Topological Representation and Operation of Motion Space Exchange Reconfiguration of Metamorphic Mechanisms

Shujun Li, Hongguang Wang, Jiansheng Dai, Xiaopeng Li, Zhaohui Ren and Shichao Xiu

Abstract The paper presents a topological representation of mechanical chains based on proposed joint-axis matrix based on our previous study[10], and motion space exchange reconfiguration metamorphic processes of multi-loop mechanisms are operated by the matrix. The matrix operations of metamorphic process are performed by replacing the joint of planar mechanisms with passive mobility joint to form the passive mobility mechanisms firstly, and then changing the orientation of metamorphic joint of the passive mobility planar mechanisms to transform the configuration of the mechanisms from planar to spatial one. The matrix operations of 8-link multi-loop mechanism are illustrated to show the motion space exchange reconfiguration processes.

Key words: motion space reconfiguration, matrix metamorphic operation, augmented adjacency matrix.

1 Introduction

Since the metamorphic mechanism was proposed a decade ago [1], interests and attentions are aroused in the field of mechanisms study. Parise et al [2] proposed a new class of mechanisms called ortho-planar mechanisms. Li et al [3] presented a method of structural synthesis of metamorphic mechanisms based on the configuration transformations. Liu and Yang [4] studied the metamorphic ways, new concept

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of metamorphic kinematic pair was introduced and two new basic metamorphic ways are presented. Generally, the adjacency matrix was used to study the variable topologies of metamorphic mechanisms via matrix operation to show the configuration changes of the mechanisms [5]. The symbols of joints are added in adjacency matrix to show the changes of type and/or number of joints in the mechanisms [6-7]. Li et al proposed a topological characteristic matrix of kinematic chains with loop relations, types of joints and orientation of joints [8], and introduced a matrix representation and of motion space exchange metamorphic operations of single-loop kinematic chains[9]. In the paper, we study the motion space exchange reconfiguration operation of multi-loop mechanisms.

2 The Topological Representation of Metamorphic Chains

2.1 Joint–Axis Matrix

The topological information of a joint J_{ij} which connects the *i*th and *j*th links can be presented by following four elements **joint-axis matrix** $\mathbf{J}_{ij}$

$$\mathbf{J}_{ij} = \begin{bmatrix} 1 & J_{ij} \\ a_{ij} & 1 \end{bmatrix}_{2X2} \tag{1}$$

where

$$J_{ij} = J_{ji} = \begin{cases} R,\ P,\ \ldots, & or\ by\ corresponding\ number\ assumed \\ 0 & when\ link\ i\ and\ \ j\ are\ not\ connected \end{cases}$$

The element a_{ij} presents the orientation of axis of turning joint and/or normal of translating joint of J_{ij}

$$a_{ij} = a_{ji} = \begin{cases} 1,\ 2,\ \ldots,\ 8 & \\ 0 & when\ link\ i\ and\ \ j\ are\ not\ connected \end{cases}$$

The values a_{ij} are assumed as in table 1.

Table 1 Proposed axis–orientation–relation and the value

a_{ij}	1	2	3	4	5	6	7	8
Orientation	parallel *z*	parallel *x*	parallel *y*	intersected on one point	arbitrary	parallel *x–y* plane	parallel *x–z* plane	parallel *y–z* plane

2.2 Augmented Adjacent Matrix with Axis-Orientation-Relationship of Kinematic Chains

A n–link kinematic chain is assembled by the putting the corresponding elements of equation (1) into the general adjacent matrix of n link kinematic chain with types of joints, the general topological form of a n link metamorphic chain with types and orientations of joints can be proposed in the following form.

$$\mathbf{A} = \begin{bmatrix} 1 & J_{12} & \cdots & J_{1i} & \cdots & J_{1j} & \cdots & J_{1,n-1} & J_{1n} \\ a_{12} & 1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & J_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1i} & \cdots & \cdots & 1 & \cdots & J_{ij} & \cdots & \cdots & J_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{1j} & \cdots & \cdots & a_{ij} & \cdots & 1 & \cdots & \cdots & J_{jn} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1,n-1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 1 & J_{n-1,n} \\ a_{1n} & a_{2,n} & \cdots & a_{in} & \cdots & a_{jn} & \cdots & a_{n-1,n} & 1 \end{bmatrix} \quad (2)$$

Where for ***i***= 1, 2,..., n, j= 1, 2, ..., n, the element J_{ij} is the joint of connecting ith and jth links of kinematic chains.

3 Motion Space Change Metamorphic Reconfiguration and Matrix Operation

3.1 Joints Replacing Operation to form the Passive Mobility of the Mechanism

To reconfigure a planar mechanism into a spatial mechanism, it is essential that passive joints have the spatial mobility be added. The operation can be performed as following: first deleting the elements of replaced joint $\mathbf{J}_{ij}$ which is equal to putting the corresponding elements of a negative joint

$$-\mathbf{J}_{ij} = \begin{bmatrix} 1 & -\mathbf{J}_{ij} \\ -\mathbf{a}_{ij} & 1 \end{bmatrix} (for\ i = 1,2,...;\ j = 1,2,...,\ and\ \mathbf{J}_{ij}\ is\ deleted\ joint)$$

into equation (2) to delete the replaced joint/joints, and then adding the elements of

$$\mathbf{J}^{p}_{ij} = \begin{bmatrix} 1 & J^{p}_{ij} \\ a^{p}_{ij} & 1 \end{bmatrix} (for\ i = 1,2,...;\ j = 1,2,...,\ and\ \mathbf{J}^{p}_{ij}\ is\ deleted\ joint)$$

in the corresponding positions of the replaced joint/joints $\mathbf{J}_{ij}$ of the equation (4) to form the topological characteristics matrix of kinematic chain with passive joints

$\mathbf{A}_p$ as

$$\mathbf{A}_p = \begin{bmatrix} 1 & J_{12} & \cdots & J_{1i} & \cdots & J_{1j} & \cdots & \cdots & J_{1,n-1} & J_{1n} \\ a_{12} & 1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & J_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1i} & \cdots & \cdots & 1 & \cdots & J_{ij}^{p} & \cdots & \cdots & \cdots & J_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{1j} & \cdots & \cdots & a_{ij}^{p} & \cdots & 1 & \cdots & \cdots & \cdots & J_{jn} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1,n-1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 1 & J_{n-1,n} \\ a_{1n} & a_{2,n} & \cdots & a_{in} & \cdots & a_{jn} & \cdots & \cdots & a_{n-1,n} & 1 \end{bmatrix} \tag{3}$$

3.2 Motion Space Change Reconfiguration Operation

The process to change the orientation of a_{ij} of $\mathbf{A}^p$ to form the metamorphic matrix can be operated as deleting the joint elements of corresponding -a_{ij} of orientation changed joint in the $\mathbf{A}^p$ first which is equal to putting the corresponding elements of

$$-\mathbf{J}_{ij}^{o} = \begin{bmatrix} 1 & J_{ij} \\ -a_{ij} & 1 \end{bmatrix} (for\ i = 1,2,...;\ j = 1,2,...,\ and\ \mathbf{J}_{ij}^{o}\ is\ orientation\ changed\ joint)$$

into equation (3) to delete the elements of changed orientations, and then changing the orientation of the metamorphic joints, i.e. adding the elements of

$$\mathbf{J}_{ij}^{c} = \begin{bmatrix} 1 & J_{ij} \\ a_{ij}^{c} & 1 \end{bmatrix} (for\ i = 1,2,...;\ j = 1,2,...,\ and\ a_{ij}^{c}\ is\ the\ new\ orientation\ of\ the\ joint)$$

in the corresponding positions of the orientation changed joint $\mathbf{J}_{ij}^{o}$ of the equation (3) to form the matrix of spatial motion kinematic chain $\mathbf{A}_S$

$$\mathbf{A}_s = \begin{bmatrix} 1 & J_{12} & \cdots & J_{1i} & \cdots & J_{1j} & \cdots & \cdots & J_{1,n-1} & J_{1n} \\ a_{12} & 1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & J_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1i} & \cdots & \cdots & 1 & \cdots & J_{ij} & J^{p} & \cdots & \cdots & J_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{1j} & \cdots & \cdots & a_{ij}^{c} & \cdots & 1 & \cdots & \cdots & \cdots & J_{jn} \\ \cdots & \cdots & \cdots & a^{p} & \cdots & \cdots & 1 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1,n-1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 1 & J_{n-1,n} \\ a_{1n} & a_{2,n} & \cdots & a_{in} & \cdots & a_{jn} & \cdots & \cdots & a_{n-1,n} & 1 \end{bmatrix} \tag{4}$$

Thus the metamorphic processes for mechanism reconfiguration from planar to motion can be managed to operate only the corresponding metamorphic joint-axis matrix by equation (4).It should be noticed that the passive joint $\mathbf{J}_{ij}^{p}$ are not to be chosen as the orientation change joint $\mathbf{J}_{ij}^{c}$ in the metamorphic process generally.

4 Motion Space Change Reconfiguration Approaches

For a multi-loop planar 3-*RRR* parallel mechanism shown in figure 1(a), the main reconfiguration procedures are as follows.

To form the topological characteristic matrix **A** *of planar 3-RRR parallel mechanism*. The topological characteristic matrix **A** of 3-*RRR* planar parallel mechanism can be formed as according to equation (3)

$$\mathbf{A}=\begin{bmatrix} 1 & J_{12} & 0 & 0 & 0 & J_{16} & 0 & J_{18} \\ a_{12} & 1 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{23} & 1 & J_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{34} & 1 & J_{45} & 0 & J_{47} & 0 \\ 0 & 0 & 0 & a_{45} & 1 & J_{56} & 0 & 0 \\ a_{16} & 0 & 0 & 0 & a_{56} & 1 & 0 & 0 \\ 0 & 0 & 0 & a_{47} & 0 & 0 & 1 & J_{78} \\ a_{18} & 0 & 0 & 0 & 0 & 0 & a_{78} & 1 \end{bmatrix}=\begin{bmatrix} 1 & R & 0 & 0 & 0 & R & 0 & R \\ 1 & 1 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & R & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & R & 0 & R & 0 \\ 0 & 0 & 0 & 1 & 1 & R & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & R \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

To form the topological characteristic matrix $\mathbf{A}^{P\text{-}3RRS}$ *of planar 3-RRS parallel mechanism with passive mobility joints*. If change the three R joints R_{34}, R_{45}, and R_{47} which connected on the moving platform of link 4 into S joints, the mechanism will become the planar 3-*RRS* mechanism with passive mobility joints. According to equation (1), the replaced passive joints are

$$\mathbf{J}_{34}^{p}=\begin{bmatrix} 1 & J_{34}^{p} \\ a_{34}^{p} & 1 \end{bmatrix}=\begin{bmatrix} 1 & S \\ 5 & 1 \end{bmatrix};\ \mathbf{J}_{45}^{p}=\begin{bmatrix} 1 & J_{45}^{p} \\ a_{45}^{p} & 1 \end{bmatrix}=\begin{bmatrix} 1 & S \\ 5 & 1 \end{bmatrix};\ \mathbf{J}_{47}^{p}=\begin{bmatrix} 1 & J_{47}^{p} \\ a_{47}^{p} & 1 \end{bmatrix}=\begin{bmatrix} 1 & S \\ 5 & 1 \end{bmatrix}$$

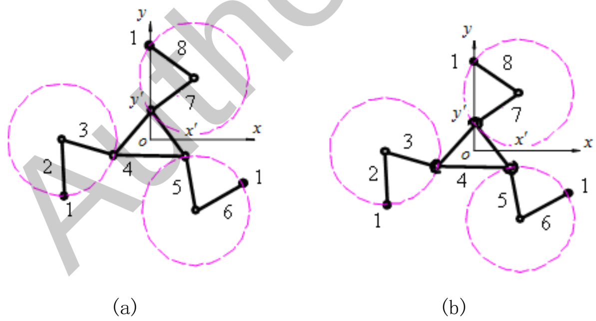


Fig. 1 Planar 3-*RRR* parallel mechanism and 3-*RRS* parallel mechanism

Adding the elements of the replaced passive joints into the corresponding positions of equation (3), to form the topological matrix $\mathbf{A}_p$ of this mechanism with passive joints

$$\mathbf{A}_p = \begin{bmatrix} 1 & J_{12} & 0 & 0 & 0 & J_{16} & 0 & J_{18} \\ a_{12} & 1 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{23} & 1 & J^p_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & a^p_{34} & 1 & J^p_{45} & 0 & J^p_{47} & 0 \\ 0 & 0 & 0 & a^p_{45} & 1 & J_{56} & 0 & 0 \\ a_{16} & 0 & 0 & 0 & a_{56} & 1 & 0 & 0 \\ 0 & 0 & 0 & a^p_{47} & 0 & 0 & 1 & J_{78} \\ a_{18} & 0 & 0 & 0 & 0 & 0 & a_{78} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R & 0 & 0 & 0 & R & 0 & R \\ 1 & 1 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & S & 0 & S & 0 \\ 0 & 0 & 0 & 5 & 1 & R & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 1 & R \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The corresponding configuration of $\mathbf{A}_p$ can be formed as shown in figure 1(b).

To form the orientation change metamorphic matrix $\mathbf{A}_S$ *of metamorphic mechanism based on the* $\mathbf{A}_p$. According to section 3, the orientation changing reconfiguration processes of this mechanism are:

Firstly, turning the link 2 in z with appropriate position, and then changing the orientation of the joint R_{12} 90 degree to form the orientation change metamorphic matrix $\mathbf{A}_S(R_{12})$. By equation (1) and (4), the orientation changed joint $\mathbf{J}^c_{12}$ and the orientation change metamorphic matrix $\mathbf{A}_S(R_{12})$ of the mechanism can be formed as

$$\mathbf{J}^c_{12} = \begin{bmatrix} 1 & J_{12} \\ a^c_{12} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 6 & 1 \end{bmatrix}$$

$$\mathbf{A}_S(R_{12}) = \begin{bmatrix} 1 & J_{12} & 0 & 0 & 0 & J_{16} & 0 & J_{18} \\ a^c_{12} & 1 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{23} & 1 & J^p_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & a^p_{34} & 1 & J^p_{45} & 0 & J^p_{47} & 0 \\ 0 & 0 & 0 & a^p_{45} & 1 & J_{56} & 0 & 0 \\ a_{16} & 0 & 0 & 0 & a_{56} & 1 & 0 & 0 \\ 0 & 0 & 0 & a^p_{47} & 0 & 0 & 1 & J_{78} \\ a_{18} & 0 & 0 & 0 & 0 & 0 & a_{78} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R & 0 & 0 & 0 & R & 0 & R \\ 6 & 1 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & S & 0 & S & 0 \\ 0 & 0 & 0 & 5 & 1 & R & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 1 & R \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The corresponding configuration is shown as in figure 2(a).

Secondly, turning the link 6 in z with appropriate position, and then changing the orientation of the joint R_{16} 90 degree to form the orientation change metamorphic matrix $\mathbf{A}_S(R_{12},R_{16})$. By equation (1) and (4), the joint orientation change matrix $\mathbf{J}^c_{16}$ and the orientation change metamorphic matrix $\mathbf{A}_S(R_{12},R_{16})$ can be formed as

$$\mathbf{J}^c_{16} = \begin{bmatrix} 1 & J_{16} \\ a^c_{16} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 6 & 1 \end{bmatrix}$$

$$\mathbf{A}_S(R_{12},R_{16}) = \begin{bmatrix} 1 & J_{12} & 0 & 0 & 0 & J_{16} & 0 & J_{18} \\ a^c_{12} & 1 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{23} & 1 & J^p_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & a^p_{34} & 1 & J^p_{45} & 0 & J^p_{47} & 0 \\ 0 & 0 & 0 & a^p_{45} & 1 & J_{56} & 0 & 0 \\ a^c_{16} & 0 & 0 & 0 & a_{56} & 1 & 0 & 0 \\ 0 & 0 & 0 & a^p_{47} & 0 & 0 & 1 & J_{78} \\ a_{18} & 0 & 0 & 0 & 0 & 0 & a_{78} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R & 0 & 0 & 0 & R & 0 & R \\ 6 & 1 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & S & 0 & S & 0 \\ 0 & 0 & 0 & 5 & 1 & R & 0 & 0 \\ 6 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 1 & R \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

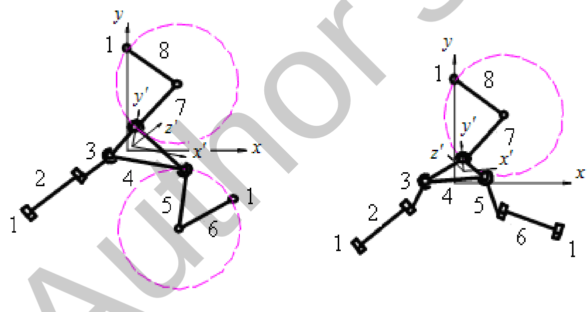
The corresponding configuration is shown as in figure 2(b).

Then, turning the link 8 in z with appropriate position, and then changing the orientation of the joint R_{18} 90 degree to form the orientation change metamorphic matrix $\mathbf{A}_S(R_{12},R_{16},R_{18})$. By equation (1) and (4), the joint orientation change matrix $\mathbf{J}^c_{18}$ and the orientation change metamorphic matrix $\mathbf{A}_S(R_{12},R_{16},R_{18})$, i.e. $\mathbf{A}_S$ can be formed as

$$\mathbf{J}^c_{18} = \begin{bmatrix} 1 & J_{18} \\ a^c_{18} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 6 & 1 \end{bmatrix}$$

$$\mathbf{A}_S(R_{12},R_{16},R_{18}) = \mathbf{A}_S = \begin{bmatrix} 1 & J_{12} & 0 & 0 & 0 & J_{16} & 0 & J_{18} \\ a^c_{12} & 1 & J_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{23} & 1 & J^p_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & a^p_{34} & 1 & J^p_{45} & 0 & J^p_{47} & 0 \\ 0 & 0 & 0 & a^p_{45} & 1 & J_{56} & 0 & 0 \\ a^c_{16} & 0 & 0 & 0 & a_{56} & 1 & 0 & 0 \\ 0 & 0 & 0 & a^p_{47} & 0 & 0 & 1 & J_{78} \\ a^c_{18} & 0 & 0 & 0 & 0 & 0 & a_{78} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R & 0 & 0 & 0 & R & 0 & R \\ 6 & 1 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & S & 0 & S & 0 \\ 0 & 0 & 0 & 5 & 1 & R & 0 & 0 \\ 6 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 1 & R \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Finally, turning the joints R_{12}, R_{16}, and R_{18} of figure 3 with some degrees to form the spatial 3-*RRS* parallel mechanism as shown in figure 3 (z direction view), so that the orientation change metamorphic mechanism is formed.



(a) (b)

Fig. 2 Configuration of the mechanism when the orientation of the joint R_{12} is to be changed 90 degree and the orientation of the joint R_{12} and R_{16} are to be changed 90 degree

5 Conclusions

Based on proposed joint-axis matrix and an augmented adjacent matrix of kinematic chains, the axis orientation change metamorphic processes are presented. The matrix operation metamorphic process can be performed both manually and be executed by computer. The matrix can be used for the topological representation both for metamorphic mechanism and general kinematic chains, and can be extended in the study of structural synthesis of kinematic chains.

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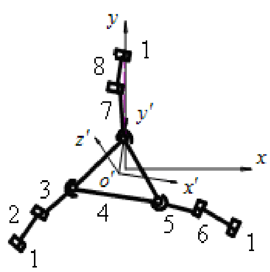

Fig. 3 Configuration of the orientation changed spatial mechanism