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A Distance-Based Formulation of the Octahedral Manipulator Kinematics

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ABSTRACT

In most practical implementations of the Gough-Stewart platform, the octahedral form is either taken as it stands or is approximated. The kinematics of this particular instance of the Gough-Stewart platform, commonly known as the octahedral manipulator, has been thoughtfully studied. It is well-known, for example, that its forward kinematics can be solved by computing the roots of an octic polynomial and that its singularities have a simple geometric interpretation in terms of the intersection of four planes in a single point. In this paper, using a distance-based formulation, it is shown that this octic polynomial can be straightforwardly derived and a whole family of platforms kinematically equivalent to the octahedral manipulator is obtained. Two Gough-Stewart parallel platforms are said to be kinematically equivalent if there is a one-to-one correspondence between their squared leg lengths for the same configuration of their moving platforms with respect to their bases. If this condition is satisfied, it can be easily shown that both platforms have the same assembly modes and their singularities, in the configuration space of the moving platform, are located in the same place. Actually, both consequences are two faces of the same coin.

Keywords: octahedral manipulator, position analysis, forward kinematics, distance-based formulations, Cayley-Menger determinants, trilateration.

1 INTRODUCTION

The Stewart-Gough platform consists of a fixed base and a moving platform connected by six ball-ended extensible legs [1]. While the kinematics analysis of the general case, that is, that in which the ball-and-socket joints are arbitrarily located on the base and the platform, is very complex, it gets greatly simplified when some of these joints, either on the base or the platform, coalesce and/or are made to be collinear or coplanar. In other words, placing constraints on the geometrical structure of the general Stewart-Gough platform offers the opportunity for obtaining a simple formulation for its forward kinematics and a simple geometrical interpretation for its singularities. The maximum simplification is obtained when all the ball-and-socket joints coalesce into only three multiple spherical joints both in the base and the platform. Only three possibilities arise whose topologies are represented in Fig. 1. These three platforms are known as the three 3-3 Stewart-Gough platforms for obvious reasons.

One of the 3-3 Stewart-Gough platforms consists of six double-ball-ended legs thereby forming a zigzag pattern. For symmetry reasons, this topology is either taken as it stands or is approximated in most implementations of the Stewart-Gough platform. Since the 12 lines that join the double-ball-joints can be interpreted as the eight triangular faces of an octahedron, the term *octahedral manipulator* was coined in [2] to name it.

Clearly, it is advantageous to have multiple spherical joints sharing the same center of rotation in a parallel manipulator to simplify its kinematics. However, difficulties always arise in constructing such spherical joints. There have been several attempts to construct them (see [3] and the references therein), but none of them use off-the-self mechanical elements. Another disadvantage of this kind of joints is that the range of action of the leg actuators is reduced because of the risk of mechanical interference. In [4], kinematic substitutions are introduced to provide a way around this problem where is it shown, for example, that the

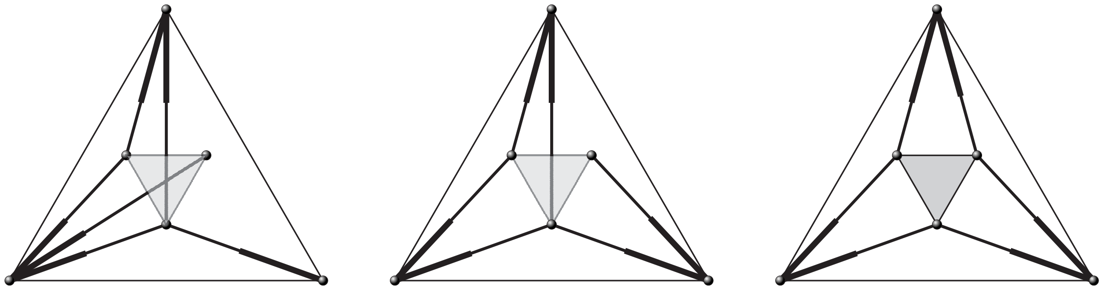

Figure 1. The three possible topologies for a 3-3 Stewart-Gough platform. The rightmost one corresponds to the octahedral manipulator.

manipulator appearing in Fig. 2(a), that avoids the double-ball-joints in the base, is kinematically equivalent to the octahedral manipulator. This particular arrangement of joints is also known as the triple arm mechanism [5].

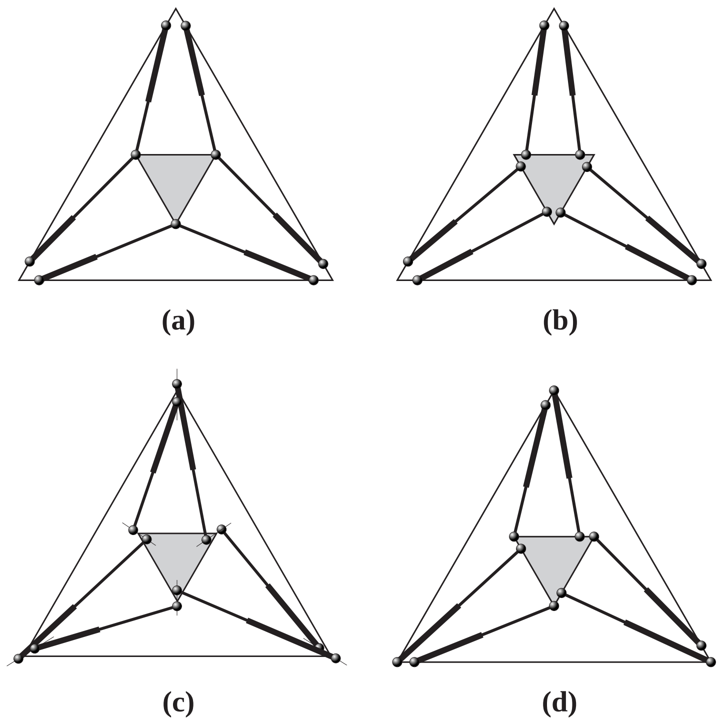

Figure 2. The triple arm mechanism **(a)**, the standard approximation to the octahedral manipulator that avoids all double-ball-joints **(b)**, the Stoughton-Arai approximation intended to also improve the dexterity of the manipulator **(c)**, and the Griffis-Duffy modification **(d)**.

Most implementations avoid the difficulty of constructing multiple spherical joints by approximating them with a collection of single spherical joints with small offsets between the centers of rotation of the links, as shown in Fig. 2(b). Such offsets change the kinematics of the mechanism, resulting in one of two possible problems, as pointed out in [3]. First, if the offsets are included in the kinematics of the mechanism, the kinematic equations may become very complex and thus very difficult to solve. Second, if the offsets are neglected, thus simplifying the kinematic equations, errors arise. These errors may have a significant impact in precision applications, or in manipulators such as the Tetrobot [6] that consists in stacking multiple octahedral manipulators resulting in the accumulation of errors if such offsets are introduced and neglected.

The modification of the octahedral manipulator proposed by Stoughton and Arai consist in separating the six double-ball joints alternatively inward and outward radially [7], as shown in Fig. 2(c). Each double-ball-joint is separated by the same amount into a pair of spherical joints whose centers are equidistant to the original center. In this paper, we show that, if this six double-ball joints are alternatively separated not radially but following the edges of the base and platform triangles, as shown in Fig. 2(d), the resulting manipulator is kinematically equivalent to the original octahedral one. This fact was already acknowledged by Griffis and Duffy in [8] (without giving an explicit formulation) but it has been overlooked, even by the same authors, in subsequent publications where alternatives to avoid these joints are discussed [4]. The formal prove to this fact can be easily derived through a formulation of the kinematics of the octahedral manipulator fully expressed in terms of distances.

This paper is organized as follows. Section 2 summarizes some basic facts about Cayley-Menger determinants and trilateration that are used throughout this paper. Section 3 briefly reviews the proposed approaches to solve the forward kinematic of the octahedral manipulator and shows how its characteristic octic polynomial can be easily obtained using a distance-based formulation. Then, using this formulation, it is shown that, when there is an affine relationship between the squared leg lengths of two platforms, a one-to-one-correspondence exits between the coefficients of their characteristic polynomials or, equivalently, between the solutions to their forward kinematics problems. Section 4 deals with the singularities of the octahedral manipulator and the relationship between the singularity locus of two platforms whose squared leg lengths are affine linearly related. In Section 5, the geometric transformations that lead to affine relationship between the squared of the leg lengths is derived. A whole family of parallel platforms kinematically equivalent to the octahedral manipulator is thus obtained. One of its members has no double-ball-joints. Section 6 analyzes this case through an example. Finally, Section 7 summarizes the main results.

2 CAYLEY-MENGER DETERMINANTS AND TRILATERATION

Let P_i and $\mathbf{p}_i$ denote a point and its position vector in a given reference frame, respectively. Then, let us define

$$D(i_1,\ldots,i_n;j_1,\ldots,j_n) = 2\left(-\frac{1}{2}\right)^n \begin{vmatrix} 0 & 1 & \ldots & 1 \\ 1 & s_{i_1,j_1} & \ldots & s_{i_1,j_n} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & s_{i_n,j_1} & \ldots & s_{i_n,j_n} \end{vmatrix}, \tag{1}$$

with $s_{i,j} = \|\mathbf{p}_i - \mathbf{p}_j\|^2$, which is independent from the chosen reference frame. This determinant is known as the *Cayley-Menger bi-determinant* of the point sequences $P_{i_1},\ldots,P_{i_n}$, and $P_{j_1},\ldots,P_{j_n}$. When the two point sequences are the same, it will be convenient to abbreviate $D(i_1,\ldots,i_n;i_1,\ldots,i_n)$ by $D(i_1,\ldots,i_n)$, which is simply called the *Cayley-Menger determinant* of the involved points.

It can be shown that the Cayley-Menger determinant $D(1,\ldots,n)$ is $((n-1)!)^2$ times the squared hypervolume of the simplex defined by $P_1,\ldots,P_n$ in $\mathbb{R}^{n-1}$. Then, when working in $\mathbb{R}^n$, all Cayley-Menger determinants involving more than $n+2$ points necessarily vanish.

Many geometric problems have an elegant and straightforward solution when expressed in terms of Cayley-Menger determinants. The trilateration problems is one of them. Given three points in space, say P_1, P_2, and P_3, the trilateration problem consists in finding the location of another point, say P_4, whose distance to these three points is known. According to Fig. 3, given the position vectors $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$, and the distances l_1, l_2, and l_3, it can be proved that [9]:

$$\mathbf{p}_{1,4} = \frac{1}{D(1,2,3)}\left(-D(1,2,3;1,3,4)\mathbf{p}_{1,2} + D(1,2,3;1,2,4)\mathbf{p}_{1,3} \pm \sqrt{D(1,2,3,4)}(\mathbf{p}_{1,2}\times\mathbf{p}_{1,3})\right), \tag{2}$$

where $\mathbf{p}_{i,j} = \mathbf{p}_j - \mathbf{p}_i$.

In the next section, we show how the forward kinematics problem of the octahedral manipulator can also

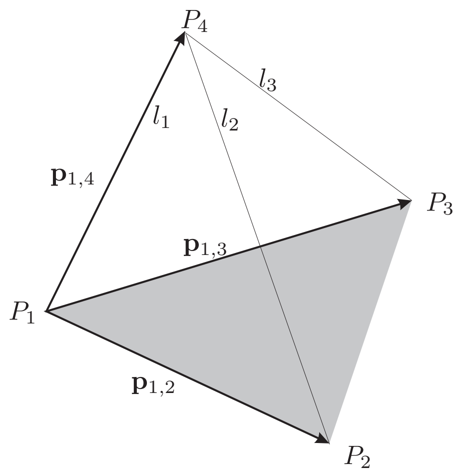

Figure 3. The trilateration problem is to find the location of a point, say P_4, given its distances to the vertices of a triangle, say $P_1P_2P_3$, whose location is known.

straightforwardly solved when formulated in terms of Cayley-Menger determinants and trilaterations.

3 FORWARD KINEMATICS OF THE OCTAHEDRAL MANIPULATOR

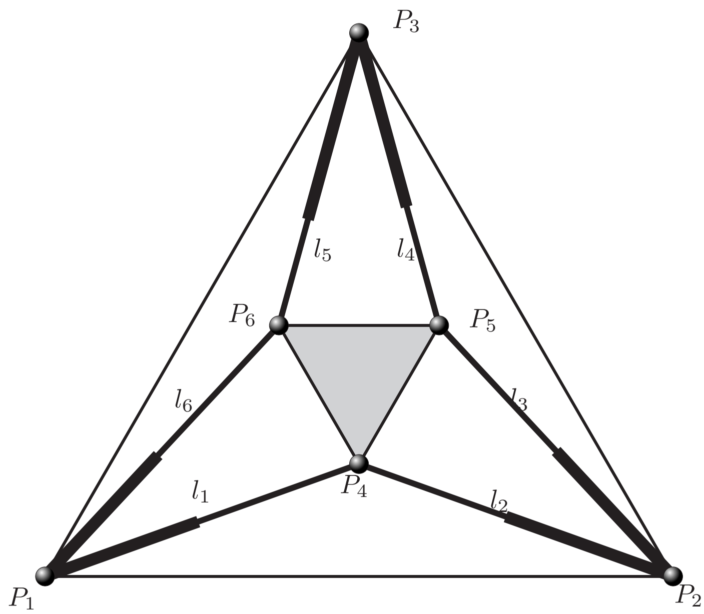

Figure 4. Octahedral manipulator and associated notation.

The forward kinematics problem is to find all poses of the platform (relative to the base) that are compatible with the six specified leg lengths. No closed-form solution to this problem is known for the octahedral manipulator, but during the late 80's and early 90's several researchers successfully addressed it giving numerical procedures that involve finding the roots of an eighth-degree univariate polynomial. In [10], Nanua *et al.* derived such a polynomial through resultant elimination and tangent-half-angle substitution techniques. A similar result, based on three spherical four-bar linkages, was obtained by Griffis and Duffy in [11]. An alternative method was also developed by Innocenti and Parenti-Castelli in [12]. In all cases the polynomial variable is the tangent of one-half the angle defined by the plane supporting $P_1P_2P_4$ (alternatively

$P_2P_3P_5$, or $P_3P_1P_6$) and the base plane. More recently, Akçali and Mutlu revisited the problem —also using resultant elimination and tangent-half-angle substitution techniques— with the aim of reducing the computational cost of evaluating the resulting univariate polynomial [13]. Finally, it is worth to mention that the forward kinematics of the octahedral manipulator has also been solved locally using Newton-Raphson iterative schemes. Liu *et al.* [14], Ku [15], and Song and Kwon [16] propose different formulations to this end.

Using Cayley–Menger determinants, though, it is possible to derive the following simple distance-based formulation. Let us consider the octahedral manipulator in Fig. 4. We already know that any Cayley-Menger determinant involving more than 4 points in $\mathbb{R}^3$ necessarily vanishes. Then, the distances between $P_1, \ldots, P_6$ must necessarily satisfy the following six equations:

$$\left.\begin{array}{l} t_1(s_{2,6}, s_{3,4}) \doteq D(2,3,4,5,6) = 0 \\ t_2(s_{1,5}, s_{3,4}) \doteq D(1,3,4,5,6) = 0 \\ t_3(s_{1,5}, s_{2,6}) \doteq D(1,2,4,5,6) = 0 \\ t_4(s_{2,6}, s_{1,5}) \doteq D(1,2,3,5,6) = 0 \\ t_5(s_{3,4}, s_{2,6}) \doteq D(1,2,3,4,6) = 0 \\ t_6(s_{1,5}, s_{3,4}) \doteq D(1,2,3,4,5) = 0 \end{array}\right\} \tag{3}$$

where $s_{2,6}$, $s_{3,4}$, and $s_{1,5}$ are unknown squared distances. All other distances are known because they correspond either to architectural parameters or leg lengths. Now, if we eliminate, for example, $s_{3,4}$ from the system formed by $t_2(s_{1,5}, s_{3,4}) = 0$ and $t_6(s_{1,5}, s_{3,4}) = 0$, an octic polynomial in $s_{1,5}$ is readily obtained. The result cannot be included here for space limitation reasons but it can be easily reproduce using a computer algebra system. The roots of this polynomial are values of $s_{1,5}$ that satisfy (3). For each of these real roots, we can determine the spatial position of the three points of the platform by computing, for example, the following sequence of trilaterations: computing $\mathbf{p}_{1,5}$ from $\mathbf{p}_{1,2}$ and $\mathbf{p}_{1,3}$, then $\mathbf{p}_{1,4}$ from $\mathbf{p}_{1,2}$ and $\mathbf{p}_{1,5}$, and finally $\mathbf{p}_{1,6}$ from $\mathbf{p}_{1,4}$ and $\mathbf{p}_{1,5}$. This leads to up to eight locations for P_6. Those locations that satisfy the distance imposed by the leg connecting P_3 and P_6 correspond to valid assembly modes.

An approach, closely related to the above one, was presented by Dedieu and Norton in [17]. They also obtained the system of six polynomial equations in (3) from which they derived three octic polynomial equations in $s_{2,6}$, $s_{3,4}$, and $s_{1,5}$ which had to simultaneously solved. The use of trilaterations clearly simplifies this distance-based approach by allowing us to realize that computing the roots of any of these three polynomials is enough to completely solve the problem.

The coefficients of the derived distance-based octic polynomial are in turn polynomials in known squared distances. Thus, this polynomial is not linked to any particular coordinate system and it does not exhibit the well-known problems derived from the tangent-half-angle substitution.

Now, let us suppose that, for a generic configuration of the moving platform with respect to the base, the location of the joints are modified so that the lengths of the legs for the new locations, say $m_1, m_2, \ldots, m_6$, are related to those of the original ones, $l_1, l_2, \ldots, l_6$, through the relation:

$$\begin{pmatrix} m_1^2 \\ m_2^2 \\ \vdots \\ m_6^2 \end{pmatrix} = \mathbf{A} \begin{pmatrix} l_1^2 \\ l_2^2 \\ \vdots \\ l_6^2 \end{pmatrix} + \mathbf{b}, \tag{4}$$

where $\mathbf{A}$ and $\mathbf{b}$ are a constant matrix and a constant vector, respectively. Then, if such a modification on the location of the joints exists, the resulting platform will have the same forward kinematics as the original one in the sense that there will be a one-to-one correspondence between the coefficients of their associated octic polynomials through (4). The effect of this kind of joint location modifications on the singularities of the moving platform is discussed in the next section.

4 SINGULARITIES

For a general Stewart-Gough platform, the linear actuators' velocities, $(\dot{l}_1, \dot{l}_2, \ldots, \dot{l}_6)$, can be expressed in terms of the platform velocity vector $(\mathbf{v}, \mathbf{\Omega})$ as follows:

$$\text{diag}(l_1, \ldots, l_6) \begin{pmatrix} \dot{l}_1 \\ \dot{l}_2 \\ \vdots \\ \dot{l}_6 \end{pmatrix} = \mathbf{J} \begin{pmatrix} \mathbf{v} \\ \mathbf{\Omega} \end{pmatrix}, \tag{5}$$

where $\mathbf{J}$ is the matrix of normalized Plücker coordinates of the six leg lines. The parallel singularities of the platform are those configurations in which det$(\mathbf{J}) = 0$. This algebraic condition have a simple geometric interpretation for the octahedral manipulator. Indeed, according to Fig. 4, when the supporting planes of the triangles $P_1P_2P_4$, $P_2P_3P_5$, $P_3P_1P_6$, and $P_4P_5P_6$ intersect in a single point, the manipulator is in a singular pose [18].

Now, as in the previous section, let us suppose that the location of the joints are modified so that the lengths of the legs in their new locations are related to those of the original legs through the relation (4). Differentiating (4) with respect to time and substituting (5) in the result, we get

$$\text{diag}(d_1, \ldots, d_6) \begin{pmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \vdots \\ \dot{d}_6 \end{pmatrix} = \mathbf{AJ} \begin{pmatrix} \mathbf{v} \\ \mathbf{\Omega} \end{pmatrix}. \tag{6}$$

Then, if a modification in the location of the joints satisfying (4) exists, the singularities of the resulting platform are those configurations in which det$(\mathbf{AJ}) =$ det$(\mathbf{A})$det$(\mathbf{J}) = 0$. In other words, the resulting platform will have the same singularities as the original one provided that det$(\mathbf{A}) \neq 0$. As a consequence, a modification in the location of the joints satisfying (4) leaves the singularities of the moving platform unaltered. Next section presents the geometric transformations that satisfy the algebraic condition (4).

5 DERIVING KINEMATICALLY EQUIVALENT MANIPULATORS

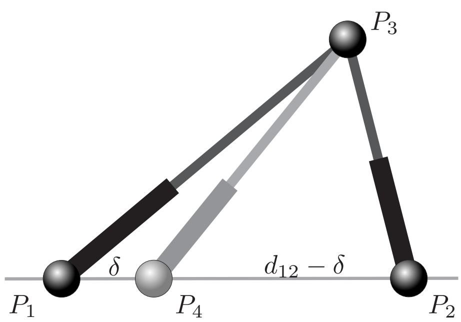

Figure 5. The squared distance $s_{3,4}$ depends affine linearly on $s_{1,3}$ and $s_{2,3}$ provided that P_4 lies in the line defined by P_1P_2.

Let us take two legs in an octahedral manipulator sharing a double-ball-joint and let us introduce an offset in the location of one of the other end spherical joints, as shown in Fig. 5. Since the Cayley-Menger determinant of P_1, P_2, P_3, and P_4 vanishes because they are coplanar, $D(1,2,3,4) = 0$ or, equivalently,

$$\delta s_{2,3} + (d_{1,2} - \delta)s_{1,3} - d_{12}s_{3,4} - d_{1,2}\delta(d_{1,2} - \delta) = 0. \tag{7}$$

Note that $s_{3,4}$ depends affine linearly on $s_{1,3}$ and $s_{2,3}$. Then, if the spherical joint centered at P_1 is moved to P_4, the resulting leg lengths, for any configuration of the moving platform, can be expressed in terms of the original leg lengths as in (4). Thus, it can be said that the introduced offset does not change the kinematics of the original octahedral manipulator.

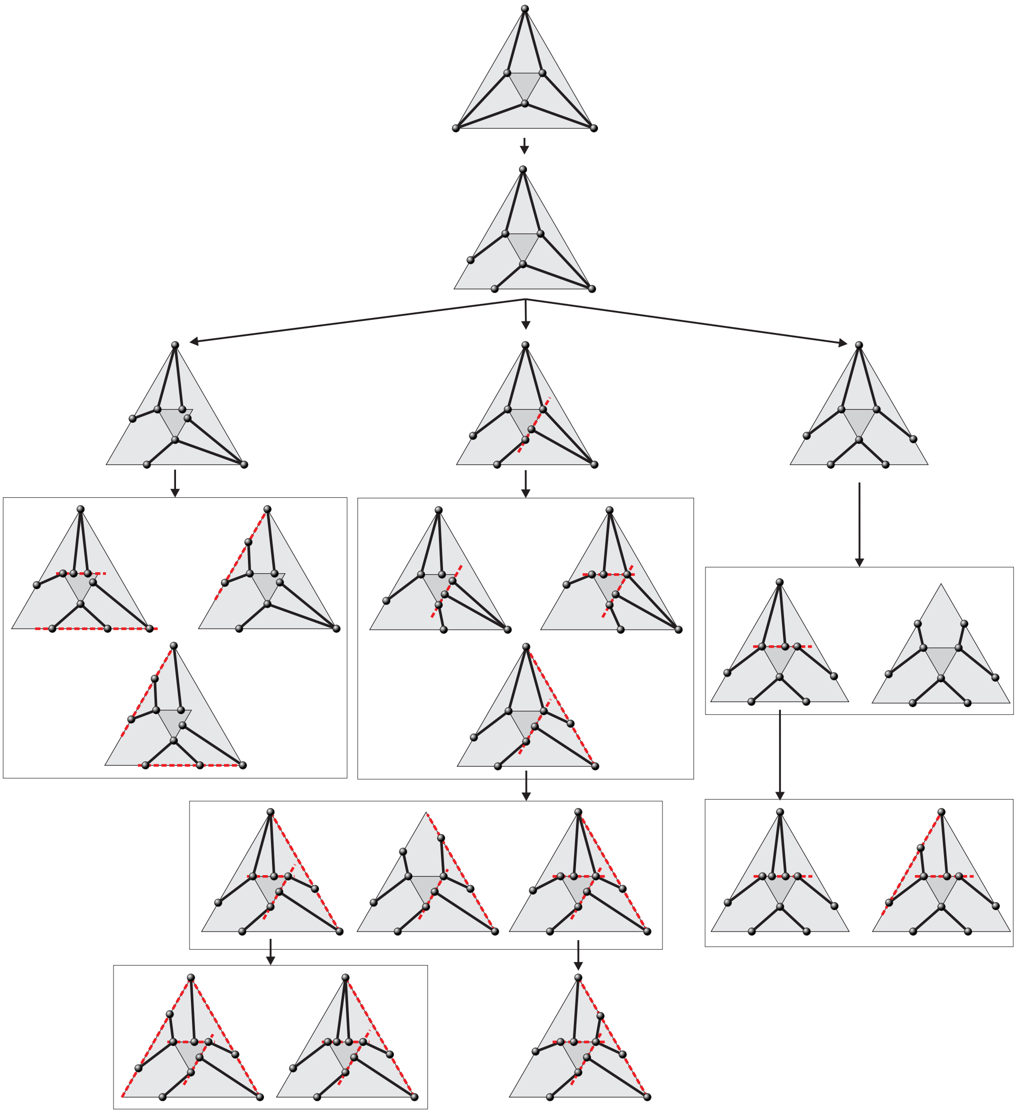

Figure 6. Family of manipulators kinematically equivalent to the octahedral manipulator obtained by sequentially applying the geometric transformation in Fig. 5. Dotted red lines indicate required alignments.

It is possible to repeat the above operation on the remaining couples of legs sharing a double-ball-joint. The family of Stewart platforms obtained from the octahedral manipulator through the sequential introduction of these offsets is depicted in Fig. 6. At the root is the octahedral manipulator and, at each level down the tree, a set of offsets is introduced that change the topology of the manipulator. Twenty different topologies up to isomorphisms is thus generated. Unfortunately, all of them include at least one double-ball-joint. Nevertheless, it is interesting to realize that these offsets can also be introduced simultaneously, not only sequentially. The details of how this operation is performed can be found in [19]. Then, if an offset is simultaneously introduced for the six sets of two legs sharing a double-ball-joint, all joints are split into single spherical joints. The result is the 6-6 platform appearing in Fig. 7.

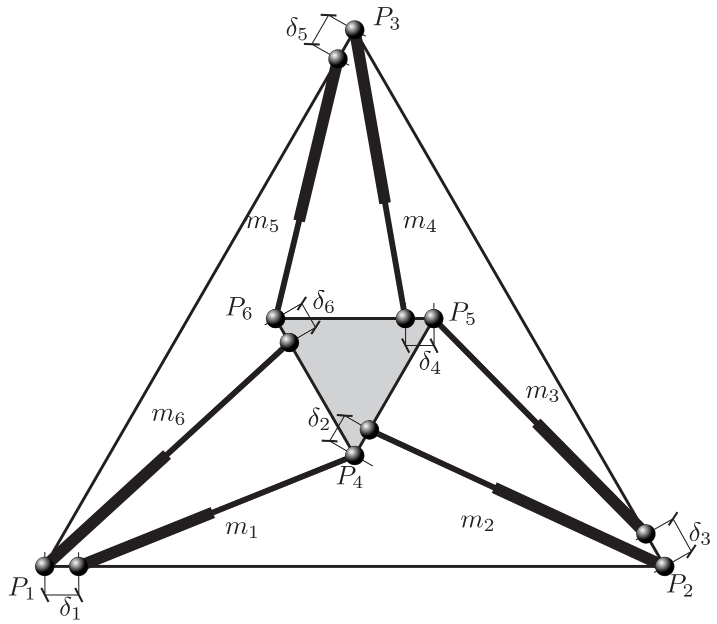

Figure 7. Contrary to what happens to the Stoughton-Arai approximation, the proposed modification leads to a 6-6 platform kinematically equivalent to the octahedral manipulator.

According to Fig. 7 and the results in [19], the affine relation between leg lengths of the resulting 6-6 platform and the original octahedral manipulator can be expressed as:

$$\begin{pmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \\ m_4^2 \\ m_5^2 \\ m_6^2 \end{pmatrix} = \mathbf{A} \begin{pmatrix} l_1^2 \\ l_2^2 \\ l_3^2 \\ l_4^2 \\ l_5^2 \\ l_6^2 \end{pmatrix} - \mathbf{b} \tag{8}$$

where

$$\mathbf{A} = \begin{pmatrix} \frac{d_{12}-\delta_1}{d_{12}} & \frac{\delta_1}{d_{12}} & 0 & 0 & 0 & 0 \\ 0 & \frac{d_{45}-\delta_2}{d_{45}} & \frac{\delta_2}{d_{45}} & 0 & 0 & 0 \\ 0 & 0 & \frac{d_{23}-\delta_3}{d_{23}} & \frac{\delta_3}{d_{23}} & 0 & 0 \\ 0 & 0 & 0 & \frac{d_{56}-\delta_4}{d_{56}} & \frac{\delta_4}{d_{56}} & 0 \\ 0 & 0 & 0 & 0 & \frac{d_{13}-\delta_5}{d_{13}} & \frac{\delta_5}{d_{13}} \\ \frac{\delta_6}{d_{46}} & 0 & 0 & 0 & 0 & \frac{d_{46}-\delta_6}{d_{46}} \end{pmatrix} \tag{9}$$

and

$$\mathbf{b} = \begin{pmatrix} \delta_1(d_{12}-\delta_1) \\ \delta_2(d_{45}-\delta_2) \\ \delta_3(d_{23}-\delta_3) \\ \delta_4(d_{56}-\delta_4) \\ \delta_5(d_{13}-\delta_5) \\ \delta_6(d_{46}-\delta_6) \end{pmatrix}$$

If det$(\mathbf{A}) \neq 0$, there is a one-to-one correspondence between $(m_1^2, \ldots, m_6^2)$ and $(l_1^2, \ldots, l_6^2)$. Remind that $\mathbf{A}$ is constant as it only depends on architectural parameters. Next, the resulting 6-6 platform is analyzed in more detail through an example.

6 EXAMPLE

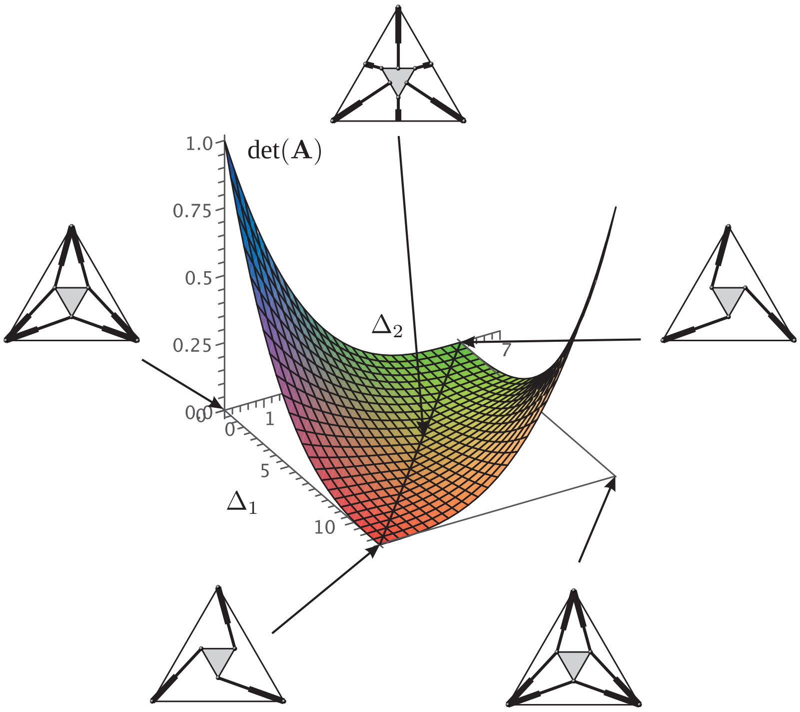

Figure 8. By properly choosing the offsets $\Delta_1 = \delta_1 = \delta_3 = \delta_5$ and $\Delta_2 = \delta_2 = \delta_4 = \delta_6$ in Fig 7, it is possible to reach architecturally singular platforms including the obvious situations in which couples of legs coincide and the architecturally singular Griffis-Duffy platform.

Let us consider a parallel manipulator with the same topology as the one depicted in Fig. 7 with the following geometric parameters: $d_{12} = d_{23} = d_{13} = 12$, $d_{46} = d_{45} = d_{56} = 6$, $\Delta_1 = \delta_1 = \delta_3 = \delta_5$, and $\Delta_2 = \delta_2 = \delta_4 = \delta_6$. Substituting these values in (9) and computing its determinant, we obtain

$$\begin{aligned}\det(\mathbf{A}) = &-\frac{1}{20736}\Delta_1^3\Delta_2^2 - \frac{1}{10368}\Delta_1^2\Delta_2^3 + \frac{1}{3456}\Delta_1^3\Delta_2 + \frac{1}{576}\Delta_1^2\Delta_2^2 + \frac{1}{864}\Delta_1\Delta_2^3 - \frac{1}{1728}\Delta_1^3 \\ &- \frac{1}{96}\Delta_1^2\Delta_2 - \frac{1}{48}\Delta_1\Delta_2^2 - \frac{1}{216}\Delta_2^3 + \frac{1}{48}\Delta_1^2 + \frac{1}{8}\Delta_1\Delta_2 + \frac{1}{12}\Delta_2^2 - \frac{1}{4}\Delta_1 - \frac{1}{2}\Delta_2 + 1.\end{aligned}$$

Fig. 8 plots $\det(\mathbf{A})$ as a function of Δ_1 and Δ_2. When $\Delta_1 + \Delta_2 = 12$, the introduced offsets lead to an architecturally singular platform as $\det(\mathbf{A}) = 0$. Now, let us suppose that we want to compute its forward kinematic solutions for the following leg lengths

$$m_1 = \frac{6}{25}\sqrt{6170}, \qquad m_3 = \frac{1}{5}\sqrt{7349}, \qquad m_5 = \frac{1}{25}\sqrt{136210},$$
$$m_2 = \frac{6}{5}\sqrt{221}, \qquad m_4 = \frac{1}{50}\sqrt{674605}, \qquad m_6 = \frac{1}{5}\sqrt{8153}.$$

with $\Delta_1 = \frac{12}{5}$ and $\Delta_2 = \frac{6}{5}$. Then, substituting these values in (8), it can be verified that this problem is equivalent to solve the forward kinematics of the octahedral manipulator defined by $P_1, \ldots, P_6$ (see Fig. 7) with leg lengths

$$l_1 = \frac{198}{10}, \qquad l_2 = 18, \qquad l_3 = 18, \qquad l_4 = 17, \qquad l_5 = \frac{149}{10}, \qquad l_6 = \frac{178}{10},$$

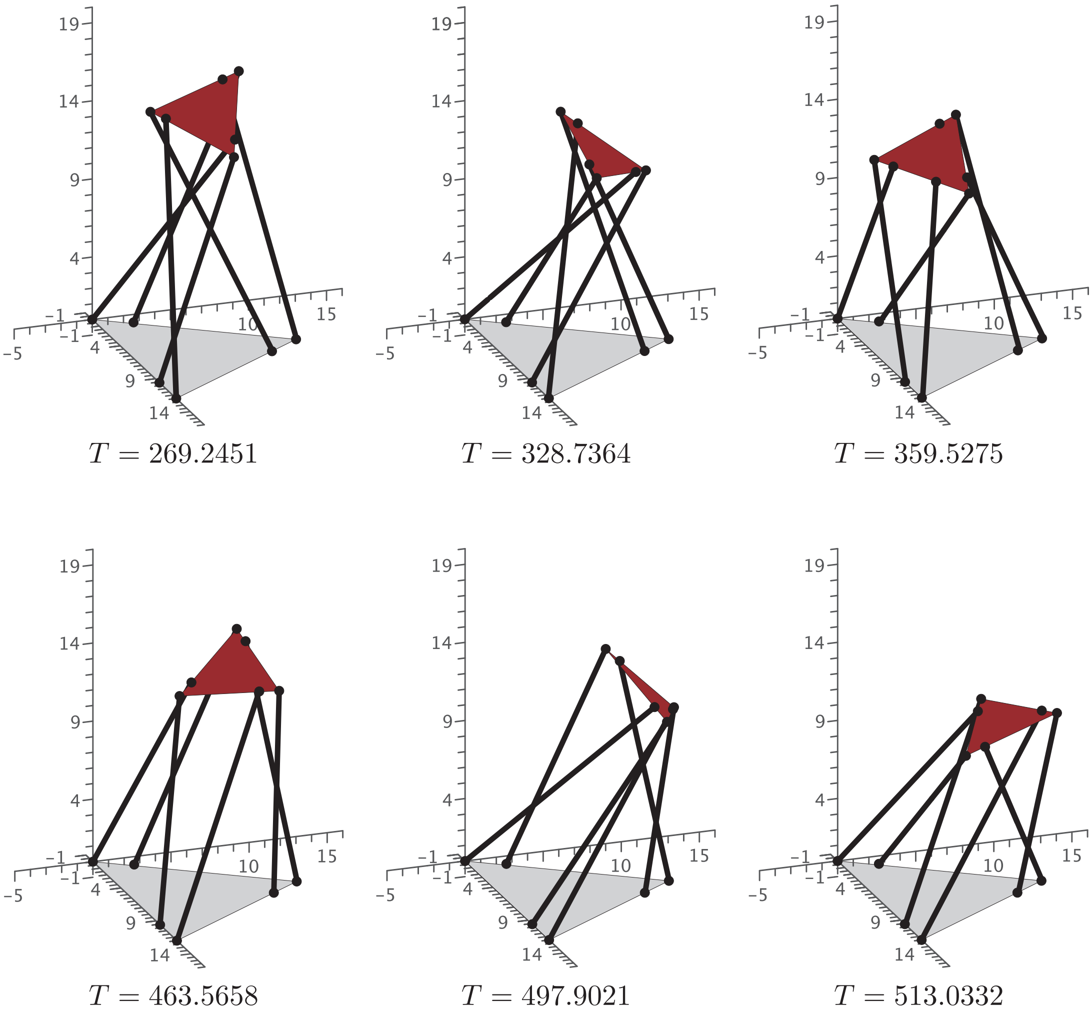

Figure 9. The forward kinematic solutions of the analyzed example. The mirror reflections with respect to the base plane are not included.

which is the same problem as the one analyzed in [11]. Substituting the above values in the resultant derived in Section 3, the following characteristic polynomial is obtained

$$\begin{aligned} &6.5844 \cdot 10^{9}\, s_{1,5}^{8} - 19.7613 \cdot 10^{12}\, s_{1,5}^{7} + 25.7996 \cdot 10^{15}\, s_{1,5}^{6} \\ &-19.1573 \cdot 10^{18}\, s_{1,5}^{5} + 8.8594 \cdot 10^{21}\, s_{1,5}^{4} - 2.6162 \cdot 10^{24}\, s_{1,5}^{3} \\ &+482.3818 \cdot 10^{24}\, s_{1,5}^{2} - 50.8263 \cdot 10^{27}\, s_{1,5} + 2.3449 \cdot 10^{30} = 0. \end{aligned} \tag{10}$$

The above polynomial has six real roots: 269.2451, 328.7364, 359.5275, 463.5658, 497.9021, and 513.0332. Each of them leads to two mirror poses with respect to the base plane. The resulting poses for the case in which $\mathbf{p}_1 = (0,0,0)^T$, $\mathbf{p}_2 = (6, \sqrt{108}, 0)^T$, and $\mathbf{p}_3 = (12,0,0)^T$, appear in Fig. 9 where the mirror reflections with respect to the base plane are not represented.

7 CONCLUSIONS

Stating the kinematics analysis of the octahedral manipulator in terms of poses introduces two major disadvantages: (a) a reference frame has to be introduced, and (b) all formulas involve translations and rotations simultaneously. This paper proposes a different approach in which, instead of directly computing the sought

Cartesian poses, a problem fully posed in terms of distances is first solved. Then, the original problem can be trivially solved by sequences of trilaterations.

The presented distance-based formulation also permits to generate a family of Stewart-Gough platforms whose members are kinematically equivalent to the octahedral manipulator. One of this members has no double-ball-joints and, hence, its important technological interest. Future developments in which an octahedral manipulator is required but double-ball-joints have to be avoided can benefit from this result.

8 ACKNOWLEDGMENTS

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