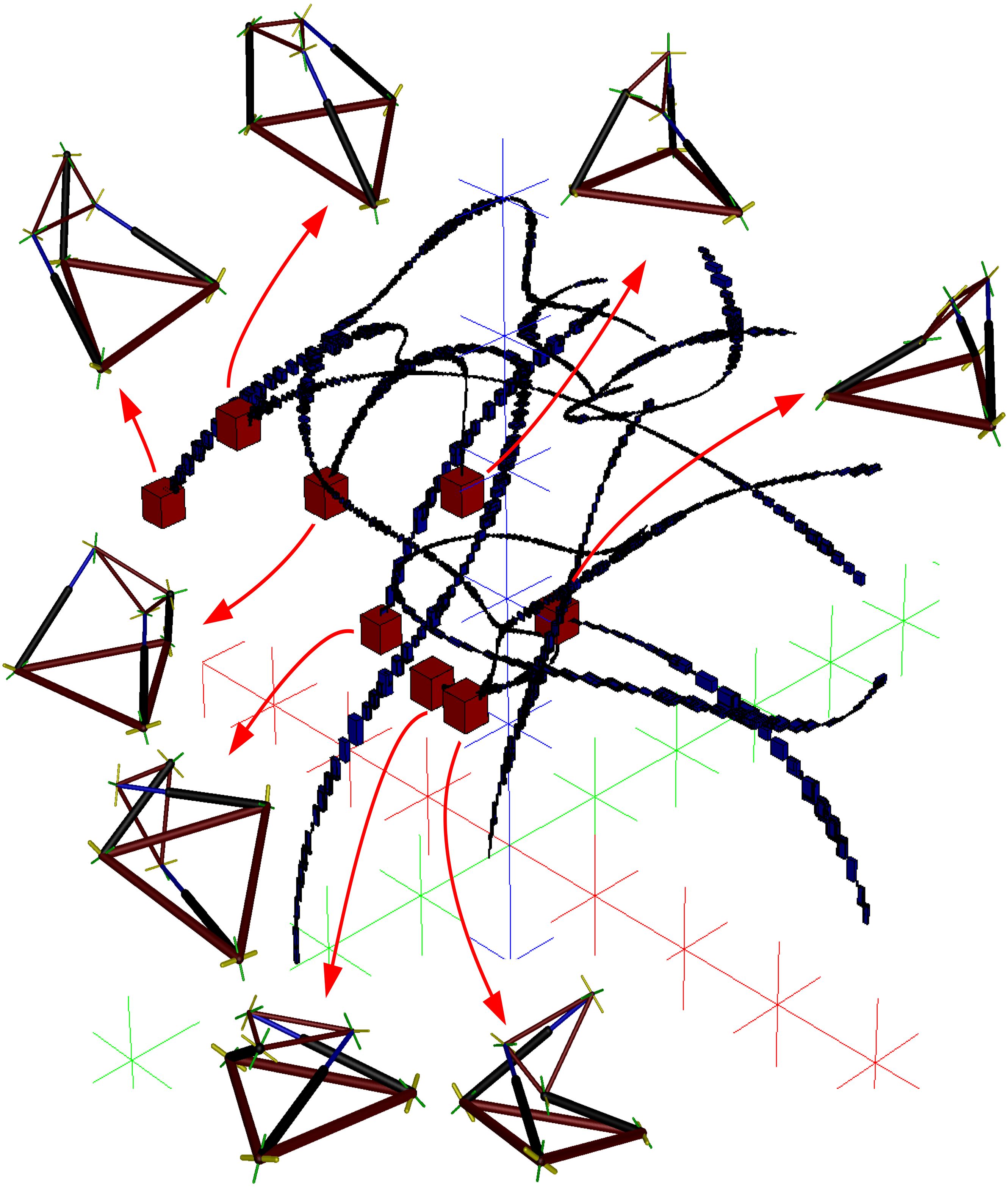

Fig. 5: Trajectories followed by the center of the moving platform of the proposed reconfigurable robot when it is assembled to work in translational mode and one leg is progressively extended. We have represented the robot next to the dark-red boxes, that correspond to the initial configurations for each assembly mode. Only in one of these configurations, the base and the moving platform are coplanar.

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