A Sufficient Condition for Parameter Identifiability in Robotic Calibration

Thibault Gayral and David Daney

Abstract Calibration aims at identifying the model parameters of a robot through experimental measures. In this paper, necessary mathematical conditions for calibration are developed, considering the desired accuracy, the sensor inaccuracy of the joint coordinates, and the measurement noise. They enable to define a physically meaningful stop criterion for the identification algorithm and a numerical bound for the observability index O_3, the minimum singular value of the observability matrix. With this bound, observability problems can be safely detected during calibration. Those conditions for calibration are illustrated through a simple example.

Key words: Conditions for calibration, Observability, Least-squares.

1 Introduction

Because of manufacturing and assembly errors, kinematic parameters of a robot are only known with uncertainties. In order to reach the desired accuracy of the robot over its workspace, a better knowledge of the model parameters is needed: this is the goal of calibration. Calibration can be decomposed in four important parts: modeling, measurements, identification and implementation [8, 11].

The required qualities of a robot model for calibration are well-known and can be found in [4]. However, necessary conditions also exist for the measurement and the identification steps. Indeed, some common assumptions have to be made: for example that the identification function mostly depends on the variations of the model parameters. Such hypotheses imply conditions both

Thibault Gayral, David Daney
INRIA Sophia Antipolis, France. e-mail: Forename.Name@inria.fr
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on the measurement device accuracy and on the measurement workspace. However, to the best of our knowledge, those conditions are not treated in the literature. Deriving those necessary conditions will lead to the calculus of the necessary accuracy of the kinematic parameters and the maximal allowed measurement inaccuracy for calibration. With these values, a numerical scaling of the observability matrix [8] involved in the identification step can be obtained. This enables to define a numerical bound for the observability index O_3 [10], the minimum singular value of the observability matrix. With this new bound, observability problems can be detected during calibration.

2 Overview and Conditions of Calibration

Without loss of generality, we will focus only on the kinematic calibration. Let consider a robot, with either a serial or parallel kinematic structure, with n' actuators controlling n degrees of freedom. The exact pose p of the end-effector is described by the n-vector $\mathbf{x}_p^*$, and $\boldsymbol{\rho}_p^*$ stands for the n'-vector of the exact joint coordinates at pose p. The kinematic model of this robot relates the joint and end-effector coordinates $\boldsymbol{\rho}_p^*$ and $\mathbf{x}_p^*$ through a function $\mathbf{f}^*$ that depends upon the m irreducible exact kinematic parameters [4] described by the vector $\boldsymbol{\xi}^*$:

$$\mathbf{f}^*(\mathbf{x}_p^*, \boldsymbol{\rho}_p^*, \boldsymbol{\xi}^*) = \mathbf{0} \tag{1}$$

However, both the joint coordinates and the kinematic parameters are not known with exactitude because of the accuracy of the actuator sensors, and manufacturing and assembly errors. Let consider measurement errors $\delta\boldsymbol{\rho}_p^*$ and $\delta\boldsymbol{\xi}^*$ between the nominal values $\boldsymbol{\rho}_p$ and $\boldsymbol{\xi}$ and the exact values $\boldsymbol{\rho}_p^*$ and $\boldsymbol{\xi}^*$ such as $\boldsymbol{\rho}_p = \boldsymbol{\rho}_p^* + \delta\boldsymbol{\rho}_p^*$ and $\boldsymbol{\xi} = \boldsymbol{\xi}^* + \delta\boldsymbol{\xi}^*$. These errors lead to a position error $\delta\mathbf{x}_p^*$ of the end-effector. The position error can be estimated by differentiating $\mathbf{f}^*(\mathbf{x}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi})$ with respect to all its parameters with the assumption of no singularity in the robot workspace [6]. Moreover, it is often complicated to obtain the exact model of a robot $\mathbf{f}^*$. Thus the modeling contains also errors $\boldsymbol{\varepsilon}_m$ that can be considered in the position error of the end-effector:

$$\delta\mathbf{x}_p^* = \mathbf{J}_\rho^p(\mathbf{x}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi})\delta\boldsymbol{\rho}_p^* + \mathbf{J}_\xi^p(\mathbf{x}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi})\delta\boldsymbol{\xi}^* \quad (+\boldsymbol{\varepsilon}_m) \tag{2}$$

where $\mathbf{J}_\rho^p$ is the kinematic Jacobian matrix, and $\mathbf{J}_\xi^p$ is the identification Jacobian matrix of pose p [8]. Note that both $\mathbf{J}_\rho^p$ and $\mathbf{J}_\xi^p$ are functions of all the parameters. For clarity reasons, this dependency will be omitted in the following equations. From (2) can be derived condition **(C1)**:

(C1) Necessary accuracy of the kinematic parameters: Assuming a given desired accuracy $\Delta\mathbf{x}_f$ and accuracy of the joint coordinate sensor $\Delta\boldsymbol{\rho}$, then the necessary accuracy $\delta\boldsymbol{\xi}^{\text{nec}}$ of the kinematic parameters can be derived from (2) under the assumption $\Delta\mathbf{x}_f >> \boldsymbol{\varepsilon}_m$:

$$\forall \mathbf{x}_p \in \mathcal{W}, \quad \text{abs}(\mathbf{J}^p_\xi)\delta\boldsymbol{\xi}^{\text{nec}} \leq \Delta\mathbf{x}_f - \text{abs}(\mathbf{J}^p_\rho)\Delta\boldsymbol{\rho} \tag{3}$$

where abs(•) stands for a matrix whose terms are the absolute values of the considered matrix, and $\mathcal{W}$ refers to the robot workspace. Note that the assumption $\Delta\mathbf{x}_f >> \boldsymbol{\varepsilon}_m$ means that the modeling errors can be neglected compared to the desired accuracy. This can be done by considering a model that contains all anticipated sources of error, before decreasing its complexity in the implementation step [9, 2]. □

Thus, (3) gives an estimation of $\delta\boldsymbol{\xi}^{\text{nec}}$ in the worst case, which is when each $\delta\rho = \pm\Delta\rho$ and each $\delta\xi = \pm\delta\xi^{\text{nec}}$: in this case, $\text{abs}(\mathbf{J}^p_\rho\delta\boldsymbol{\rho} + \mathbf{J}^p_\xi\delta\boldsymbol{\xi}) = \text{abs}(\mathbf{J}^p_\rho)\Delta\boldsymbol{\rho} + \text{abs}(\mathbf{J}^p_\xi)\delta\boldsymbol{\xi}^{\text{nec}}$. However, matrices $\mathbf{J}^p_\xi$ and $\mathbf{J}^p_\rho$ are calculated with the non-exact values $\boldsymbol{\xi}$: a better estimation of $\delta\boldsymbol{\xi}^{\text{nec}}$ can be obtained using interval analysis [7]. Another point is that $\mathbf{J}^p_\xi$ is an $n \times m$ matrix, with most of the time $m > n$, which yields to an under-determined system of equations. So, several sets of $\delta\boldsymbol{\xi}^{\text{nec}}$ satisfying (3) can be chosen.

The second step of calibration is the identification process. From the kinematic model, measurements are taken on the robot and the kinematic parameters are estimated so that an objective function depending on both the measurements and the model parameters is minimized. Let consider that all the m kinematic parameters can be identified and that d measurements are taken for N_p different poses of the end-effector. A trivial condition for being able to perform calibration is:

(C2) Sufficient number of measurements: $d.N_p > m$ □

Each d-vector of exact measurements $\mathbf{y}^*_p$, with $p = 1..N_p$, is then compared to an estimation resulting from the kinematic model through the identification function $\mathbf{g}_p$ of pose p, $\boldsymbol{\epsilon}_p$ being a residual considering the modeling errors:

$$\mathbf{g}_p(\mathbf{y}^*_p, \boldsymbol{\rho}^*_p, \boldsymbol{\xi}^*) = \boldsymbol{\epsilon}_p \tag{4}$$

Considering the measurement noises $\delta\mathbf{y}^*_p$ and $\delta\boldsymbol{\rho}^*_p$, with $\mathbf{y}_p = \mathbf{y}^*_p + \delta\mathbf{y}^*_p$, and the parameter errors $\delta\boldsymbol{\xi}^*$, differentiating (4) with respect to all its parameters yields to:

$$\mathbf{g}_p(\mathbf{y}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi}) = \boldsymbol{\epsilon}_p + \mathbf{G}^p_y\delta\mathbf{y}^*_p + \mathbf{G}^p_\rho\delta\boldsymbol{\rho}^*_p + \mathbf{G}^p_\xi\delta\boldsymbol{\xi}^* \tag{5}$$

where matrices $\mathbf{G}^p_y$, $\mathbf{G}^p_\rho$ and $\mathbf{G}^p_\xi$ all depend on the robot pose p. However, since the exact values of $\delta\mathbf{y}^*_p$ and $\delta\boldsymbol{\rho}^*_p$ are not known, the identification process is always performed under the hypothesis that $\mathbf{g}_p$ mostly depends on the variation $\delta\boldsymbol{\xi}^*$ of the kinematic parameters. This hypothesis can be written as in (6) and yields to **(C3)**.

$$\forall p = 1..N_p, \quad \text{abs}(\mathbf{G}^p_y\delta\mathbf{y}^*_p + \mathbf{G}^p_\rho\delta\boldsymbol{\rho}^*_p) << \text{abs}(\mathbf{G}^p_\xi\delta\boldsymbol{\xi}^*) \tag{6}$$

(C3) Necessary condition on measurement inaccuracy: Considering the maximal inaccuracy $\Delta\boldsymbol{\rho}$ of the joint coordinates and the necessary accuracy of the kinematic parameters $\delta\boldsymbol{\xi}^{\text{nec}}$, (6) can be rewritten as:

$$\forall p = 1..N_p, \quad \text{abs}(\mathbf{G}_y^p)\Delta\mathbf{y}^{\max} + \text{abs}(\mathbf{G}_\rho^p)\Delta\boldsymbol{\rho} < \text{abs}(\mathbf{G}_\xi^p \delta\boldsymbol{\xi}^{\text{nec}}) \tag{7}$$

with $\Delta\mathbf{y}^{\max}$ the maximal measurement inaccuracy, which enables to choose the appropriate measurement device. □

Condition **(C3)** leads to an estimation of $\Delta\mathbf{y}^{\max}$ in the worst case. If the obtained value of $\Delta\mathbf{y}^{\max}$ is too strong, first another set of $\delta\boldsymbol{\xi}^{\text{nec}}$ can be chosen accordingly to **(C1)**. Then, (7) has to be valid for the N_p measurement configurations only. Thus, the inequality of (7) can also be verified through a proper choice of the observability matrices $\mathbf{G}_\xi^p$. This property is well-known and has already been studied through observability indexes [3, 10]. From (7), we can remark that analyzing $\mathbf{G}_y^p$ and $\mathbf{G}_\rho^p$ can also be of prime interest.

With **(C3)**, we obtain $\mathbf{g}_p(\mathbf{y}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi} + \delta\boldsymbol{\xi}) \approx \mathbf{g}_p(\mathbf{y}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi}) + \mathbf{G}_\xi^p \delta\boldsymbol{\xi}$, with $\delta\boldsymbol{\xi} \neq \delta\boldsymbol{\xi}^*$ since $\delta\boldsymbol{\xi}$ considers the measurement noise, sensor inaccuracies and modeling errors. The objective of identification is to find the best set of parameter errors $\delta\boldsymbol{\xi}$ that minimizes functions $\mathbf{g}_p$ for $p = 1..N_p$. Setting $h = \sum_{p=1}^{N_p} \mathbf{g}_p^T \mathbf{g}_p = \mathbf{g}^T \mathbf{g}$ as the objective function of identification, with $\mathbf{g} = [\mathbf{g}_1^T \ldots \mathbf{g}_{N_p}^T]^T$, yields to the following normal equations (8) at iteration j:

$$\mathbf{G}_\xi^T \mathbf{G}_\xi \delta\boldsymbol{\xi}^{j+1} = \mathbf{G}_\xi^T \mathbf{g}(\mathbf{y}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi} + \sum_{i=1}^{j} \delta\boldsymbol{\xi}^i) \;\; \text{, with } \; \mathbf{G}_\xi = [{\mathbf{G}_\xi^1}^T \ldots {\mathbf{G}_\xi^{N_p}}^T]^T \tag{8}$$

An estimation of the kinematic parameters is given by $\boldsymbol{\xi}_{\text{est}} = \boldsymbol{\xi} + \sum_{i=1}^{j_{\max}} \delta\boldsymbol{\xi}^i$, $j_{\max}$ being the number of the last iteration. Most of the time, the optimization algorithm is stopped when the residual of h is under a certain threshold. The measurement errors have to be considered in this threshold. However, in practice, this threshold is manually adjusted to obtain the algorithm convergence and has no physical meanings. From the above conditions, the necessary accuracy $\delta\boldsymbol{\xi}^{\text{nec}}$ was derived. This value considers the desired accuracy and can be reached by the identification algorithm considering the measurement noise due to **(C3)**. Thus, the stop criterion of the optimization algorithm can be set as $\text{abs}(\delta\boldsymbol{\xi}^{j_{\max}}) \leq \delta\boldsymbol{\xi}^{\text{nec}}$ which is physically meaningful.

However, results of identification also depend on the scaling of the identification function $\mathbf{g}$ [5]. Thus, the developed conditions are necessary but not sufficient because of observability issues. They will however be useful to define a physical bound for the observability index O_3 [10].

3 Observability Issues

In practice, even if the previous conditions of calibration are fulfilled, only k model parameters among m can be identified, with $k \leq m$, because some model parameter errors $\delta\xi$ cannot be observed during identification. Such observability problems depend on properties of the observability matrix $\mathbf{G}_\xi$.

Three types of identifiability problems can occur [5]: unidentifiable, weakly identifiable or identifiable only in linear combination.

Non-Observability: It occurs when a kinematic parameter is not involved in the identification function $\mathbf{g}$. In this case, its corresponding column of $\mathbf{G}_\xi$ is zero. If the rank of $\mathbf{G}_\xi^T\mathbf{G}_\xi$ is r, then $m-r$ kinematic parameter errors are non-observable. This phenomenon appears when measurements are only partial or when measurement configurations do not involve all the kinematic parameters, and can be observed with a QR-decomposition. A parameter whose variation is non-observable is called non-identifiable.

Low-Observability: A model parameter is said to be low-identifiable if its corresponding column of $\mathbf{G}_\xi$ is close to zero: an important variation of its value only has small consequences on the identification function $\mathbf{g}$, compared to the influence of measurement and modeling errors. The identification of such a parameter often leads to an important variation of its initial value and degrades the robustness of calibration [5].

Linked-Observability: The linked-observability occurs when two or more parameter errors $\delta\xi$ appear in the normal equations only as a linear combination whose variation is less than the measurement noise. Their corresponding columns of $\mathbf{G}_\xi$ are linearly related and only the linear combination of those parameters can be observed. The linked-observability often appears for robot having a small workspace, or when measures are not generic enough (with constant orientation for example).

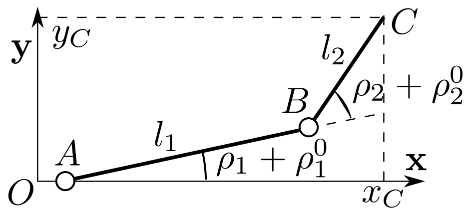
Non-observability is very easy to check. In order to tackle low- and linked-observability problems, observability indexes were proposed [10]. Those indexes are based on properties of the singular values of the observability matrix $\mathbf{G}_\xi$. However, a proper scaling of this matrix is compulsory for a comparison of its singular values [5]. A proper scaling of $\mathbf{G}_\xi$ can be obtained due to the above developed conditions of calibration. Let consider the worst case of observability during identification. This occurs when the parameter variation is minimum, that is $\delta\boldsymbol{\xi}^{\mathrm{nec}}$, and when the identification function $\mathbf{g}$ is highly noisy, that is $\delta\mathbf{g}^{\min} = [\delta\mathbf{g}_1^T ... \delta\mathbf{g}_{N_p}^T]^T$ with $\delta\mathbf{g}_p = \mathrm{abs}(\mathbf{G}_y^p)\Delta\mathbf{y}^{\max} + \mathrm{abs}(\mathbf{G}_\rho^p)\Delta\boldsymbol{\rho}$. Let define the scaling vectors $\Delta\boldsymbol{\xi}^i = \mathrm{diag}(\delta\boldsymbol{\xi}^{\mathrm{nec}})^{-1}.\delta\boldsymbol{\xi}^i$ and $\Delta\mathbf{g} = \mathrm{diag}(\delta\mathbf{g}^{\min})^{-1}.\mathbf{g}$, where $\mathrm{diag}(\mathbf{x})$ stands for a matrix whose diagonal is the vector $\mathbf{x}$. Thus, in the worst case, $\Delta\boldsymbol{\xi}^i$ and $\Delta\mathbf{g}$ become vectors whose terms are approximately equal to 1. The normal equations (8) can be written as:

$$\mathbf{H}_\xi\Delta\boldsymbol{\xi}^{j+1} = \Delta\mathbf{g} \quad \text{with} \quad \mathbf{H}_\xi = \mathrm{diag}(\delta\mathbf{g}^{\min})^{-1}.\mathbf{G}_\xi.\mathrm{diag}(\delta\boldsymbol{\xi}^{\mathrm{nec}}) \qquad (9)$$

In this case, if a singular value σ_L of $\mathbf{H}_\xi$ is under 1, the measurement noise considered in $\Delta\mathbf{g}$ will be amplified, which may lead to a wrong estimation of $\Delta\boldsymbol{\xi}^{j+1}$. So, the condition $\sigma_L(\mathbf{H}_\xi) \geq 1$ is sufficient to prevent observability issues. This condition is related to the observability index O_3 [10]. However, it will be seen in the following example that this condition is sufficient but not necessary for observability.

4 Application Example

The 2-bar serial planar mechanism, presented in Fig. 1, consists of two bars of lengths l_1 and l_2, actuated by two motors at angles ρ_1 and ρ_2 from their initial poses ρ_1^0 and ρ_2^0, respectively. The objective is to calibrate this serial manipulator considering measurements x_C of its end-effector in the measurement frame $(O, \mathbf{x}, \mathbf{y})$. The nominal (initial guess) and real values of the kinematic parameters $\boldsymbol{\xi} = [x_A, y_A, l_1, l_2, \rho_1^0, \rho_2^0]^T$ are given in Fig. 1.



Nominal values	Real values
x_A=0.1 m	x_A=0 m
y_A=0 m	y_A=0 m
l_1 =1 m	l_1 =0.99 m
l_2 =0.5 m	l_2 =0.45 m
ρ_1^0 =0°	ρ_1^0 =1°
ρ_2^0 =0°	ρ_2^0 =−1°

Fig. 1: The serial planar 2-bar mechanism and its kinematic parameter definition

With vectors of the actuator positions $\boldsymbol{\rho}_p = [\rho_1^p, \rho_2^p]^T$ and end-effector locations $\mathbf{x}_p = [x_C^p, y_C^p]^T$ of pose $p \in \mathcal{W}$, the exact kinematic model $\mathbf{f}^*$ can be written, with $\alpha_1^p = \rho_1^p + \rho_1^0$, $\alpha_2^p = \alpha_1^p + \rho_2^p + \rho_2^0$, $s_\alpha = \sin(\alpha)$, and $c_\alpha = \cos(\alpha)$:

$$\mathbf{f}^*(\mathbf{x}_p, \boldsymbol{\rho}_p, \boldsymbol{\xi}) = \begin{bmatrix} x_A + l_1 c_{\alpha_1^p} + l_2 c_{\alpha_2^p} - x_C^p \\ y_A + l_1 s_{\alpha_1^p} + l_2 s_{\alpha_2^p} - y_C^p \end{bmatrix}, \text{ which yields to:} \tag{10}$$

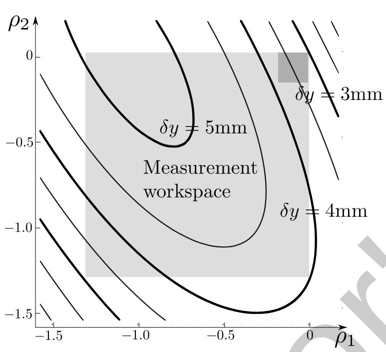
$$\mathbf{J}_\rho^p = \begin{bmatrix} -l_1 s_{\alpha_1^p} - l_2 s_{\alpha_2^p} & -l_2 s_{\alpha_2^p} \\ l_1 c_{\alpha_1^p} + l_2 c_{\alpha_2^p} & l_2 c_{\alpha_2^p} \end{bmatrix}, \mathbf{J}_\xi^p = \begin{bmatrix} 1 & 0 & c_{\alpha_1^p} & c_{\alpha_2^p} & -l_1 s_{\alpha_1^p} - l_2 s_{\alpha_2^p} & -l_2 s_{\alpha_2^p} \\ 0 & 1 & s_{\alpha_1^p} & s_{\alpha_2^p} & l_1 c_{\alpha_1^p} + l_2 c_{\alpha_2^p} & l_2 c_{\alpha_2^p} \end{bmatrix}$$

Considering the desired accuracy $\Delta\mathbf{x}_f = [0.01\text{m}, 0.01\text{m}]^T$, the accuracy of the actuator sensors $\Delta\boldsymbol{\rho} = [0.01°, 0.01°]^T$, the allowed motor positions ρ_1 and $\rho_2 = \pm 90°$, and setting that all the distance parameters (respectively the orientation parameters) must have the same necessary accuracy δx^{nec} (respectively $\delta\theta^{\text{nec}}$), we obtain from **(C1)**: $\delta x^{\text{nec}} = 1\text{mm}$ and $\delta\theta^{\text{nec}} = 0.1°$. These values allow to reach $\Delta\mathbf{x}_f$ over the entire robot workspace. The detailed calculus can be found in the Maple® or Mathematica® worksheets of [1].

As for the number of measurements, **(C2)** gives $N_p \geq 6$. However, this is a lower bound and the required number of measurements allowing good calibration results is still an open issue. This number can be minimized through a proper choice of the measurement configurations [3]. The number of measurements is not limited for simulation. Thus, two cases will be studied: $N_p = 250$ and $N_p = 20$ configurations randomly chosen in the measurement workspace.

Since measurements are directly the output x_C of the mechanism, the identification function $\mathbf{g}_p$ of pose p can be chosen as the first row of the kinematic model $\mathbf{f}^*$ of (10). This enables the matrices $\mathbf{G}_\rho^p$ and $\mathbf{G}_\xi^p$ to be equal to the first row of $\mathbf{J}_\rho^p$ and $\mathbf{J}_\xi^p$ respectively, and $\mathbf{G}_y^p = [-1]$. With those matrices, the last condition of calibration **(C3)** can be derived. Thus, the accuracy of the measurement device can be chosen accordingly to the measurement workspace as shown in Fig. 2. In this figure are plotted the curves for ρ_1 and $\rho_2 \in [-\frac{\pi}{2}; 0]$ for which (7) is exactly satisfied considering different values of the measurement device accuracy δy. Setting a measurement workspace such as ρ_1 and $\rho_2 \in [-75°; 0]$, represented as a light-gray square in Fig. 2, a necessary accuracy δy^{nec} of 3mm can be chosen from **(C3)**.

Finally, calibration is performed using a least-square algorithm with normal equations of (9). Obviously, y_A is non-identifiable since it does not appear in the identification function $\mathbf{g}_p$, and is removed from the set of identifiable parameters. For 4 different cases presented in Fig. 2, N_p random poses are taken in the measurement workspace. Considering the actuator sensor inaccuracies, the pose x_C^p is calculated and a uniformly distributed measurement noise of $\pm\delta y$ is added. Calibration is repeated 10 000 times and results are presented in Fig. 2 as the maximal obtained inaccuracies Δx_C and Δy_C on the 100 verification poses, randomly taken in the manipulator workspace $\mathcal{W}$.



Case	1	2	3	4
N_p	250	250	20	250
ρ_1 (°)	$[-75, 0]$	$[-75, 0]$	$[-75, 0]$	$[-10, 0]$
ρ_2 (°)	$[-75, 0]$	$[-75, 0]$	$[-75, 0]$	$[-10, 0]$
δy (mm)	3	4	3	3
Δx_C (mm)	10.6	14.3	66.8	436
Δy_C (mm)	7.7	10.1	46.9	457
$\overline{\sigma_L}(\mathbf{H}_\xi)$	0.239	0.179	0.058	0.004

Fig. 2: Choice of the necessary measurement accuracy as a function of the measurement workspace and results of calibration for the 4 cases.

Case 1 is the ideal case: all conditions are fulfilled and the accuracy $\Delta\mathbf{x}_f$ is reached over $\mathcal{W}$. Case 2 shows that **(C3)**, the maximal measurement inaccuracy, is a necessary condition to reach $\Delta\mathbf{x}_f$. Case 3 confirms the necessity to optimize the configurations of measurement when decreasing their number N_p. Finally, the linked-identifiability issues are addressed in case 4: because of the small measurement workspace and the measurement noise, x_A, l_1 and l_2 appear in $\mathbf{g}_p$ only as a linear combination and cannot be properly identified. The average of the minimum singular values $\overline{\sigma_L}(\mathbf{H}_\xi)$ of the last iteration is also given. Since identifiability is possible for cases 1 and 2, the physically meaningful bound for O_3, $\sigma_L(\mathbf{H}_\xi) \geq 1$, seems to be a too strong condition.

5 Conclusion and Discussions

Necessary mathematical conditions were developed to ensure the quality of calibration with respect to the final accuracy. According to those conditions, a physically meaningful stop criterion for the optimization algorithm can be derived. Those conditions were illustrated on a simple serial example but also stand for parallel manipulators.

The developed conditions also enable the calculus of a lower bound for the minimum singular value of the observability matrix, after a proper scaling. This bound leads to a sufficient condition for observability but seems overestimated since it considers the worst case. More studies are needed to derive the most appropriate formula for this threshold. However, the developed scaled observability matrix must be of prime interest for a proper choice of the configurations of measurement. In the same field, **(C3)** shows that considering the sensitivity of the identification function to measurement noise and sensor inaccuracy through matrices $\mathbf{G}_y^p$ and $\mathbf{G}_\rho^p$ must also be of prime interest.

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