# Solving Multi-Loop Linkages by Iterating 2D Clippings

## The Cuik Algorithm

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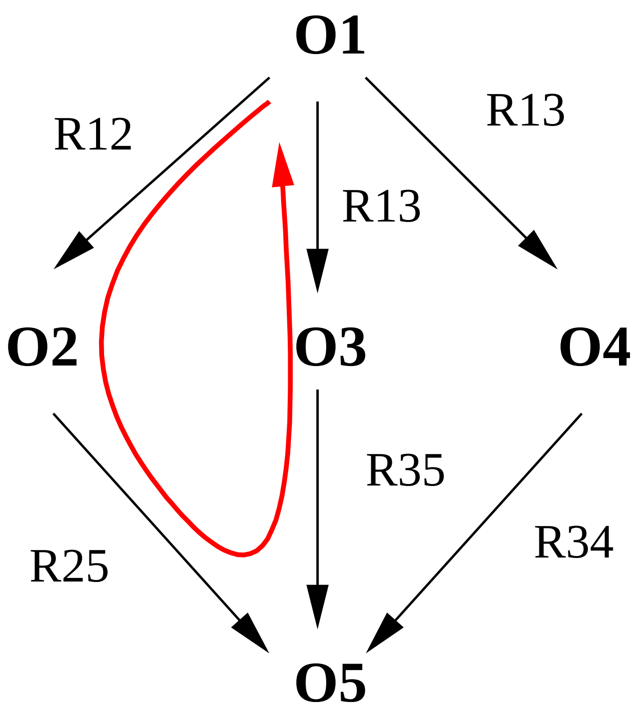
# Problem Definition




$R_{i,j} = t_1 \, t_2 \, \ldots$

- $t_i = Rot(ct/var : [\alpha_l, \alpha_u])$
- $t_i = Trans(ct/var : [l_l, l_u])$

## Equations

- **Loops in the graph:**
  - $R12 \; R25 \; R35^{-1} \; R13^{-1} = Id$
- **Solution:** Assignment of values to variables.
- **Obtain a basis of loops.**
- **One matrix equations per loop in the basis.**
- **12 scalar equations per matrix equation.**

# Different Approaches

**To solve the system of equations**

- ***Algebraic Geometry***

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- ***Interval-based Methods***
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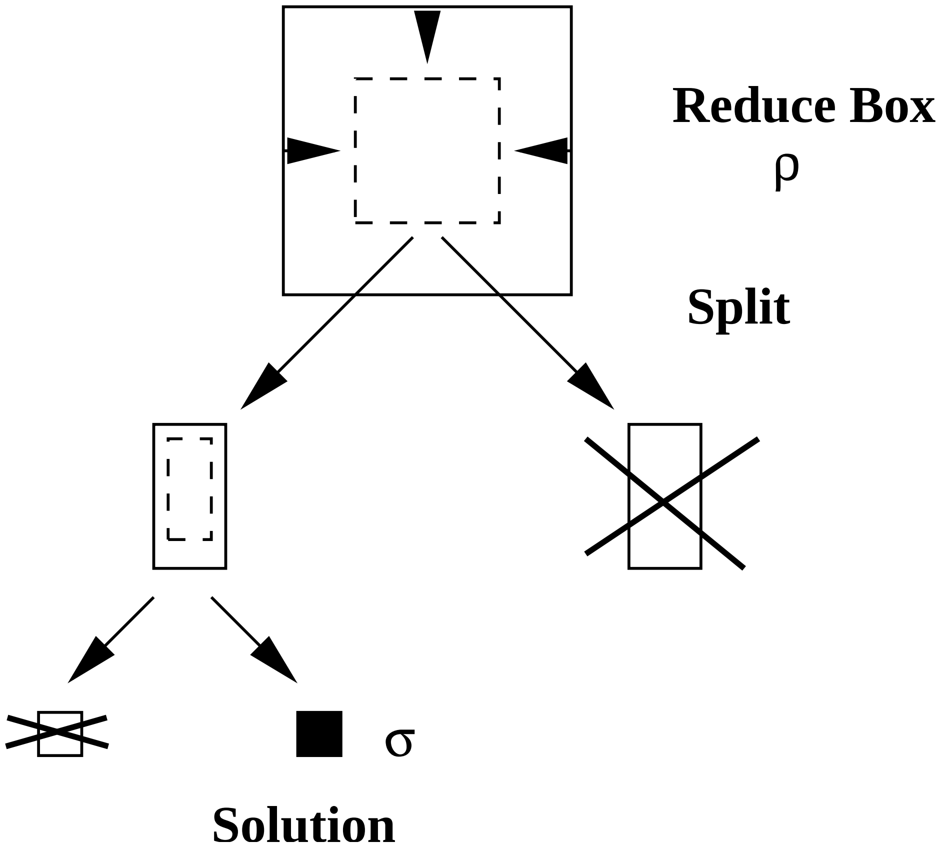
  - Subdivision
  - 2-D Clippings (CUIK)

# Interval-Based Methods



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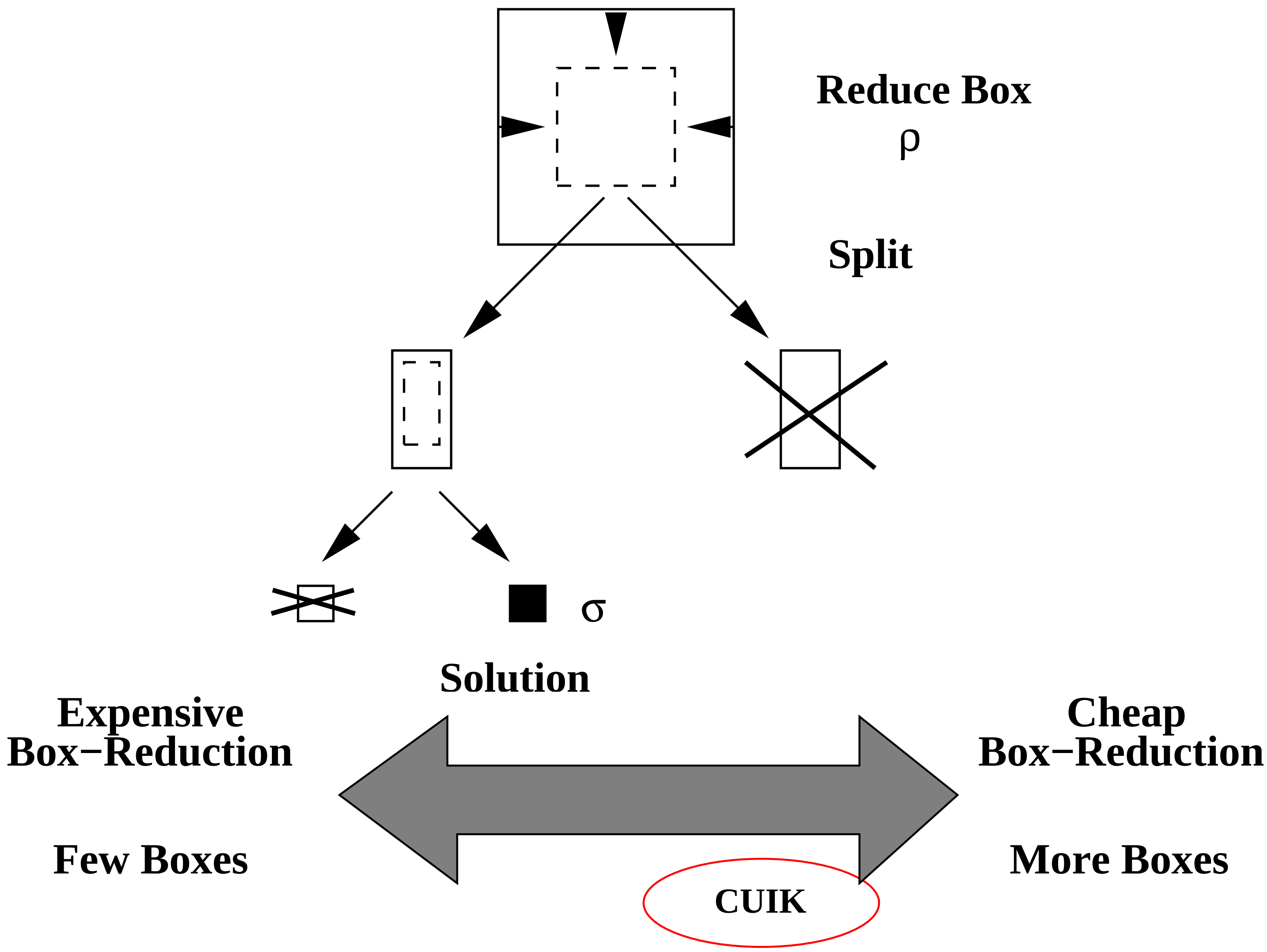

# Simple Example

**A trivial mechanism with 1 rotational dof**

$$Rz(\alpha) = M$$

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## In homogeneous coordinates

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,1} & m_{2,3} & m_{2,4} \\ m_{3,1} & m_{3,1} & m_{2,3} & m_{3,4} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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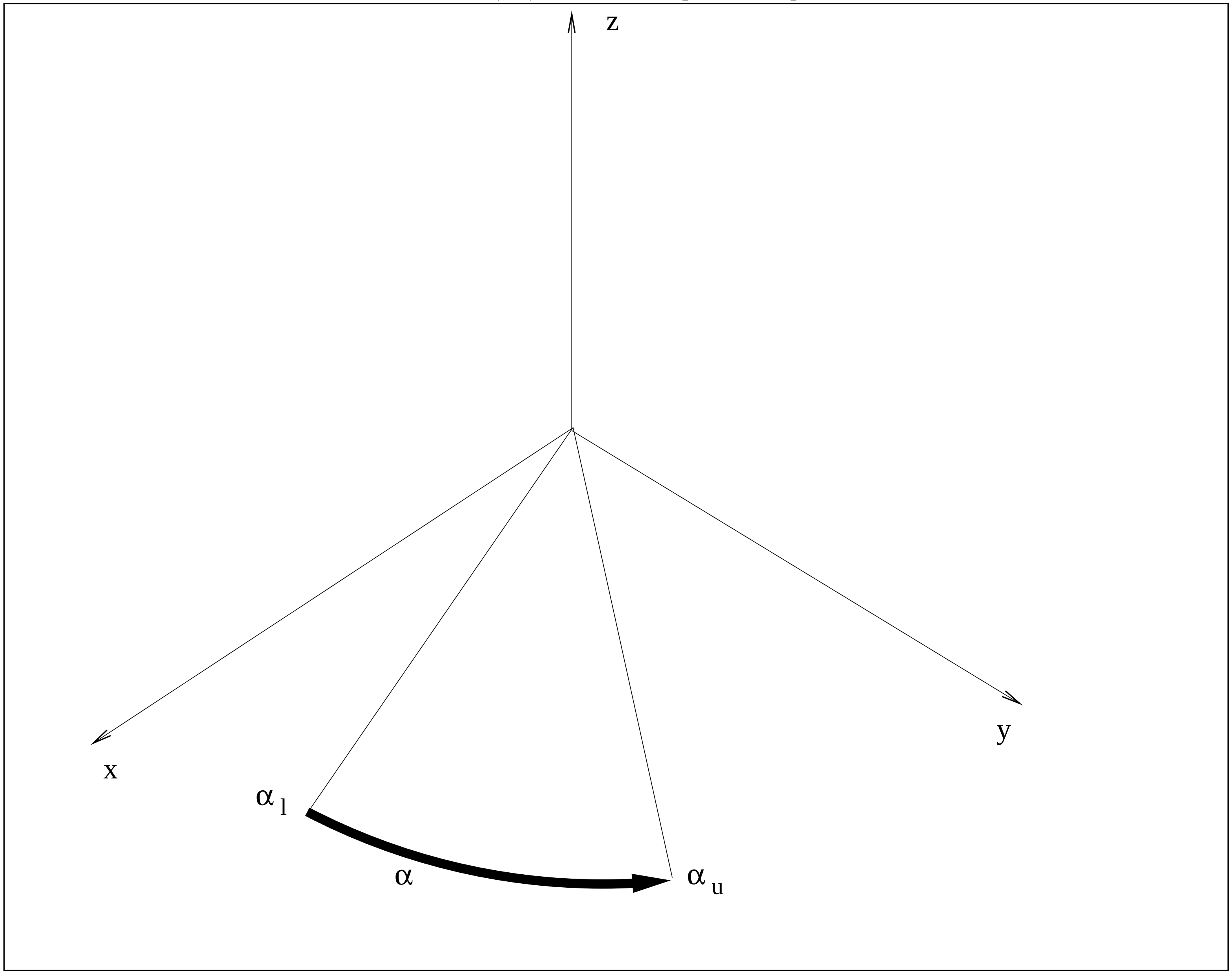
$$\dots$$

## Variable substitution: $x = \cos(\alpha)$, $y = \sin(\alpha)$

$$x\, m_{1,1} - y\, m_{2,1} = 1$$
$$y\, m_{1,2} + x\, m_{2,2} = 0$$
$$\dots$$

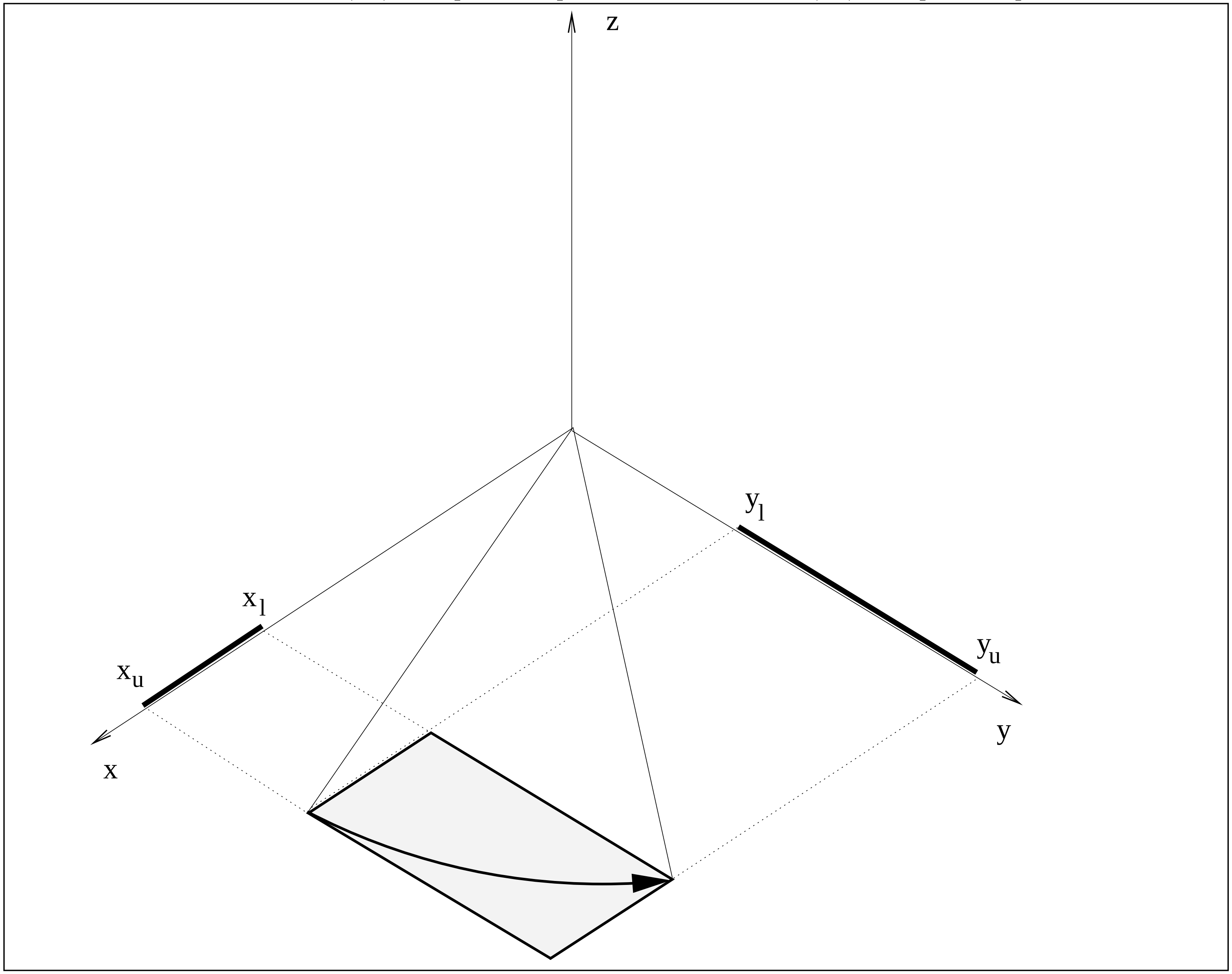
# Box Reduction (I)

$$Rz(\alpha)\ ,\ \ \alpha \in [\alpha_l, \alpha_u]$$



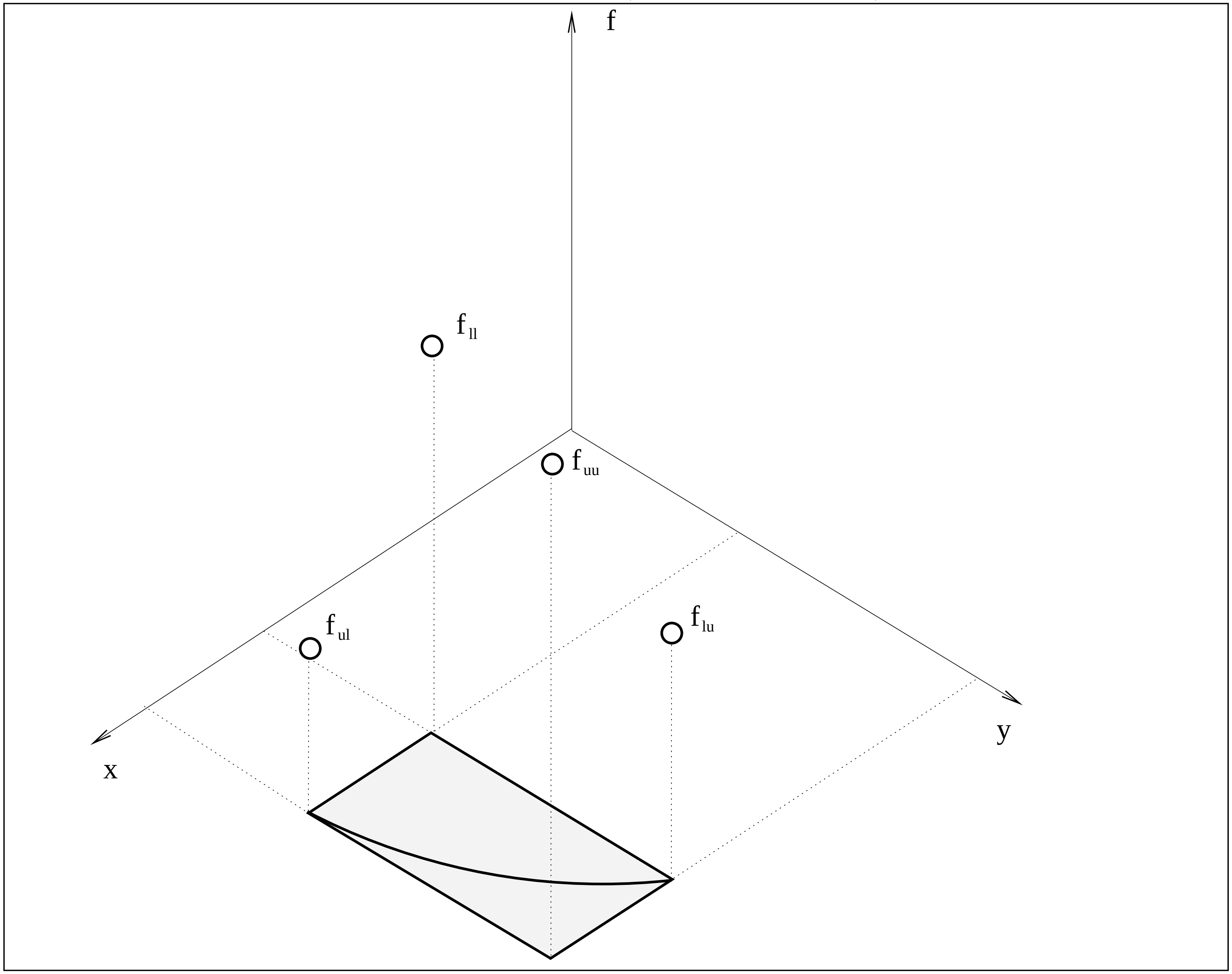
# Box Reduction (II)

$x = \cos(\alpha) \in [x_l, x_u]$ **and** $y = \sin(\alpha) \in [y_l, y_u]$



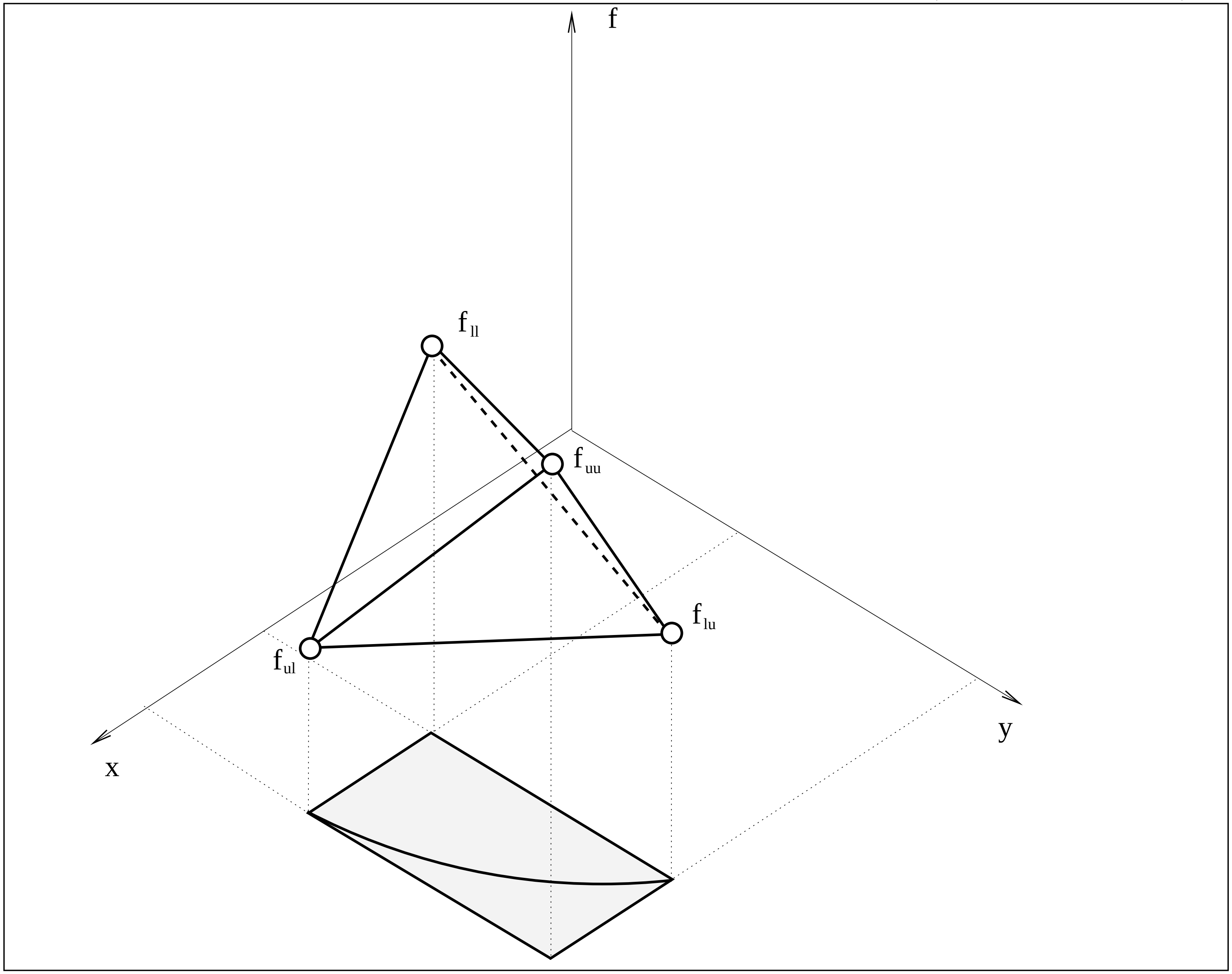
# Box Reduction (III)

$$f_0 = \cos(\alpha)\, m_{1,1} + \sin(\alpha)\, m_{2,1}$$



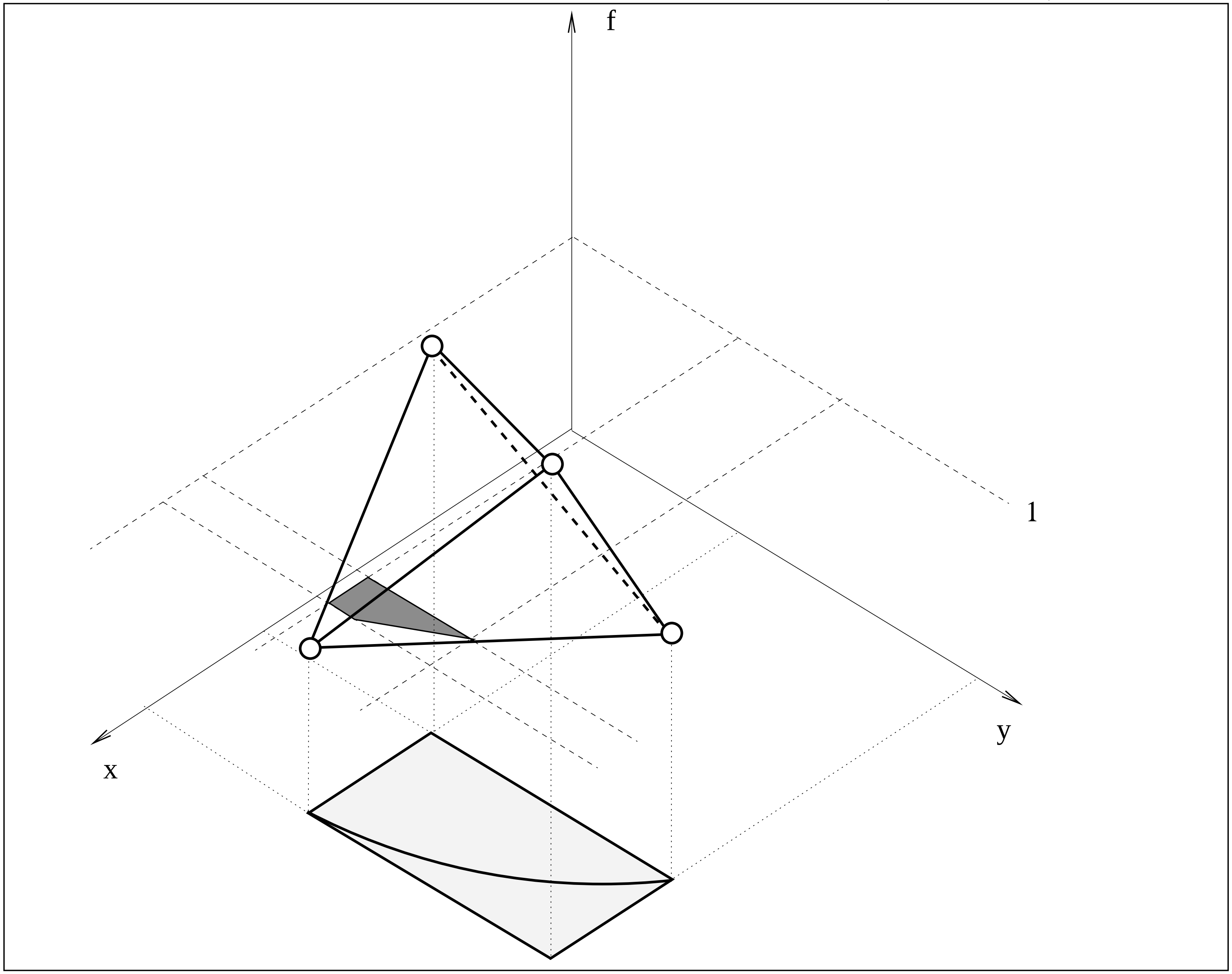
# Box Reduction (IV)

**The convex-hull property on** $f_0 = \cos(\alpha)\, m_{1,1} + \sin(\alpha)\, m_{2,1}$**.**



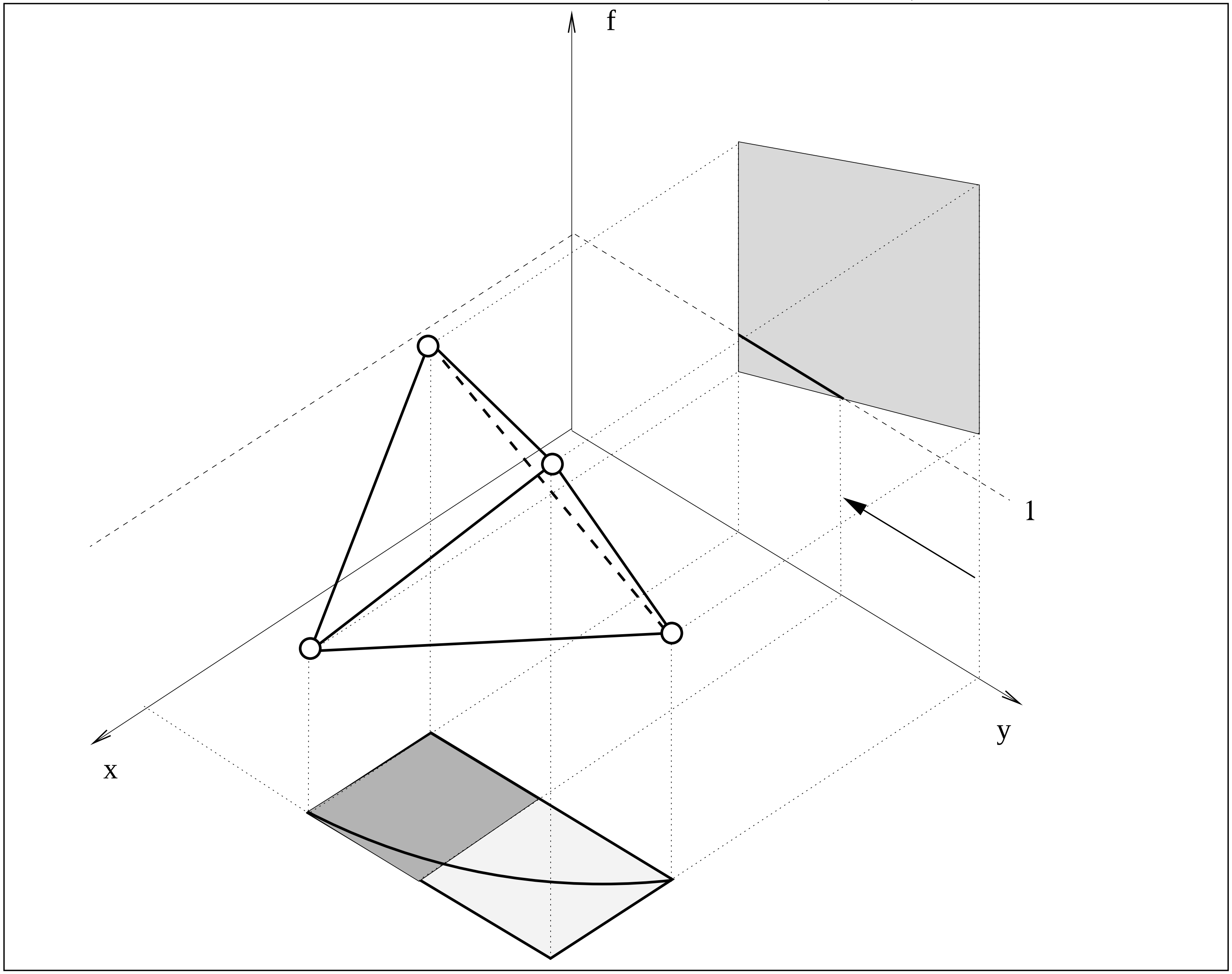
# Box Reduction (V)

**Intersection with the plane $f_0 = 1$ ( $x\, m_{1,1} + y\, m_{1,1} \ = \ 1$).**



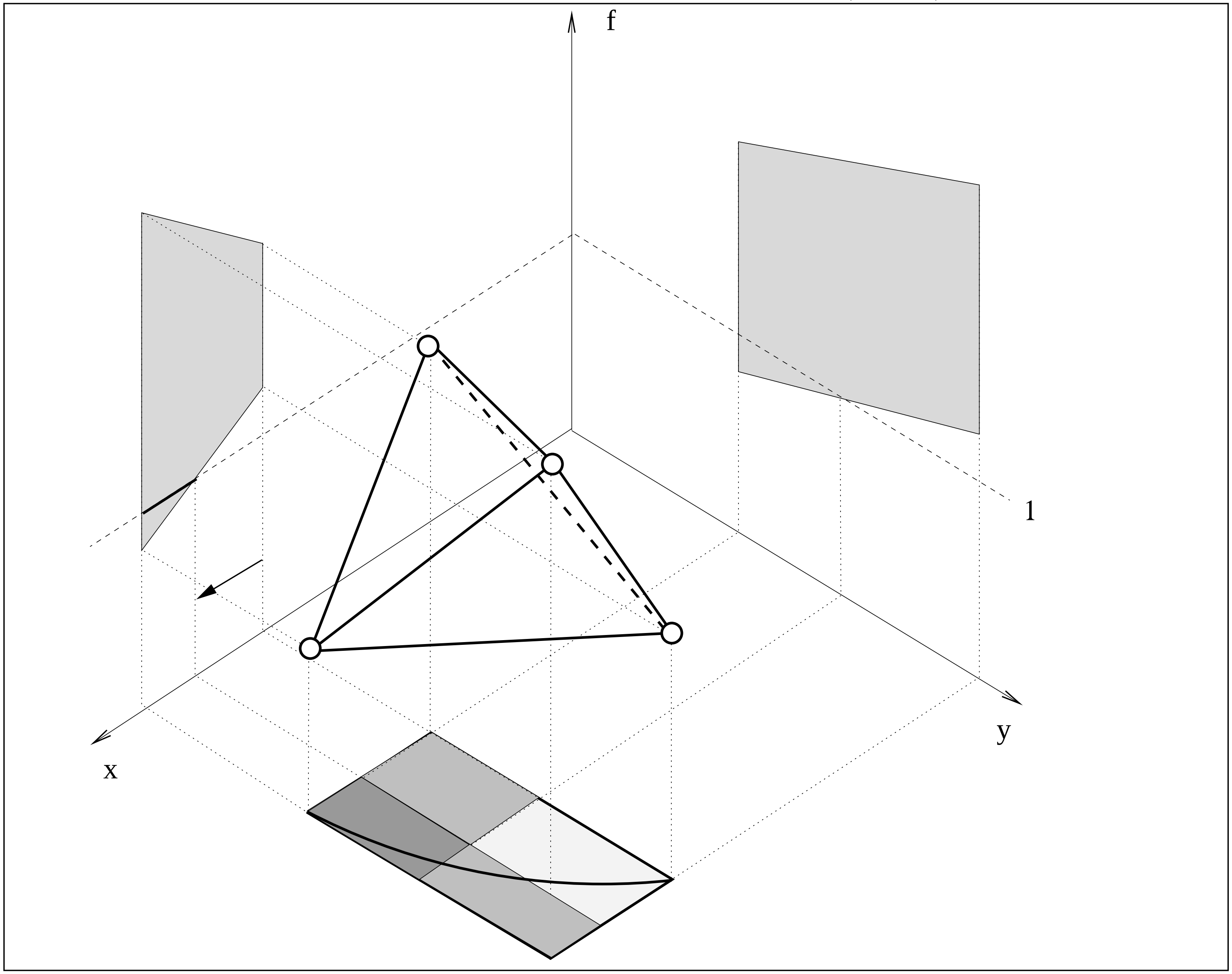
# Box Reduction (VI)

**Projection on the plane $(f_0, y)$.**



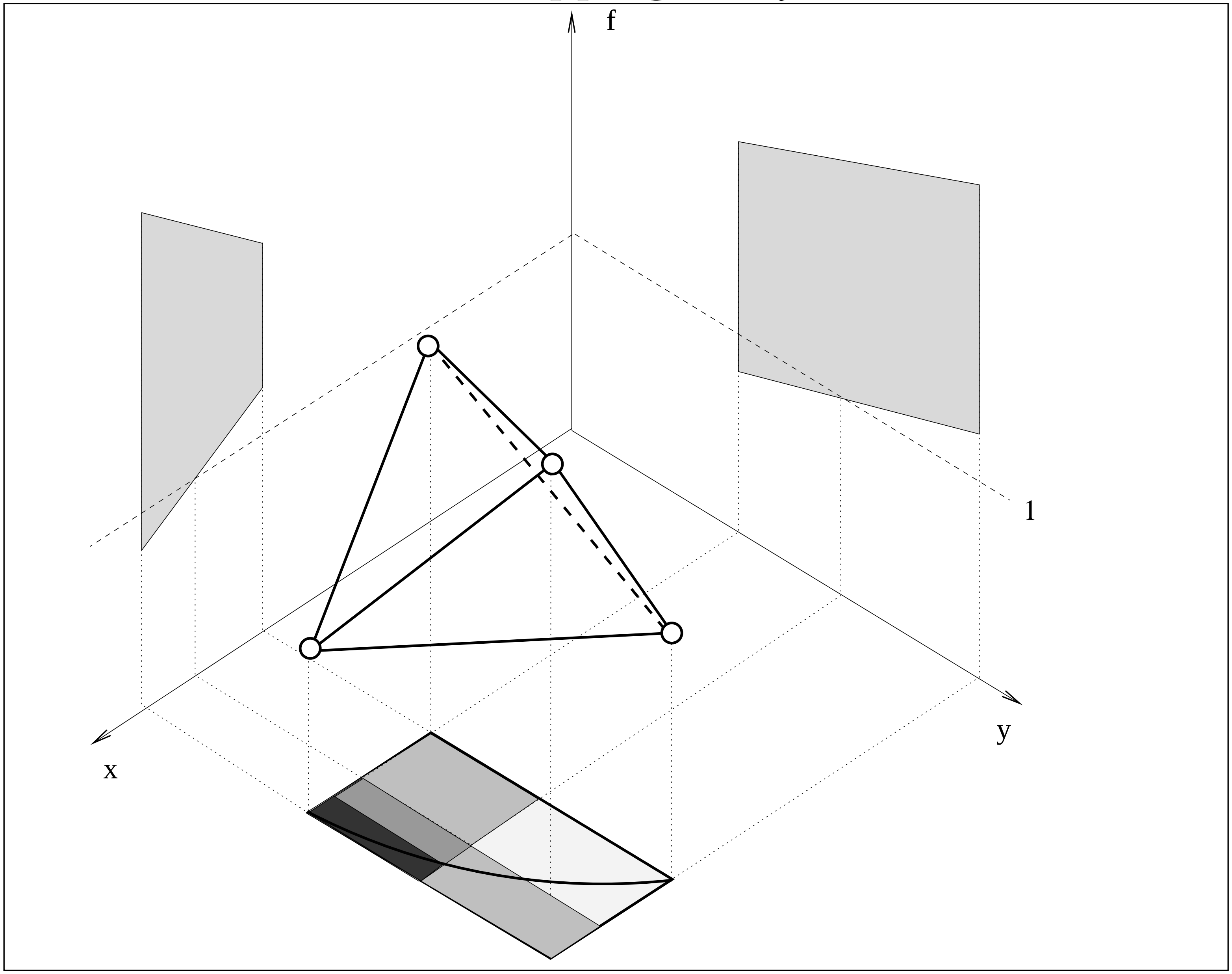
# Box Reduction (VII)

**Second projection. Plane $(f_0, x)$**



# Box Reduction (VIII)

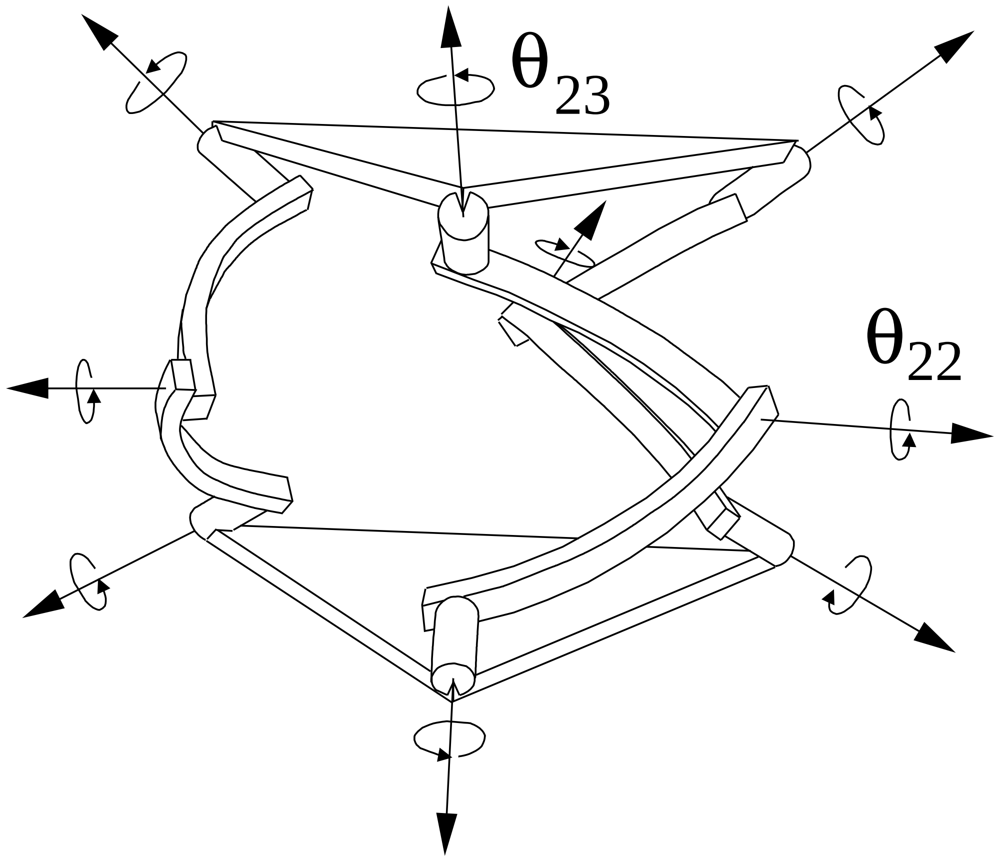
**Circle Clipping** $x^2 + y^2 = 1$

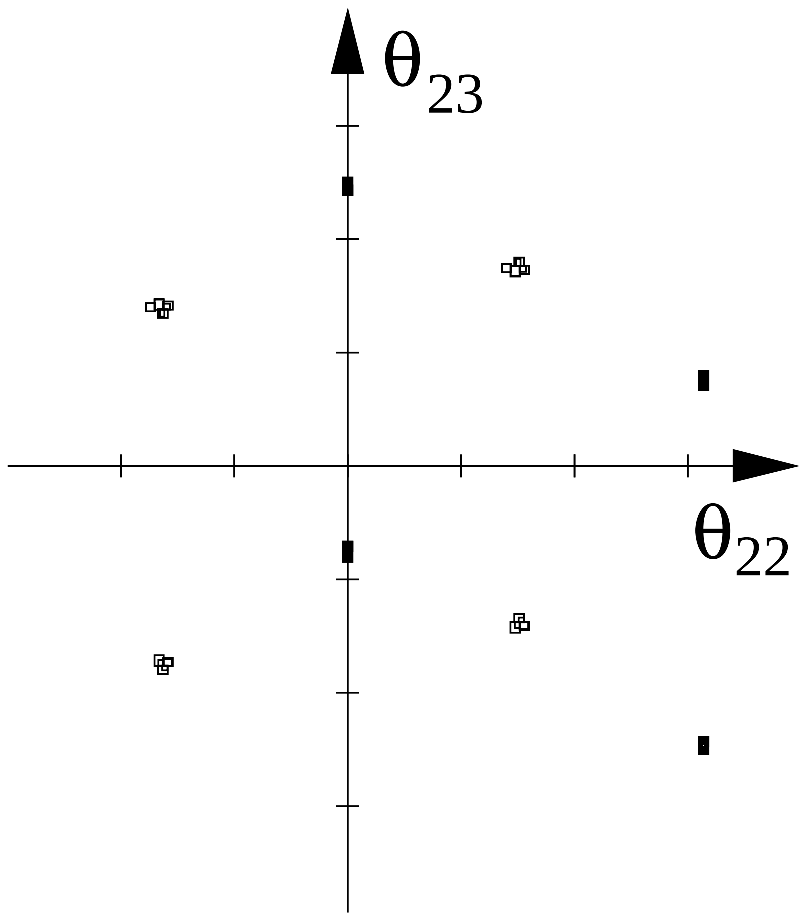


# The Cuik Algorithm

***Compute a cycle basis of the graph***
$S \leftarrow \emptyset$
$L \leftarrow$ ***Initial list of boxes***
**while not** ***empty(*** $L$ ***)***
  $\mathcal{B} \leftarrow$ ***first box(*** $L$ ***)***
  **do**
    $s \leftarrow$ ***size(*** $\mathcal{B}$ ***)***
    **Reduce_Box(** $\mathcal{B}$ **)**
  **Until** ***empty(*** $\mathcal{B}$ ***)*** **or** ***size(*** $\mathcal{B}$ ***)*** $< \sigma$ **or** ***size(*** $\mathcal{B}$ ***)*** $/s > \rho$
  **if not** ***empty(*** $\mathcal{B}$ ***)*** **then**
    **if** ***size(*** $\mathcal{B}$ ***)*** $\leq \sigma$ **then**
      $S \leftarrow S \cup \{\mathcal{B}\}$
    **else**
      ***Split*** $\mathcal{B}$ ***into two sub-boxes:*** $\mathcal{B}_1, \mathcal{B}_2$
      ***Add*** $\mathcal{B}_1$ ***and*** $\mathcal{B}_2$ ***to*** $L$
    **endif**
  **endif**
**endwhile**

# The Gosselin Platform

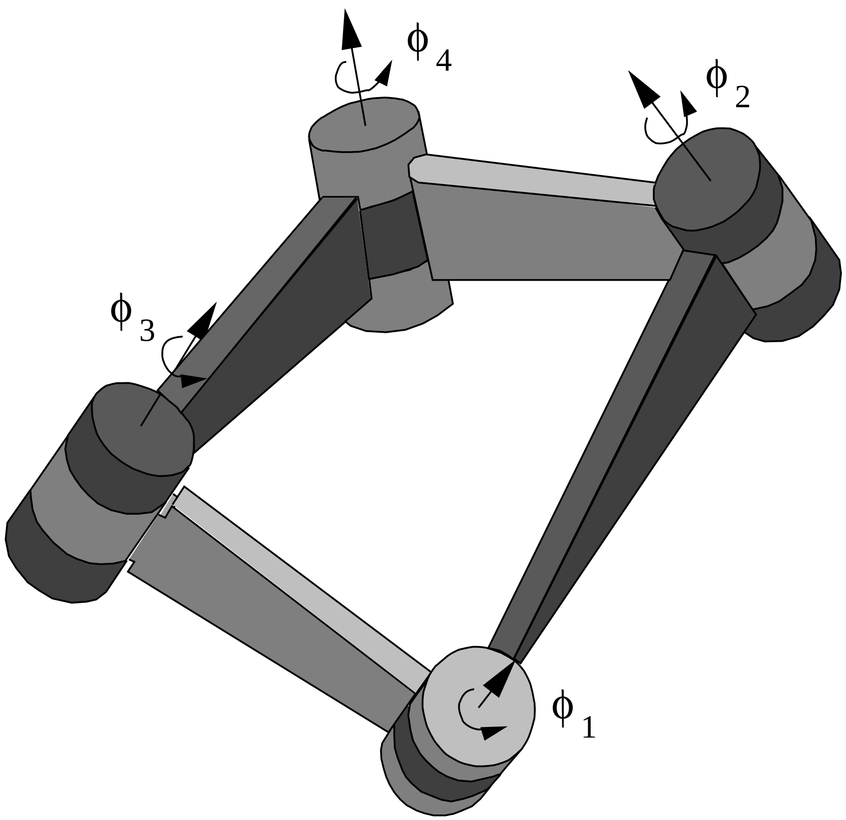


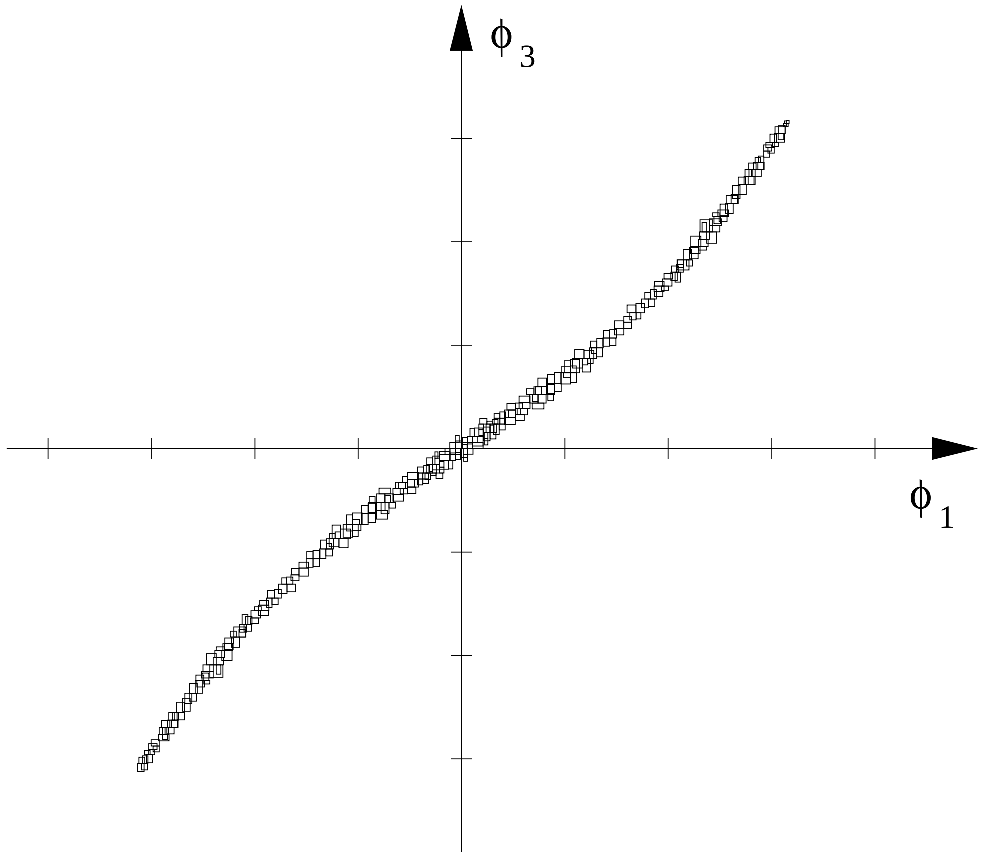




## Results

- Time: 2 sec
- Solution Boxes: 58 (8 clusters)

# The Bennett Linkage

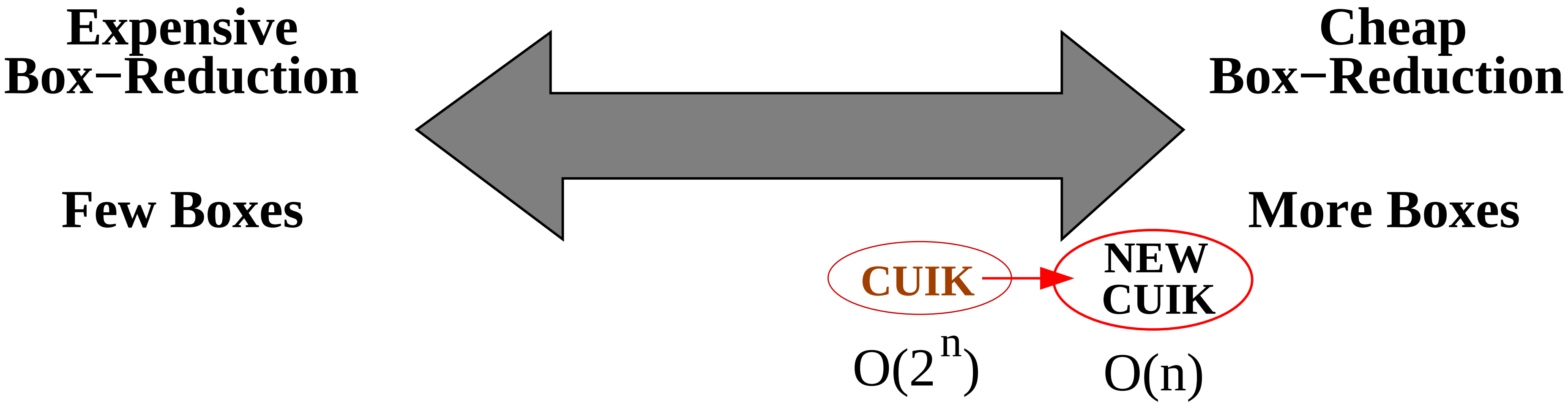



## Results

- Time: 1 sec
- Solution Boxes: 300

# On Going Work



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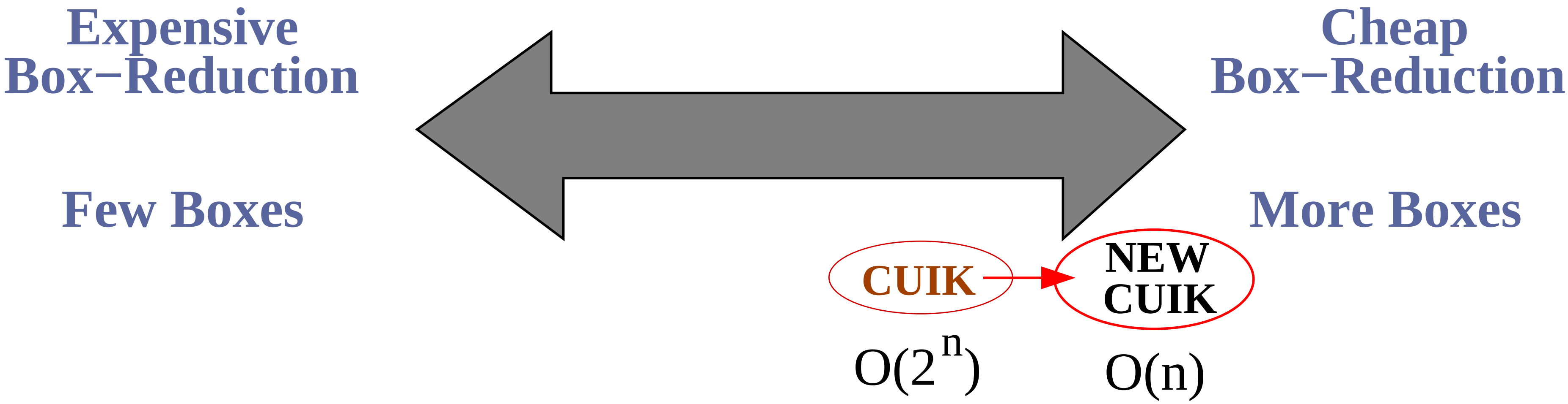


The Puma case:

- Bezier Method: Days

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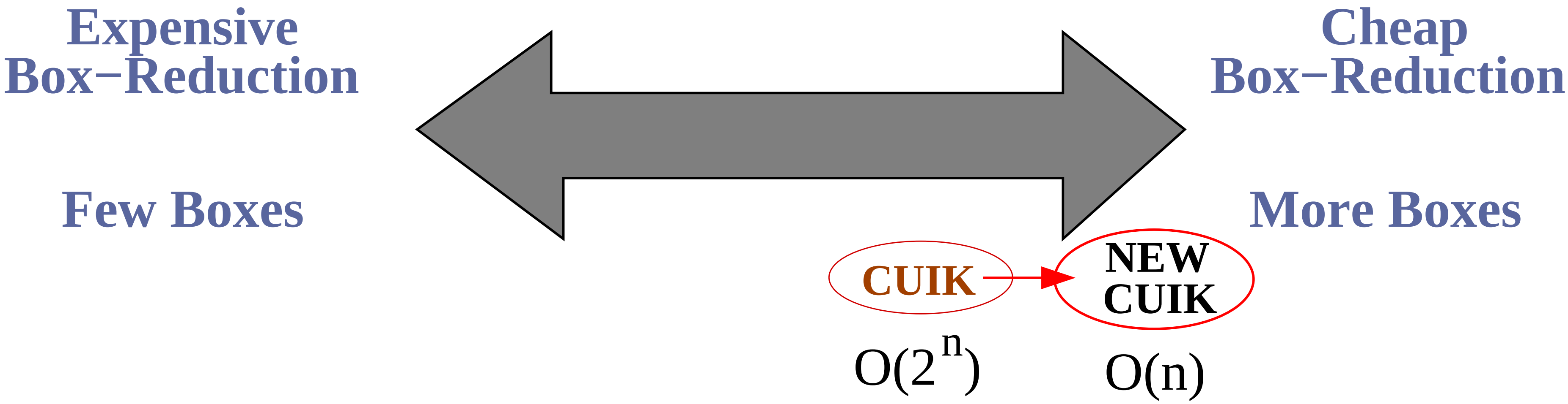


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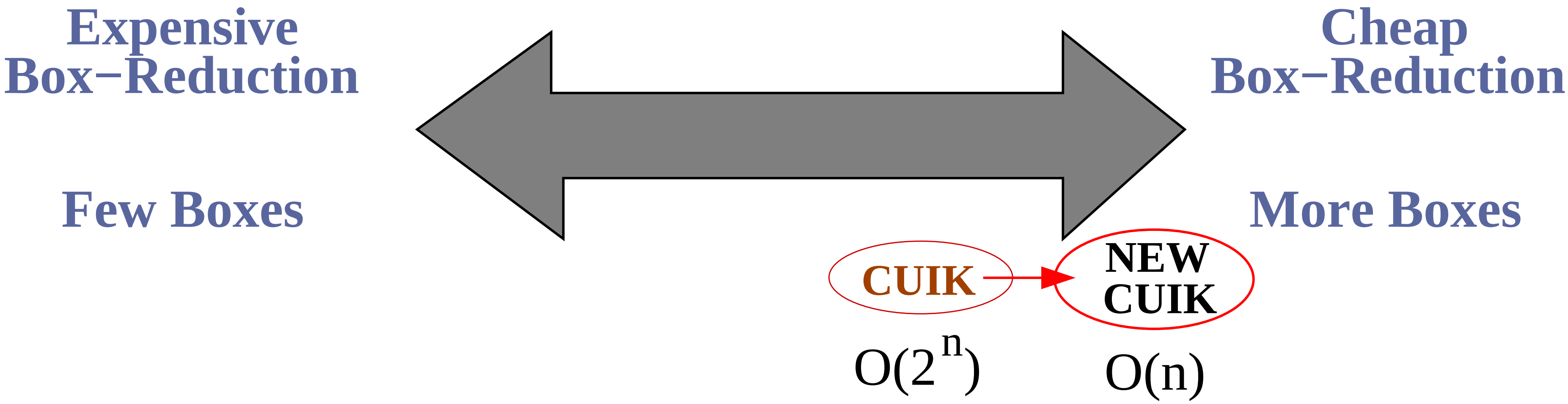


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- January 2002 (ARK): 1 hour

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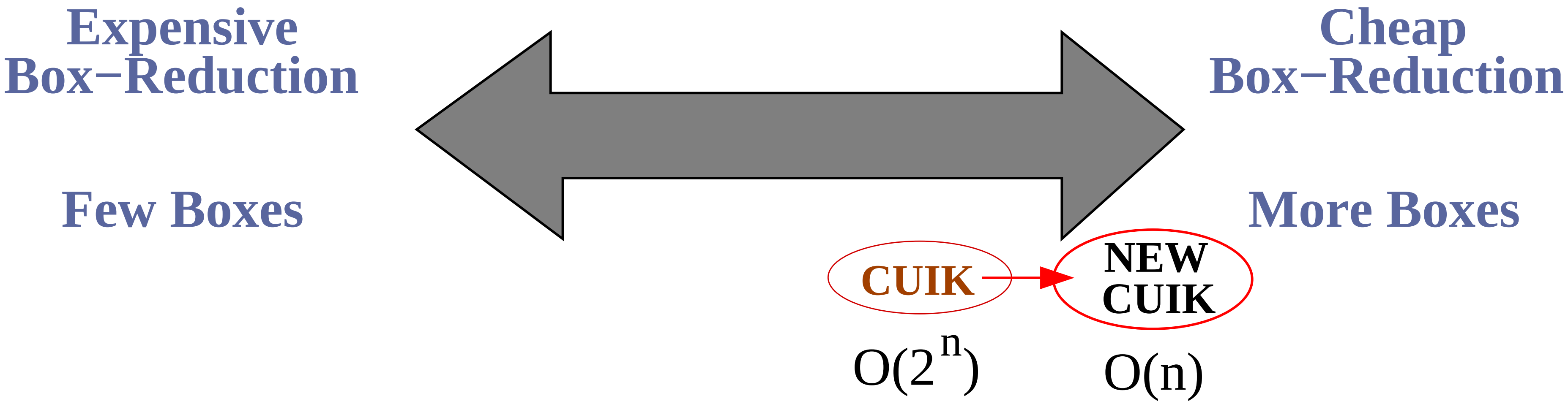


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- December 2001: 10 hours
- January 2002 (ARK): 1 hour
- March 2002: 20 min

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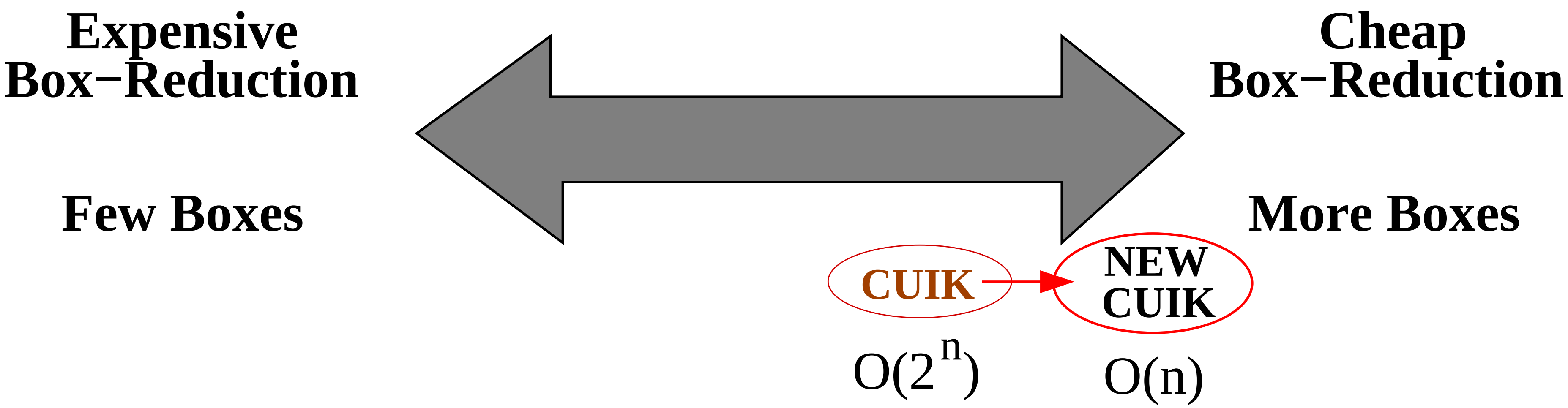


The Puma case:

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- December 2001: 10 hours
- January 2002 (ARK): 1 hour
- March 2002: 20 min
- May 2002 (New Cuik): 30 seg

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  - Algebraic methods.

# The End

Typesetting Software: TEX, *Textures*, LATEX, hyperref, texpower, Adobe Acrobat 4.05

**ARK. Caldes de Malabella. July 2002**

The Geometry and Kinematics Group

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LATEX Slide Macro Packages: Wendy McKay, Ross Moore