Type Synthesis of Two DOF Hybrid Translational Manipulators

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Abstract This paper introduces a new methodology for the type synthesis of two degrees of freedom hybrid translational manipulators with identical legs. The type synthesis method is based upon screw theory. Three types of two degrees of freedom hybrid translational manipulators with two identical legs are identified based upon their wrench decomposition. Each leg of the manipulators is composed of a proximal module and a distal module mounted in series. The assembly conditions and the validity of the actuation scheme are also defined. Finally, some novel two degrees of freedom hybrid translational manipulators are synthesized with the proposed procedure.

Key words: Type synthesis; Parallel manipulators; Hybrid Legs, Screw theory.

1 Introduction

At the conceptual design stage of manipulator architectures, the idea is to construct several design alternatives by following a systematic approach. However, the information at this stage is usually qualitative and not quantitative, which makes the design process quite difficult and challenging.

A manipulator is a mechanical system that aims at *manipulating* objects. For simple task such as pick-and-place operations, the two degrees of freedom (*dof*) parallel manipulators may be sufficient. Several two-*dof* translational parallel manipulators (*TPM*) are composed of a planar architecture that yields their stiffness quite low along the normal to the plane of motion [3]. Moreover, those manipulators are usually not composed of identical legs.

In order to increase the stiffness properties of the two-*dof TPM*, some researchers have proposed a new manipulator architecture named the Par2 [2]. This architecture has the particularity to be spatial instead of planar and thus is stiffer along the normal to the plane of motion. Two legs amongst the four legs of the manipulator are

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linked to each other with a rigid belt in order to constrain the rotation of the moving platform. As a consequence, it leads to a robot with a poor accuracy. To avoid the design problems of the Par2, a new robot with spatial architecture and two legs has been proposed: the IRSBot-2 [4]. Each leg of the IRSBot-2 is hybrid, i.e. it is composed of a proximal module (PM) and a distal module (DM) mounted in series, each module containing two kinematic chains. This mechanism has exhibited interesting stiffness properties. Therefore, it is of interest to focus on the type synthesis of two-*dof TPM* by considering architectures with hybrid legs.

The subject of the paper is about the type synthesis of of two-*dof* hybrid translational manipulators with identical legs. Each leg is composed of a proximal module and a distal module mounted in series. Those modules contain one kinematic chain or two kinematic chains mounted in parallel. These kinematic chains are called sub legs. This research work has been carried out in the framework of the French National Project[1] that aims to develop some fast and accurate robots with a large operational workspace.

In this paper, the general approach for the type synthesis of manipulators is developed based on [5]. The screw systems used in this paper are adopted from [5]. The approach is decomposed into five steps: (*i*) Classification of the leg-constraint wrench system; (*ii*) Decomposition of the constraint wrench system of a proximal and distal module; (*iii*) Type synthesis of the sub legs; (*iv*) Assembly of the sub legs and legs; (*v*) Selection of the actuated joints.

2 Two DOF Hybrid Translational Manipulators with Two Identical Legs

The general approach for the type synthesis of two *dof* hybrid translational manipulators with two identical legs are presented using the following procedure.

2.1 Step 1: Classification of the Leg-Constraint Wrench System

The moving platform of two-*dof* hybrid translational manipulators is intended to perform a two translational motion in plane ($\mathbf{x}O\mathbf{z}$). In general configuration, the twist system of the moving-platform amounts to a $2\xi_\infty-$system[2]. Therefore, the overall constraint wrench system $\mathscr{W}^c$ is a $1\zeta_0 - 3\zeta_\infty-$system[3], containing one zero-pitch wrench along $\mathbf{y}$-axis and three independent infinite-pitch wrenches. Such manipulators can be obtained by a combination of any leg-constraint wrench system of order $c^i (1 \leq c^i \leq 4)$, decomposed as follows:

- $c^i = 4 \rightarrow 1\zeta_0 - 3\zeta_\infty-$system
- $c^i = 3 \rightarrow 1\zeta_0 - 2\zeta_\infty-$system, $3\zeta_\infty-$system
- $c^i = 2 \rightarrow 1\zeta_0 - 1\zeta_\infty-$system, $2\zeta_\infty-$system

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[2] ξ_∞ denotes an infinite-pitch twist, namely, a pure translation.

[3] ζ_0 and ζ_∞ denote a zero-pitch wrench (a pure force) and an infinite-pitch wrench (a pure moment), respectively.

- $c^i = 1 \rightarrow 1\zeta_0-$system, $1\zeta_\infty-$system

However, only three types of two-*dof* hybrid translational manipulators can be properly assembled with two identical legs, namely:

- **Type 1**: $c^i = 4 \rightarrow 1\zeta_0 - 3\zeta_\infty-$system
- **Type 2**: $c^i = 3 \rightarrow 1\zeta_0 - 2\zeta_\infty-$system
- **Type 3**: $c^i = 2 \rightarrow 1\zeta_0 - 1\zeta_\infty-$system

Type 3 is included in this paper, since the combination of two parallel legs with $1\zeta_0 - 1\zeta_\infty-$system can generate a manipulator with overall constraint wrench system $1\zeta_0 - 3\zeta_\infty-$system from [1].

The overall constraint wrench system of one leg is determined by the intersection of constraint wrench systems associated with the proximal and distal modules as they are mounted in series. Therefore, the constraint wrench systems associated with the proximal and distal modules are decomposed thereafter. Note that these constraint wrench decompositions are interchangeable between proximal and distal modules.

2.1.1 Type 1: $c^i = 4 \rightarrow 1\zeta_0 - 3\zeta_\infty-$system

The overall constraint wrench system of one leg for Type 1, is $c^i = 4$, $1\zeta_0 - 3\zeta_\infty-$system. Thus, the feasible constraint wrenches are:

Proximal module:

1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$

Distal module:

1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$

At least one ζ_0 from the constraint wrench systems described above is along **y**-axis.

2.1.2 Type 2: $c^i = 3 \rightarrow 1\zeta_0 - 2\zeta_\infty-$system

The overall constraint wrench system of one leg for Type 2, is $c^i = 3$, $1\zeta_0 - 2\zeta_\infty-$system. Hence, the possible constraint wrenches are:

Proximal module:

1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2}, \zeta_{\infty 3})$
3. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2})$
4. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2})$

Distal module:

1. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_{01}, \zeta_{02}, \zeta_{\infty 1}, \zeta_{\infty 2})$
2. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W} = \text{span}(\zeta_0, \zeta_{\infty 1}, \zeta_{\infty 2})$

At least one ζ_0 from the constraint wrench systems described above is along **y**-axis.

2.1.3 Type 3: $c^i = 2 \rightarrow 1\zeta_0 - 1\zeta_\infty-$system

The overall constraint wrench system of one leg for Type 3, is $c^i = 2$, $1\zeta_0 - 1\zeta_\infty-$system. Hence, the possible constraint wrenches are:

Proximal module:
1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty1}, \zeta_{\infty2}, \zeta_{\infty3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty1}, \zeta_{\infty2}, \zeta_{\infty3})$
3. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty1}, \zeta_{\infty2})$
4. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty1}, \zeta_{\infty2})$
5. $2\zeta_0 - 1\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{01}, \zeta_\infty)$
6. $1\zeta_0 - 1\zeta_\infty-$system, $\mathscr{W}$ = span(ζ_0, ζ_∞)

Distal module:
1. $2\zeta_0 - 1\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{01}, \zeta_\infty)$
2. $1\zeta_0 - 1\zeta_\infty-$system, $\mathscr{W}$ = span(ζ_0, ζ_∞)

At least one ζ_0 from the constraint wrench systems described above is along **y**-axis.

2.2 Step 2: Decomposition of the Constraint Wrench for Proximal and Distal Modules

The next step for type synthesis of two *dof* hybrid translational manipulators is the decomposition of the constraint wrench for proximal and distal modules. It is noteworthy that Types 1, 2 and 3 have the following constraint wrench systems for proximal and distal modules:

1. $2\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty1}, \zeta_{\infty2}, \zeta_{\infty3})$
2. $1\zeta_0 - 3\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty1}, \zeta_{\infty2}, \zeta_{\infty3})$
3. $2\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{02}, \zeta_{\infty1}, \zeta_{\infty2})$
4. $1\zeta_0 - 2\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_0, \zeta_{\infty1}, \zeta_{\infty2})$
5. $2\zeta_0 - 1\zeta_\infty-$system, $\mathscr{W}$ = span$(\zeta_{01}, \zeta_{01}, \zeta_\infty)$
6. $1\zeta_0 - 1\zeta_\infty-$system, $\mathscr{W}$ = span(ζ_0, ζ_∞)

Such module constraint wrench systems emerge from the vector sum of the subleg-constraint wrench systems, due to the in-parallel arrangement of two identical sublegs. All potential identical subleg combinations are presented in Tab. 1.

2.3 Step 3: Type Synthesis of Sublegs

Once the different combinations of subleg-constraint wrench systems have been achieved (Tab. 1), the kinematic chains that instantiate proximal and distal modules can be determined. The concept of virtual chain [5] is used to generate subleg types. A virtual chain is a serial kinematic chain associated with the motion pattern of the proximal and distal modules. The number of joints in the subleg is defined based on the following mobility criterion [5]:

$$f = F + (6 - c) \tag{1}$$

where f is the number of 1-*dof* joints, F is the mobility of a single-loop kinematic chain, and c is the order of the wrench system.

2.3.1 Step 3a: Type Synthesis of Single-loop Kinematic Chains that Involve a Virtual Chain and Have a Specified Subleg-Constraint Wrench System

Type synthesis of single-loop kinematic chains is illustrated for modules with $2\zeta_0 - 3\zeta_\infty-$system and $1\zeta_0 - 3\zeta_\infty-$system only due to space limitation.

Module with $2\zeta_0 - 3\zeta_\infty-$system

This module has 1-*dof* and can be represented by a P[4]-virtual chain. Let us consider a subleg whose constraint wrench system is $2\zeta_0 - 3\zeta_\infty-$system (from Tab. 1). The number of joints that involves a P-virtual chain and has a $2\zeta_0 - 3\zeta_\infty-$system is defined as follows:

$$c^i = 5 \quad , \quad f = F + (6 - c) = 1 + (6 - 5) = 2 \text{ joints} \tag{2}$$

This single-loop kinematic chain can only be formed by one P joint, whose direction is perpendicular to the axes of $2\zeta_0$ as depicted in Fig. 1.

Module with $1\zeta_0 - 3\zeta_\infty-$system

This module has 2-*dof* and can be represented by a PP-virtual chain. Let us consider a subleg whose constraint wrench system is $1\zeta_0 - 2\zeta_\infty-$system (from Tab. 1). The number of joints that involves a PP-virtual chain and has a $1\zeta_0 - 2\zeta_\infty-$system is:

$$c^i = 3 \quad , \quad f = F + (6 - c) = 2 + (6 - 3) = 5 \text{ joints} \tag{3}$$

Such a single-loop kinematic chain contains two cases: perpendicular case and general case. For perpendicular case, all R joint axes are parallel to the axis of ζ_0 and all directions of P joints are perpendicular to the axis of ζ_0. In the general case, all R joint axes are perpendicular to the directions of ζ_∞ and coplanar with the axis of ζ_0. One example of RRR subleg is given in Fig. 2 for perpendicular case.

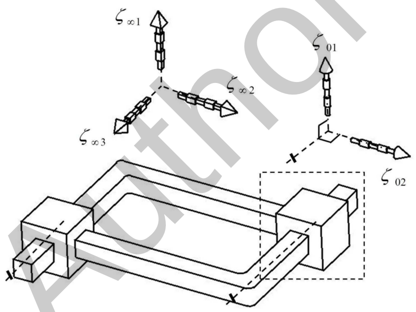

Fig. 1: P subleg with a P-virtual chain

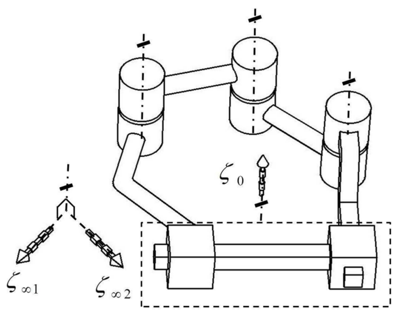

Fig. 2: RRR subleg with PP-virtual chain

[4] P stands for a prismatic joint and R stands for a revolute joint.

2.3.2 Step 3b: Generation of Types of Sublegs

The types of sublegs are derived by removing the P-virtual chain and PP-virtual chain from Figs. 1 and 2. Those types of sub legs are given in Tab. 1. Nonetheless, several types of sub legs that contain a non-invariant sub leg-wrench system (kinematic chains with varying wrench system) [5] are still kept as they can produce two-*dof* translational motions. All these types of sub legs can be assembled to generate either proximal modules or distal modules.

2.4 Step 4: Assembly of Sublegs and Legs

The assembly process of two-*dof* hybrid translational manipulators is performed for Types 1, 2 and 3. Each leg of manipulator is realized by mounting a proximal module and a distal module in series. Then, each leg becomes a hybrid manipulator and is attached to the base at one end and to the moving-platform at the other end. Therefore, the assembly process consists of two steps as explained thereafter.

2.4.1 Step 4a: Assembly of Sublegs → Proximal and a Distal Modules

The proximal and distal modules can be obtained by assembling some sublegs from Tab. 1. Nevertheless, the following conditions should be respected:

1. The overall wrench system of a module should constitute the desired wrench system, as explained in Step 1.
2. At least one translational twist generated by the module should lie in plane ($\mathbf{x}O\mathbf{z}$).

2.4.2 Step 4b: Assembly of Legs → Two-dof Hybrid Manipulator

The legs of the two-*dof* hybrid translational manipulators are synthesized by mounting in series the proximal and distal modules derived in Step. 4a. However, the following conditions should be fulfilled:

1. The wrench system of the leg should of Type 1, Type 2, or Type 3 namely, it should be a $1\zeta_0 - 3\zeta_\infty-$system, a $1\zeta_0 - 2\zeta_\infty-$system, or a$1\zeta_0 - 1\zeta_\infty-$system.
2. The linear combination of the wrench systems associated with the legs should be a $1\zeta_0 - 3\zeta_\infty-$system.

Figure 3 illustrates a novel two-*dof* hybrid translational manipulators with identical legs. This is a Type 1 mechanism that has been synthesized with the proposed approach. Each leg has a $1\zeta_0 - 3\zeta_\infty-$wrench system. Both proximal and distal modules have a $2\zeta_0 - 3\zeta_\infty-$wrench system and are composed of two RRR legs, known as *Sarrus Linkage*. This novel mechanism is named Q-Sarrus, Q standing for Quadruple.

Figure 4 depicts another mechanism synthesized with the proposed type-synthesis approach. This mechanism is named IRSBot-2 [4] and is of Type 2. Its proximal modules have a $2\zeta_0 - 3\zeta_\infty-$system and are made up of a Π joint. Its distal modules have a $2\zeta_0 - 2\zeta_\infty-$system and are composed of 2-UU kinematic chains.

Table 1: Type of Sublegs for Proximal and Distal Modules Free of Inactive Joint

c	Wrench System of PM and DM	Subleg Wrench System	Type	Note
5	$2\zeta_0 - 3\zeta_\infty$	$2\zeta_0 - 3\zeta_\infty$	Permutation P	
		$2\zeta_0 - 2\zeta_\infty$	Permutation RR	Non-invariant
		$1\zeta_0 - 2\zeta_\infty$	Permutation RRR	
			Permutation PRR	
4	$1\zeta_0 - 3\zeta_\infty$	$1\zeta_0 - 3\zeta_\infty$	Permutation PP	
		$1\zeta_0 - 2\zeta_\infty$	Permutation RRR	
			Permutation PRR	
		$1\zeta_0 - 1\zeta_\infty$	Permutation RRŔŔ	Non-invariant
	$2\zeta_0 - 2\zeta_\infty$	$2\zeta_0 - 2\zeta_\infty$	Permutation PR	
			Permutation RR	Non-invariant
		$1\zeta_0 - 2\zeta_\infty$	Permutation RRR	
			Permutation PRR	
		$2\zeta_0 - 1\zeta_\infty$	Permutation RRŔ	Non-invariant
			Permutation RŔŔ	Non-invariant
		$1\zeta_0 - 1\zeta_\infty$	Permutation RRŔŔ	Non-invariant
		$2\zeta_0$	Permutation RRŔŘ	Non-invariant
			Permutation PRŔŘ	
3	$1\zeta_0 - 2\zeta_\infty$	$1\zeta_0 - 2\zeta_\infty$	Permutation RRR	
			Permutation PRR	
			Permutation PPR	
		$1\zeta_0 - 1\zeta_\infty$	Permutation RRŔŔ	Non-invariant
	$2\zeta_0 - 1\zeta_\infty$	$2\zeta_0 - 1\zeta_\infty$	Permutation PRŔ	
		$1\zeta_0 - 1\zeta_\infty$	Permutation RRRŔ	
			Permutation PRRŔ	
			Permutation PRŔŔ	
		$2\zeta_0$	Permutation RRŔŘ	
			Permutation PRŔŘ	
2	$1\zeta_0 - 1\zeta_\infty$	$1\zeta_0 - 1\zeta_\infty$	Permutation RRRŔ	
			Permutation PRRŔ	
			Permutation PRŔŔ	
			Permutation PPRŔ	
		$1\zeta_0$	Permutation RRRŔŘ	
			Permutation RRŔŔŘ	
			Permutation RŔŔŘŘ	
			Permutation RRŔŘŘ	
			Permutation PRRŔŘ	
			Permutation PRŔŔŘ	
			Permutation PRŔŘŘ	
			Permutation PPRŔŘ	

2.5 Step 5: Selection of the Actuated Joints

Let assume that the condition of constraint wrench system is satisfied, namely, the assembly of legs applies a $1\zeta_0 - 3\zeta_\infty-$system on the moving-platfom. In a general configuration, a set of constraint wrench system, $\mathscr{W}^c$, together with an actuation wrench system, $\mathscr{W}^a$, constitute a 6-system. Ultimately, the selection of actuated

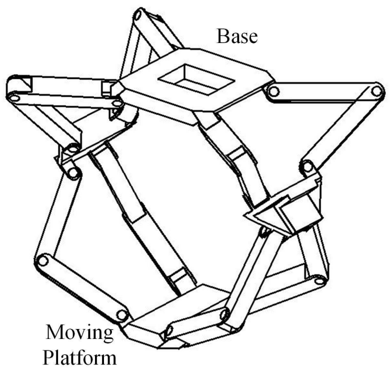

Fig. 3: Q-Sarrus

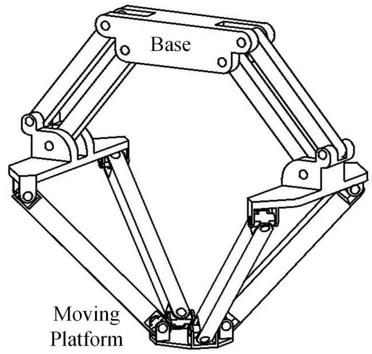

Fig. 4: IRSBot-2

joints for two-*dof* hybrid translational manipulators can be made in such a way that a basis of the actuation wrench system $\mathscr{W}^a$ contains at least two actuation forces.

3 Conclusion

A general approach has been introduced in this paper for the type synthesis of two-*dof* hybrid translational manipulators with identical legs. The proposed approach is based on the screw theory and the method described in [5]. Two types of two-*dof* hybrid translational manipulators have been highlighted with regard to the wrench system associated with their legs. Moreover, many novel two-*dof* hybrid translational manipulators have been obtained and two of them have been illustrated, namely, the Q-Sarrus and the IRSBot-2. The comparison of the synthesized manipulators with regard to their complexity and intrinsic stiffness is part of the future work.

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