A Reconfigurable 5-DoF 5-SP̲U Parallel Platform

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Abstract—*This paper presents a 5-SP̲U platform whose base leg attachments can be easily reconfigured, statically or dynamically, without altering its singularity locus. This permits to adapt the platform's geometry to particular tasks without increasing the complexity of its control. The allowed reconfigurations permit to reduce the risk of collisions between legs, or even improving the stiffness of the platform, in a given region of its configuration space. It is also shown that no architectural singularities are introduced by the proposed reconfigurations.*

Index Terms—*kinematics and dynamics of reconfiguration*

1. Introduction

The Stewart-Gough platform is defined as a 6-DoF parallel mechanism with six identical UP̲S legs [1], [2]. Although it is certainly the most celebrated parallel mechanism, platforms with a lower number of UP̲S legs are also of interest both from the theoretical and practical point of view. Kong and Gosselin refer to them as *components* as they can always be considered as rigid subassemblies in a standard Stewart-Gough platform [3].

A parallel platform with only five UP̲S legs is not architecturally singular, in general, if the attachments on the platform are aligned. The resulting platform has obvious interest, for example, as a robot manipulator with axisymmetric tool (a 5-axis milling machine is a good example).

Zhang and Song were the first to solve the forward kinematics of a general Gough-Stewart platform containing a five-legged rigid subassembly with collinear attachments in the platform and coplanar in the base [4]. They showed how the line defined by the five attachments in the platform can attain, in the general case, up to eight configurations with respect to the base for a given set of leg lengths. Husty and Karger studied the conditions for this subassembly being architecturally singular and found two algebraic conditions that must be simultaneously satisfied [5]. Borràs and Thomas recently showed that the location of the attachments determine a one-to-one correspondence between points in the line and lines in the base plane [6]. They showed that, if the attachments on the plane are moved along their corresponding lines, the singularities of the platform remain unaltered. This theoretical result is exploited here in a design in which these lines are radially arranged passing though the vertices of a regular pentagon. The base leg attachments of such a robot can be easily reconfigured so that its geometry can be modified to adapt it to particular tasks. This includes the possibility of reducing the risk of collisions between legs, or even improving the stiffness of the robot, in a given region of its workspace. Moreover, if the possible locations for the attachments are limited to the radii passing through the vertices of the pentagon (that is, to semi-lines instead of the possible whole lines), it is shown that no architectural singularities can be introduced.

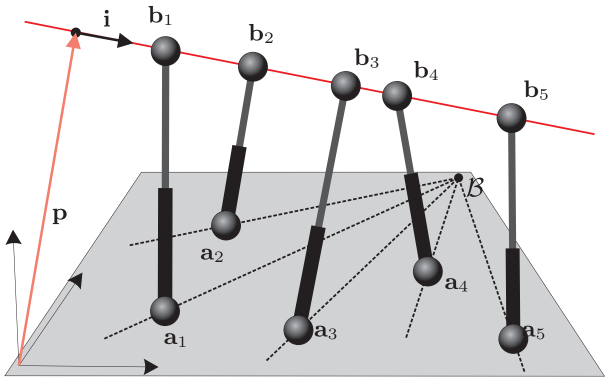

Fig. 1. A 5-DoF 5-SP̲U parallel manipulator with collinear attachments in the platform and coplanar in the base.

The investment cost to purchase a parallel robot for a particular task could be worth if there is the possibility to reconfigure it for another task. Static and dynamic reconfigurations can be distinguished [7], [8], [9]. Static reconfiguration denotes a manual rebuilding of a robot which might lead to a robot with new kinematic characteristics and a new workspace. In this work, we follow the less radical approach in which some leg attachments can be rearranged so that the geometry of the robot is modified but its singularity locus remains unaltered. This kind of reconfigurations can be carried out not only statically but also dynamically without increasing significantly the control of the platform because a singularity-invariant reconfiguration guarantees the existence of a one-to-one mapping between the leg lengths of the robot before and

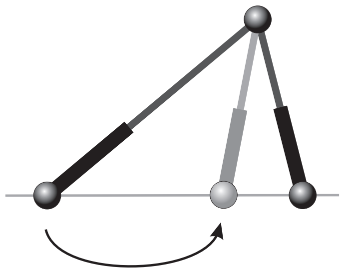

Fig. 2. When two legs in a Stewart-Gough platform share an attachment, the other two attachments can be reconfigured without altering the singularities of the platform.

after the reconfiguration.

The simplest reconfiguration in the location of the attachments of a Stewart-Gough platform without changing its singularity locus arises when two legs share a multiple spherical joint, as shown in Fig. 2. In this particular case, the other two attachments can be displaced along the line they define provided that both legs are not made coincident, in which case a trivial architectural singularity is introduced [10]. In more complex configurations, like the one considered herein, the possible singularity-invariant reconfigurations, if any, follow much more complex rules and the configurations that lead to architectural singularities are non obvious.

This paper is divided into two main parts. In the first one, the kinematics of a general 5-SPU parallel platform is reviewed. Particular attention is paid to the characterization of both those configurations of the attachments that leave the platform singularity locus invariant and those that introduce architectural singularities. In the second part, based on these theoretical results, a particular reconfigurable architecture is proposed an analyzed.

2. Kinematics of the 5-DoF 5-SPU parallel mechanism

Let us consider the 5-DoF 5-SPU parallel mechanism appearing in Fig. 1. We assume that no four attachments on the plane are collinear, otherwise this subassembly would contain a four-legged rigid subassembly that can be studied separately [11].

The attachments on the plane have coordinates $\mathbf{a}_i = (x_i, y_i, 0)$, for $i = 1, \ldots, 5$. The pose of the line with respect to the plane can be described by the position vector $\mathbf{p} = (p_x, p_y, p_z)$ and the unit vector $\mathbf{i} = (u, v, w)$ in the direction of the line. Thus, the coordinates of the attachments on the line, expressed in the base reference frame, can be written as $\mathbf{b}_i = \mathbf{p} + z_i\mathbf{i}$.

It has been shown that the singularities of this mechanism correspond to those configurations in which the determinant of the following matrix is zero [6]:

$$\mathbf{T} = \begin{pmatrix} wp_z & wk_1 & wk_2 & -p_zk_1 & -p_zk_2 & -w^2 \\ z_1 & x_1 & y_1 & x_2z_2 & y_1z_1 & 1 \\ z_2 & x_2 & y_2 & x_2z_2 & y_2z_2 & 1 \\ z_3 & x_3 & y_3 & x_3z_3 & y_3z_3 & 1 \\ z_4 & x_4 & y_4 & x_4z_4 & y_4z_4 & 1 \\ z_5 & x_5 & y_5 & x_5z_5 & y_5z_5 & 1 \end{pmatrix}, \quad (1)$$

where $k_1 = (p_zu - p_xw)$ and $k_2 = (p_zv - p_yw)$.

In other words, when

$$\det(\mathbf{T}) = C_1wp_z + C_2w(p_zu - p_xw) + C_3w(p_zv - p_yw) + C_4p_z(p_xw - p_zu) + C_5p_z(p_yw - p_zv) - C_6w^2 = 0, \quad (2)$$

where C_i, for $i = 1, \ldots 6$, is the cofactor of the element i of the first row of $\mathbf{T}$.

2.1. Singularity-invariant reconfigurations

Let us consider the multilinear equation

$$ax + by + cz + dxz + eyz + f = 0, \quad (3)$$

which implicitly defines a hypersurface in the space defined by $(x, y, z) \in \mathbb{R}^3$. Let us assume that the attachments of leg i of our platform define a point, (x_i, y_i, z_i), in this hypersurface. Since we have five legs (i.e., five points in this hypersurface), the coefficients a, b, c, e, and f are uniquely determined. Actually, (3) can be explicitly expressed in terms of these five points as:

$$\begin{vmatrix} z & x & y & xz & yz & 1 \\ z_1 & x_1 & y_1 & x_1z_1 & y_1z_1 & 1 \\ z_2 & x_2 & y_2 & x_2z_2 & y_2z_2 & 1 \\ z_3 & x_3 & y_3 & x_3z_3 & y_3z_3 & 1 \\ z_4 & x_4 & y_4 & x_4z_4 & y_4z_4 & 1 \\ z_5 & x_5 & y_5 & x_5z_5 & y_5z_5 & 1 \end{vmatrix} = 0, \quad (4)$$

or, alternatively, as

$$C_1z + C_2x + C_3y + C_4xz + C_5yz + C_6 = 0 \quad (5)$$

where C_i are the cofactors referred in the previous section, i.e., the same coefficients appearing in (2).

Now, observe that, if we substitute one of the five points by any other point in the hypersurface, the polynomial equation (5) will have the same set of coefficients up to a scalar multiple. Then, if we change the attachments of one leg so that the coordinates of the new attachments satisfy (4), the coefficients of the singularity polynomial in (2) remain the same up to a constant multiple and, as a consequence, its root locus remains invariant. This simple observation gives us the clue to change the attachments locations without changing the platform singularity locus.

Equation (4) implicitly defines a one-to-one correspondence between points in the line and lines in the plane. Indeed, given an attachment on the plane with coordinates $(x, y, 0)$, we conclude from equation (4) that there is a unique corresponding attachment on the line with coordinate z. On the way round, given an attachment on the line, a line is defined in the plane.

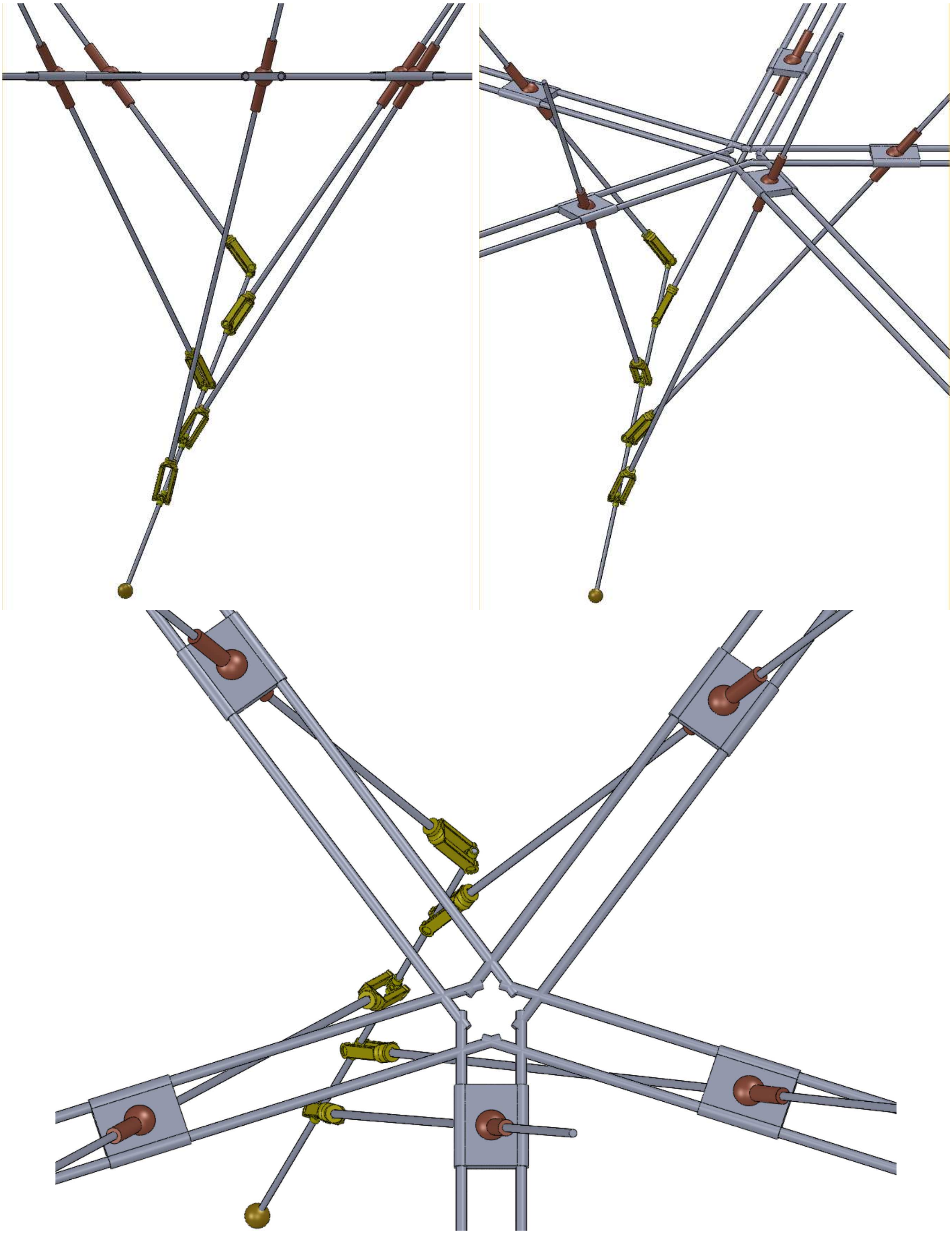

Fig. 3. Three different views of the proposed 5-DoF 5-S̲PU parallel platform in the same configuration. The base attachments can be independently moved along radial guides without altering the singularity locus of the platform.

It is important to realize that, as z varies, the generated lines intersect at a single point whose coordinates are:

$$\mathcal{B} = \left(\frac{C_3C_1 - C_6C_5}{C_2C_5 - C_4C_3}, -\frac{C_2C_1 - C_4C_6}{C_2C_5 - C_4C_3}, 0 \right). \qquad (6)$$

Each line on the base passing through $\mathcal{B}$ is called a $\mathcal{B}$-line (Fig. 1).

Point $\mathcal{B}$ is located at the origin of the reference frame if the following design conditions are satisfied:

$$C_3C_1 = C_6C_5, \quad C_2C_1 = C_4C_6, \quad \text{and} \quad C_2C_5 \neq C_4C_3. \quad (7)$$

Summing up, the coordinates of the attachments of leg l_i are given by (x_i, y_i, z_i), where (x_i, y_i) are the coordinates of the attachments on the base and z_i the local coordinate of the attachment on the line. Then, if the base attachments are moved along their corresponding $\mathcal{B}$-lines, the resulting new attachments (x'_i, y'_i, z_i) satisfy (4), and the resulting singularity polynomial is the same, up to a scalar factor, which does not modify its zeros provided that this scalar factor is different form zero.

2.2. Assembly modes

In order to obtain the assembly modes of a 5-SPU parallel platform, it is possible to apply the procedure proposed in [4]. Next, the main steps of this procedure are summarized.

The leg lengths of the platform can be expressed as $l_i^2 = \|\mathbf{b}_i - \mathbf{a}_i\|$, for $i = 1, \ldots, 5$. If we subtract from these expressions the equation $u^2 + v^2 + w^2 = 1$, quadratic terms in u, v and w cancel yielding

$$\begin{aligned} \tfrac{1}{2}(p_x^2 + p_y^2 + p_z^2) + z_i t - x_i p_x - y_i p_y - x_i z_i u - y_i z_i v \\ + \tfrac{1}{2}(x_i^2 + y_i^2 + z_i^2 - l_i^2) = 0, \end{aligned} \qquad (8)$$

for $i = 1, \ldots, 5$, where $t = \mathbf{p} \cdot \mathbf{i}$. Subtracting the equation for $i = 1$ from the others, quadratic terms in p_x, p_y and p_z cancel as well, and the resulting equations can be written in matrix form as:

$$\begin{pmatrix} x_2 - x_1 & y_2 - y_1 & x_2z_2 - x_1z_1 & y_2z_2 - y_1z_1 \\ x_3 - x_1 & y_3 - y_1 & x_3z_3 - x_1z_1 & y_3z_3 - y_1z_1 \\ x_4 - x_1 & y_4 - y_1 & x_4z_4 - x_1z_1 & y_4z_4 - y_1z_1 \\ x_5 - x_1 & y_5 - y_1 & x_5z_5 - x_1z_1 & y_5z_5 - y_1z_1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ u \\ v \end{pmatrix} = \begin{pmatrix} (z_2 - z_1)t + N_2 \\ (z_3 - z_1)t + N_3 \\ (z_4 - z_1)t + N_4 \\ (z_5 - z_1)t + N_5 \end{pmatrix}, \qquad (9)$$

where $N_i = 1/2(x_i^2 + y_i^2 + z_i^2 - l_i^2 - x_1^2 - y_1^2 - z_1^2 + l_1^2)$.

The system determinant is C_1, that is, the value of the cofactor of the (1,1) entry of $\mathbf{T}$ in (1). If this cofactor is zero, we can always choose as parameter either p_x, p_y, u or v to reformulate the above linear system. Since not all cofactors can be zero, otherwise the platform becomes architecturally singular, we can always find a non-singular linear system by choosing the right parameter.

The solution of the above system, using Crammer's rule, can be written in terms of the cofactors of the first row of $\mathbf{T}$ as:

$$\left\{ p_x = \frac{E_2 - C_2t}{C_1}, p_y = \frac{E_3 - C_3t}{C_1}, u = \frac{E_4 - C_4t}{C_1}, v = \frac{E_5 - C_5t}{C_1} \right\}, \qquad (10)$$

where E_i results from substituting the $(i-1)$th column of C_1 by $(N_2, N_3, N_4, N_5)^T$.

Now, since $t = \mathbf{p} \cdot \mathbf{i}$,

$$(p_z w)^2 = (t - p_x u - p_y v)^2. \qquad (11)$$

From equation $u^2 + v^2 + w^2 = 1$, and equation (8) for $i = 1$, we have

$$\begin{aligned} w^2 &= 1 - u^2 - v^2, \\ p_z^2 &= 2(-z_1 t + x_1 p_x + y_1 p_y + z_1 y_1 v + z_1 x_1 u) \\ &\quad - p_x^2 - p_y^2 - x_1^2 - y_1^2 - z_1^2 + l_1^2. \end{aligned} \qquad (12)$$

Then, substituting the above expressions for w^2 and p_z^2, and the values of p_x, p_y, u and v in (10), in equation (11), a fourth-degree polynomial in t is obtained. For each root of this polynomial, when substituted in equations (11) and (12), we obtain two values for $\{z, w\}$. Thus, a total number of eight assembly modes is obtained.

3. Architectural singularities

When a manipulator is architecturally singular, it is singular in all the points of its configuration space [12]. It is important to characterize architectural singularities to avoid them in the design process but, when working with reconfigurable robots, such characterization become crucial to design the leg rearrangements.

Architectural singularities of the presented manipulator were fully characterized in [13], where it was shown that this kind of singularities arise either when

(a) four attachments in the plane are collinear, or
(b) a base attachment is located on the conic formed by the other four base attachments and point $\mathcal{B}$.

When all base attachments are located on different $\mathcal{B}$-lines (as in the proposed manipulator), no other architectural singularities can arise (see [13] for details).

Let us consider the conic passing through any four base attachments, with coordinates (x_i, y_i), (x_j, y_j), (x_k, y_k), and (x_l, y_l), and point $\mathcal{B}$ (in our case, the origin of the reference frame). Its equation can be expressed as:

$$\begin{vmatrix} x^2 & xy & y^2 & x & y \\ x_i^2 & x_iy_i & y_i^2 & x_i & y_i \\ x_j^2 & x_jy_j & y_j^2 & x_j & y_j \\ x_k^2 & x_ky_k & y_k^2 & x_k & y_k \\ x_l^2 & x_ly_l & y_l^2 & x_l & y_l \end{vmatrix} = 0. \qquad (13)$$

Provided that no four attachments are collinear, the remaining base attachment can be freely moved along its associated line without introducing an architectural singularity if, and only if, it is not located on this conic [13].

In the next section, it is shown that, by arranging the lines radially and constraining the attachments locations to lie on half of these lines, it is possible to completely avoid such kind of singularities.

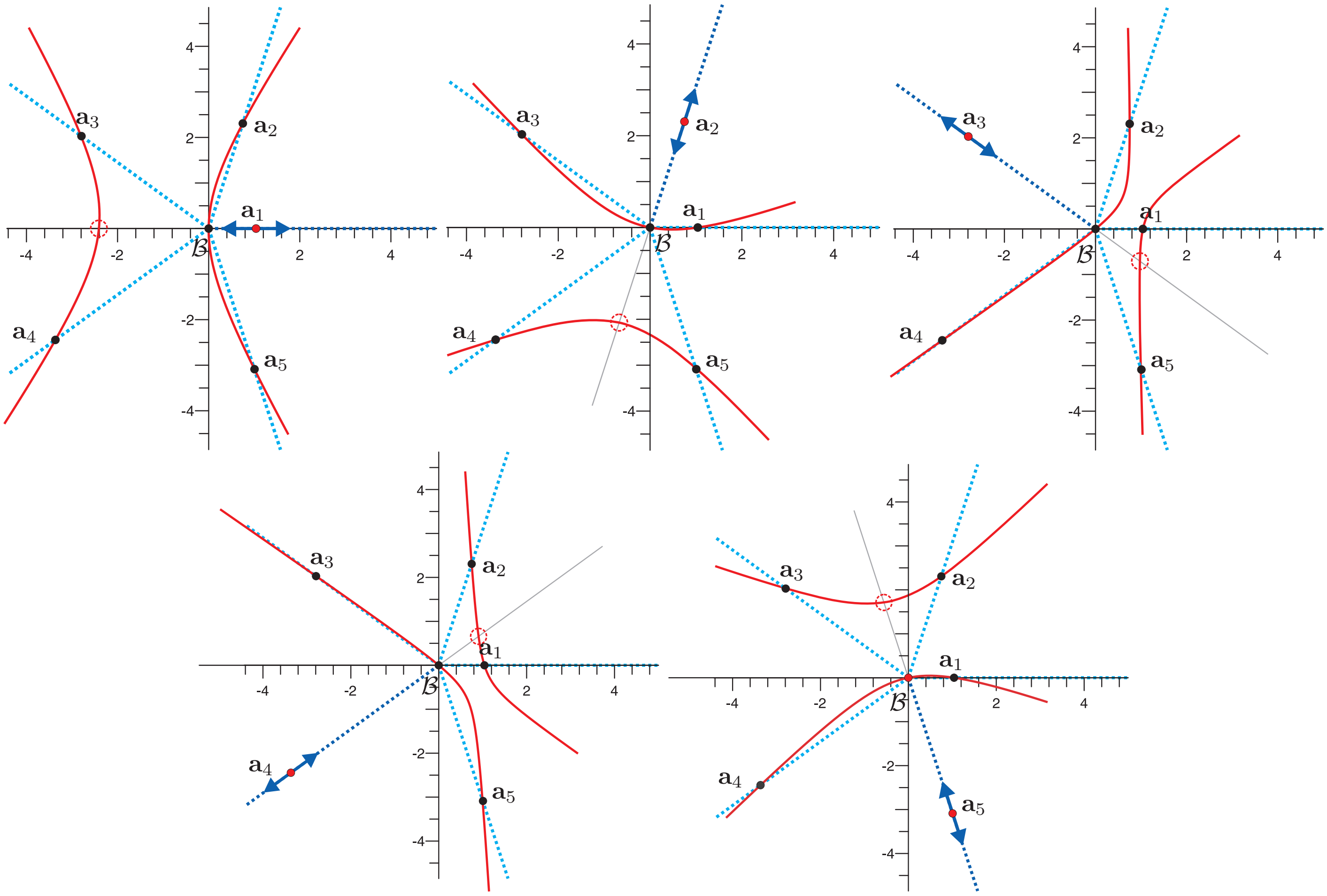

Fig. 4. The base attachments in the proposed architecture can be independently moved along radial guides (shown in dotted lines) without leading to architectural singularities because each attachment cannot be located on the conic (shown in solid lines) defined by the other four attachments and the origin. Moreover, no four attachments can be made collinear. If the attachments could move along their whole associated $\mathcal{B}$-lines, they could overpass the origin giving rise to the possibility of intersect the conic (shown in dotted circles) thus leading to architectural singularities. Limiting the possible reconfigurations to radial guides, this possibility is totally excluded.

4. The proposed architecture

Let us consider the 5-SP̲U parallel mechanism in Fig. 3 whose attachments in their local reference frames are given in Table I. The value of the resulting cofactors for the elements of the first row of $\mathbf{T}$ are:

$$C_1 = 0,$$
$$C_2 = 1130.928486,$$
$$C_3 = -532.2016037,$$
$$C_4 = -665.2520496,$$
$$C_5 = 66.52518034,$$
$$C_6 = 0.$$

Since $C_1 = C_6 = 0$, the design conditions in (7) are satisfied. As a consequence, point $\mathcal{B}$, according to equation (6), is located at the origin of the base reference frame, and the relationship between the coordinates of the attachments given by Eq. (4) simplifies to

$$(C_4 x_i + C_5 y_i) z_i + C_2 x_i + C_3 y_i = 0. \tag{14}$$

Likewise, the expression for the singularity locus, given by the root locus of the polynomial in Eq. (2), can also be simplified resulting in the set of configurations satisfying

$$(C_4 p_z - C_2 w)(p_x w - p_z u) + (C_5 p_z - C_3 w)(p_y w - p_z v) = 0. \tag{15}$$

For fixed orientations, the singularity locus is a ruled surface, and for fixed positions it is a quadratic curve on the sphere (see Fig. 5 for two particular cases).

When moving the base attachments along their associated $\mathcal{B}$-lines, we must avoid possible architectural singularities. As explained in the previous section, this is achieved by ensuring that each base attachment does not lie in the conic defined by the other four attachments and point $\mathcal{B}$ (Fig. 4).

If the possible locations for the attachments are limited to the radii passing through the vertices of a pentagon, any four base attachments and the origin define, independently of the

TABLE I
COORDINATES OF THE ATTACHMENTS IN THEIR LOCAL REFERENCE FRAMES ($\mathbf{a}_i = (x_i, y_i, 0)$ AND $\mathbf{b}_i = \mathbf{p} + z_i\mathbf{i}$) FOR THE PROPOSED ARCHITECTURE.

i	(x_i, y_i)	z_i
1	$(3, 0)$	$\frac{17}{10}$
2	$\left(3\cos\left(\frac{2\pi}{5}\right), 3\sin\left(\frac{2\pi}{5}\right)\right)$	$\dfrac{17\cos\left(\frac{2\pi}{5}\right) - 8\sin\left(\frac{2\pi}{5}\right)}{10\cos\left(\frac{2\pi}{5}\right) - \sin\left(\frac{2\pi}{5}\right)}$
3	$\left(-3\cos\left(\frac{\pi}{5}\right), 3\sin\left(\frac{\pi}{5}\right)\right)$	$\dfrac{8\sin\left(\frac{\pi}{5}\right) + 17\cos\left(\frac{\pi}{5}\right)}{10\cos\left(\frac{\pi}{5}\right) + \sin\left(\frac{\pi}{5}\right)}$
4	$\left(-3\cos\left(\frac{\pi}{5}\right), -3\sin\left(\frac{\pi}{5}\right)\right)$	$\dfrac{17\cos\left(\frac{\pi}{5}\right) - 8\sin\left(\frac{\pi}{5}\right)}{10\cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{\pi}{5}\right)}$
5	$\left(3\cos\left(\frac{2\pi}{5}\right), -3\sin\left(\frac{2\pi}{5}\right)\right)$	$\dfrac{8\sin\left(\frac{2\pi}{5}\right) + 17\cos\left(\frac{2\pi}{5}\right)}{\sin\left(\frac{2\pi}{5}\right) + 10\cos\left(\frac{2\pi}{5}\right)}$

location of the four attachments on their radii, a non-convex planar set of points. As a consequence, the conic passing through them all is a hyperbola that does not intersect the radius associated with the remaining attachment but at the origin (Fig. 4). Thus, by allowing the attachments to independently slide along radial guides, the obtained reconfigurations can never be architecturally singular.

5. THE EFFECT OF RECONFIGURING

J-P. Merlet showed that non of the dexterity indices defined for serial robots, such as the condition number or the manipulability index, are appropriate for parallel robots [14].

In the example presented in [14], it is shown how the determinant of the inverse Jacobian matrix can be used as a measure of the maximal positioning errors. Therefore, here the value of $\det(\mathbf{T})$ can be taken as a valid index to analyze the variation of the dexterity of the manipulator when performing reconfigurations.

As it has been proved in Section 2.1, for any given pose of the manipulator, when performing a rearrangement of the leg attachments along their radial guides, the singularity polynomial (15) is multiplied by a constant factor. An analytical expression for this factor is next obtained.

Consider as parameters the distances of each attachment to the origin, say $\lambda_i = \|\mathcal{B} - \mathbf{a}_i\|$ (see Fig.6). Using a symbolic algebraic manipulator it can be checked that, when rearranging leg attachments along their guides, (15) is multiplied by the factor

$$\xi(\lambda_1, \ldots, \lambda_5) = \frac{1}{135(2\cos(\frac{\pi}{5}) - 3)} \cdot$$

$$\Big(2(\cos(\tfrac{\pi}{5}) - 1)(\lambda_1(\lambda_2\lambda_4 + \lambda_3\lambda_4 + \lambda_3\lambda_5) + \lambda_2(\lambda_3\lambda_5 + \lambda_4\lambda_5))$$
$$-(\lambda_1(\lambda_2\lambda_3 + \lambda_2\lambda_5 + \lambda_4\lambda_5) + \lambda_3(\lambda_2\lambda_4 + \lambda_4\lambda_5))\Big).$$

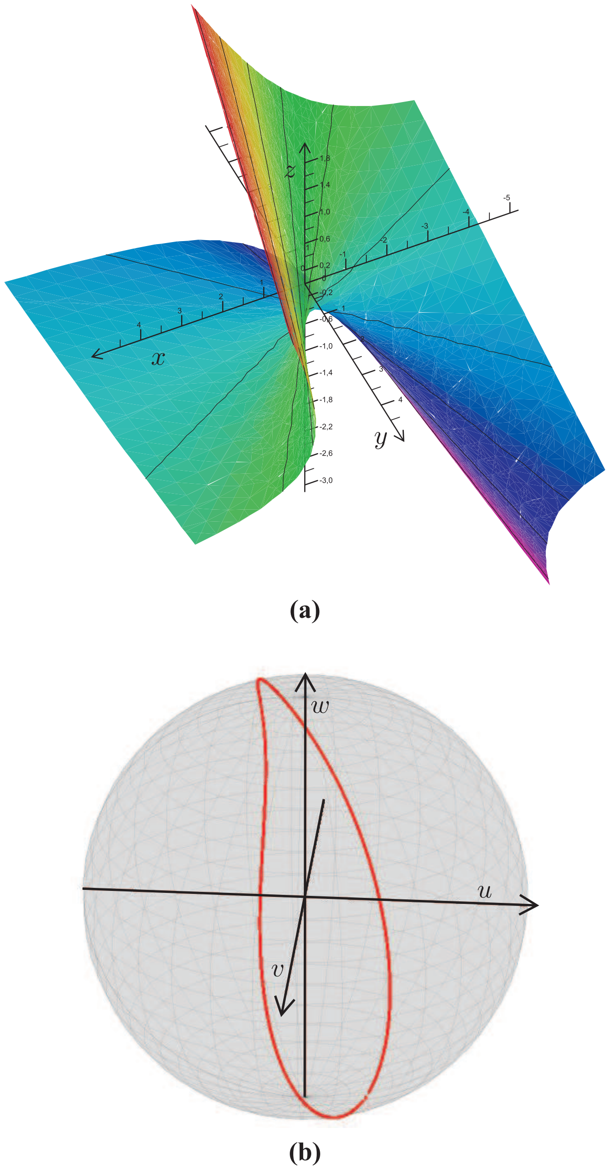

Fig. 5. Singularity locus for **(a)** a fixed orientation ($u = \frac{\sqrt{3}}{3}$, $v = \frac{\sqrt{3}}{3}$, $w = \frac{\sqrt{3}}{3}$), and **(b)** a fixed position ($p_x = 1, p_y = -1, p_z = -4$).

It can be checked that this factor cannot be zero for any positive value of λ_i, $i = 1, \ldots, 5$, which is consistent with the fact that no architectural singularity can be attained with the proposed reconfigurations.

In the initial location of the base attachments given by the coordinates in Table I, $\lambda_i = 3$, for $i = 1, \ldots, 5$. Then, $\xi(\lambda_1, \ldots, \lambda_5) = 1$. When moving one attachment along its

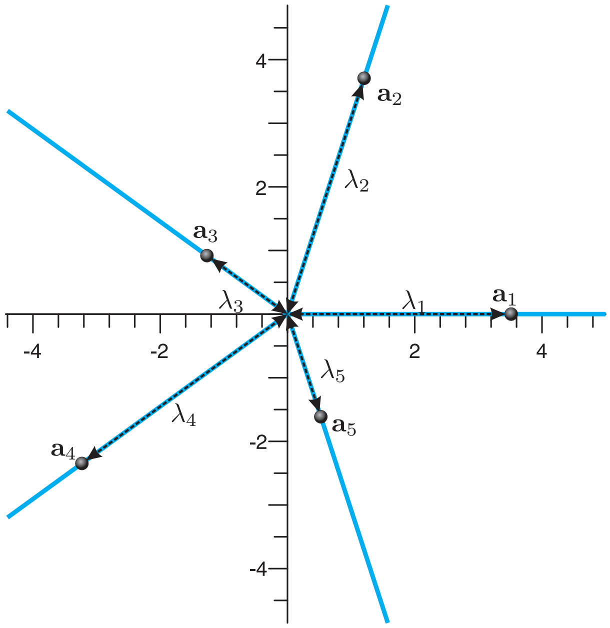

Fig. 6. The value of $\det(\mathbf{T})$ is multiplied by a constant that only depends on the distances of the attachments to the origin.

guide, the value of this factor increases linearly with the distance of the attachment to the origin.

In conclusion, for a given pose of the manipulator, the influence of a reconfiguration on the variation of the dexterity can be measured using ξ. Thus, for a given position of the base attachments, the global dexterity of the workspace can be improved by reconfiguring the robot.

6. Conclusion

This paper presents a 5-SPU platform whose attachments on the base can be independently displaced radially without modifying the singularity locus of the platform. This permits changing the geometry of the platform, statically or dynamically, to adapt it to different tasks and, as a consequence, increasing its flexibility. The resulting reconfigurations would permit to avoid some leg collisions, thus enlarging the workspace of the platform, or even to modify the stiffness of the robot in a given region of its workspace.

Since the resulting reconfigurations do not modify the singularity locus of the platform, it is possible to guarantee that there will always be a one-to-one mapping between the leg lengths of the robot before and after the reconfiguration. This obviously simplifies the control of the robot even when the reconfigurations are performed dynamically.

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