Compact Form of the Pseudoinverse Matrix in the Approximation of a Star Graph Using the Conductance Electrical Model (CEM) ⋆

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Abstract. The Conductance Electrical Model (CEM) translate a graph into a circuit. After applying the model, in many cases, it is necessary to approximate the circuit obtained by a star circuit and this requires the calculation of the Moore–Penrose pseudoinverse of a matrix for which there is a general formula that requires transpose, multiply and invert matrices. But in this particular case, the matrix has a peculiar structure, exploited this peculiar structure in this paper show that the pseudoinverse can be obtained without recourse to the general formula. We demonstrate a closed formula that gives the values of the elements of the pseudoinverse directly without iteration, no longer necessary to multiply or inverter matrices. This improved method eliminates the problems due to computer rounding and due to bad-conditioned problems in mathematical terms.

1 Introduction

Graphs have been successfully applied in various fields such as chemistry and biochemistry; transportation, telephony and computers networks, speech recognition and computer vision [1]. In this paper we concentrate in those graphs coming from the field of computer vision. In this case, graphs have labeled nodes and/or edges [7] and they usually have a large number of nodes and/or edges. The methods for graph and sub-graph matching are based on enumerative techniques [2, 3], edit operations [4–6], spectral methods [8], expectation-maximization [9], random walks [10], genetics algorithms [11] and probabilistic approximations [12]. The time complexity in the enumerative and edit operation methods is NP–complete while in the other methods it is polynomially bounded. Only in the

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enumerative solutions we have the exact solution, in the other cases we can get only graph and sub–graph matching approximations.
In [13] was proposed a model to replace a graph for a circuit. This will apply theories and methods of Circuit Theory to solve some of the problems of graph approximately. In that paper they approached the circuit (N nodes) by a star circuit ($N+1$ nodes and N branches) seeking the value of N resistors of star circuit minimizing the mean square error of the $N(N-1)/2$ values equivalent resistances between the two circuits. This operation required the calculation of the Moore–Penrose pseudoinverse (hereinafter simply pseudoinverse) of a matrix by the general formula involving the product and inversion of matrices. In this work we demonstrate that the use of that general formula is not necessary to calculate the pseudoinverse as described in [13].

2 Conductance Electrical Model (CEM)

As explained in [13] becomes an undirected weighted graph in a passive resistive circuit where the weights of the edges are converted to values of conductance (siemens), note that at [14] there is another approach to the problem apparently similar but the initial idea is different because there the edges are replaced by resistances (Ω) and here the edges are replaced by conductances (S).
The circuit is characterized by its Indefinite Admitance Matrix that will always be real (the circuit is resistive) and symmetric (no sources dependent) that in the case the step function, as defined in [13], is the identity will coincide with the Laplacian of the adjacency matrix of the original graph.
Once the model can be applied consolidated Circuit Theory in order to obtain valid results for graph problems.

3 Star approximation using CEM

Based on the CEM is obtained from a graph of N nodes and M branches its counterpart electrical resistive circuit. In this circuit, using Circuit Theory, we obtain the $N(N-1)/2$ equivalent resistances ($r_{eq_{ij}}$) these values can be represented by a column vector

$$\boldsymbol{r_{eq}} = (r_{eq_{1,2}}, r_{eq_{1,3}}, \dots, r_{eq_{1,N}}, r_{eq_{2,3}}, \dots, r_{eq_{2,N}}, \dots\dots, r_{eq_{N-3,N-1}}, r_{eq_{N-2,N-1}}, r_{eq_{N-1,N}})^t$$

At work [13] was proposed to approximate the original circuit by a star circuit (N branches and $N+1$ nodes with the central as the reference) with one resistance (r_i) for each branch, these values can be written as column vector

$$\boldsymbol{r} = (r_1, r_2, \dots, r_N)^t$$

Also there are $N(N-1)/2$ equivalent resistances in the star circuit, note that the central node is not involved in the calculation of the equivalent resistances. These can be written as column vector

$$\boldsymbol{r'_{eq}} = (r'_{eq_{1,2}}, r'_{eq_{1,3}}, \dots, r'_{eq_{1,N}}, r'_{eq_{2,3}}, \dots, r'_{eq_{2,N}}, \dots\dots, r'_{eq_{N-3,N-1}}, r'_{eq_{N-2,N-1}}, r'_{eq_{N-1,N}})^t$$

It easy to see that $r'_{eq_{ij}} = r_i + r_j$ since the equivalent resistance between two nodes in the star circuit is the association of two series resistances. Then we have $\boldsymbol{r}'_{eq} = B\boldsymbol{r}$ where B is the matrix show in (1)

$$B = \left(\begin{array}{c} 1\,1\,0\,0\cdots0\,0\,0 \\ 1\,0\,1\,0\cdots0\,0\,0 \\ 1\,0\,0\,1\cdots0\,0\,0 \\ 1\,0\,0\,0\cdots0\,0\,0 \\ \vdots\;\vdots\;\vdots\;\vdots\;\ddots\;\vdots\;\vdots\;\vdots \\ 1\,0\,0\,0\cdots0\,1\,0 \\ 1\,0\,0\,0\cdots0\,0\,1 \\ \hline 0\,1\,1\,0\cdots0\,0\,0 \\ 0\,1\,0\,1\cdots0\,0\,0 \\ 0\,1\,0\,0\cdots0\,0\,0 \\ \vdots\;\vdots\;\vdots\;\vdots\;\ddots\;\vdots\;\vdots\;\vdots \\ 0\,1\,0\,0\cdots0\,1\,0 \\ 0\,1\,0\,0\cdots0\,0\,1 \\ \hline 0\,0\,1\,1\cdots0\,0\,0 \\ 0\,0\,1\,0\cdots0\,0\,0 \\ \vdots\;\vdots\;\vdots\;\vdots\;\ddots\;\vdots\;\vdots\;\vdots \\ 0\,0\,1\,0\cdots0\,1\,0 \\ 0\,0\,1\,0\cdots0\,0\,1 \\ \hline 0\,0\,0\,1\cdots0\,0\,0 \\ \vdots\;\vdots\;\vdots\;\vdots\;\ddots\;\vdots\;\vdots\;\vdots \\ 0\,0\,0\,1\cdots0\,1\,0 \\ 0\,0\,0\,1\cdots0\,0\,1 \\ \hline \vdots\;\vdots\;\vdots\;\vdots\;\ddots\;\vdots\;\vdots\;\vdots \\ \vdots\;\vdots\;\vdots\;\vdots\;\ddots\;\vdots\;\vdots\;\vdots \\ \hline 0\,0\,0\,0\cdots0\,1\,1 \end{array}\right) \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array}\right\} N-1 \text{ rows} \\ \left.\begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array}\right\} N-2 \text{ rows} \\ \left.\begin{array}{l} \\ \\ \\ \\ \\ \end{array}\right\} N-3 \text{ rows} \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} N-4 \text{ rows} \\ \\ \\ \left.\right\} 1 \text{ row} \end{array} \tag{1}$$

The approximation discussed above paragraphs is to be understood as the search for values of $\boldsymbol{r}$ such that $\boldsymbol{r}'_{eq}$ is approximately equal to $\boldsymbol{r}_{eq}$ in the sense of minimizing the mean square error between $\boldsymbol{r}_{eq}$ and $\boldsymbol{r}'_{eq}$ in this case the solution is given by

$$\boldsymbol{r} = (B^t B)^{-1} B^t \boldsymbol{r}_{eq}$$

where

$$B^+ = (B^t B)^{-1} B^t \tag{2}$$

is known as the pseudoinverse of B, note that B^+ has N rows and $N(N-1)/2$ columns. Thanks to the above equation we can finally write

$$\boldsymbol{r} = B^+ \boldsymbol{r_{eq}}$$

4 Compact form of the pseudoinverse

To obtein the pseudoinverse (B^+) of any matrix B by the (2) expresion is necessary to make a matrix inversion, two products of matrices and matrix transpose. But for the particular case that the matrix B is of the form given in (1) it is not necessary to use (2) expresion. This substantially simplifies calculations as discussed in the following theorem.

Theorem 1. *Let B be the matrix with the structure shown in (1) with $N \neq 1$ and $N \neq 2$ then its pseudoinverse is*

$$B^+ = \frac{1}{(N-1)(N-2)} \left[(N-1)B^t - \mathbb{1}_{N,N(N-1)/2} \right] \tag{3}$$

where N is the number of columns of matrix B and $\mathbb{1}_{N,N(N-1)/2}$ is the all ones matrix with N rows and $N(N-1)/2$ columns.

Proof. We call M the result of B^tB then it is easy to see that

$$M = \begin{pmatrix}
1&1&1&\cdots&1&1&0&0&\cdots&0&0&0&\cdots&0&0&0&\cdots&0&0&\cdots&\cdots&0\\
1&0&0&\cdots&0&0&1&1&\cdots&1&1&0&\cdots&0&0&0&\cdots&0&0&\cdots&\cdots&0\\
0&1&0&\cdots&0&0&1&0&\cdots&0&0&1&\cdots&1&1&0&\cdots&0&0&\cdots&\cdots&0\\
0&0&1&\cdots&0&0&0&1&\cdots&0&0&1&\cdots&0&0&1&\cdots&1&1&\cdots&\cdots&0\\
\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\ddots&\ddots&\vdots\\
0&0&0&\cdots&1&0&0&0&\cdots&1&0&0&\cdots&1&0&0&\cdots&1&0&\cdots&\cdots&1\\
0&0&0&\cdots&0&1&0&0&\cdots&0&1&0&\cdots&0&1&0&\cdots&0&1&\cdots&\cdots&1
\end{pmatrix}
\begin{pmatrix}
1&1&0&0&\cdots&0&0\\
1&0&1&0&\cdots&0&0\\
1&0&0&1&\cdots&0&0\\
\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\
1&0&0&0&\cdots&1&0\\
1&0&0&0&\cdots&0&1\\
0&1&1&0&\cdots&0&0\\
0&1&0&1&\cdots&0&0\\
\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\
0&1&0&0&\cdots&1&0\\
0&1&0&0&\cdots&0&1\\
0&0&1&1&\cdots&0&0\\
\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\
0&0&1&0&\cdots&1&0\\
0&0&1&0&\cdots&0&1\\
0&0&0&1&\cdots&0&0\\
\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\
0&0&0&1&\cdots&1&0\\
0&0&0&1&\cdots&0&1\\
\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\
\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\
0&0&0&0&\cdots&1&1
\end{pmatrix} =$$

$$= \begin{pmatrix}
N-1&1&1&\cdots&1\\
1&N-1&1&\cdots&1\\
1&1&N-1&\cdots&1\\
\vdots&\vdots&\vdots&\ddots&\vdots\\
1&1&1&\cdots&N-1
\end{pmatrix} \tag{4}$$

where M is a square matrix of order N. To calculate M^{-1} we will use that

$$M^{-1} = \frac{M^*}{|M|}$$

where M^* is the adjugate matrix and $|M|$ is the determinant that must necessarily be non-zero so that the inverse exists. Applying formula (9) obtained in Annex substituting n for N and k for $N-1$ remains for the determinant

$$|M| = 2(N-1)(N-2)^{N-1} \tag{5}$$

Performing the same substitutions in (10) for the adjugate matrix we obtain the following

$$M^* = (N-2)^{N-2} \begin{pmatrix} 2N-3 & -1 & -1 & \cdots & -1 \\ -1 & 2N-3) & -1 & \cdots & -1 \\ -1 & -1 & 2N-3) & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & 2N-3) \end{pmatrix} \tag{6}$$

Dividing the expressions (5) and (6) we obtein

$$M^{-1} = \frac{1}{2(N-1)(N-2)} \begin{pmatrix} 2N-3 & -1 & -1 & \cdots & -1 \\ -1 & 2N-3 & -1 & \cdots & -1 \\ -1 & -1 & 2N-3 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & 2N-3 \end{pmatrix} \tag{7}$$

Note that M^{-1} can be written as

$$M^{-1} = \frac{1}{2(N-1)(N-2)} \left[2(N-1)\mathbb{I}_N - \mathbb{1}_{N,N}\right]$$

where $\mathbb{I}_N$ is the identity matrix of order N. We finally have

$$\begin{aligned} B^+ = M^{-1}B^t = & \frac{1}{2(N-1)(N-2)} \left[2(N-1)\mathbb{I}_N - \mathbb{1}_{N,N}\right] B^t = \\ = & \frac{1}{2(N-1)(N-2)} \left[2(N-1)\mathbb{I}_N B^t - \mathbb{1}_{N,N} B^t\right] = \\ = & \frac{1}{2(N-1)(N-2)} \left[2(N-1)B^t - 2\mathbb{1}_{N,N(N-1)/2}\right] = \\ = & \frac{1}{(N-1)(N-2)} \left[(N-1)B^t - \mathbb{1}_{N,N(N-1)/2}\right] \end{aligned}$$

The last step is because all the columns of B^t add 2. □

5 Advantages of the compact form of the pseudoinverse

The advantages of the calculation of pseudoinverse by compact formula (3) versus general formula (2) are:
1) The computacional complexity is reduced from $O(N^3)$ to $O(N^2)$.
2) This improvement is immune to problems of numerical resolution on a computer and bad–conditioned problems in mathematical terms.

6 Annex

For a matrix Q of order n as follows

$$Q = \begin{pmatrix} k & 1 & 1 & \cdots & 1 \\ 1 & k & 1 & \cdots & 1 \\ 1 & 1 & k & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & k \end{pmatrix} \tag{8}$$

are fulfilled the two following theorems

Theorem 2. *The determinant of matrix Q is*

$$|Q| = (k+n-1)(k-1)^{n-1} \tag{9}$$

Proof. In effect, this will be to obtain an upper triangular matrix. For each row adds all the columns to first column

$$|Q| = \begin{vmatrix} k+n-1 & 1 & 1 & \cdots & 1 \\ k+n-1 & k & 1 & \cdots & 1 \\ k+n-1 & 1 & k & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k+n-1 & 1 & 1 & \cdots & k \end{vmatrix}$$

then

$$|Q| = (k+n-1) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & k & 1 & \cdots & 1 \\ 1 & 1 & k & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & k \end{vmatrix}$$

Is replaced each row except the first so that results from subtracting the first row

$$|Q| = (k+n-1) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & k-1 & 0 & \cdots & 0 \\ 0 & 0 & k-1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k-1 \end{vmatrix}$$

As the determinant of a triangular matrix is the product of the diagonal elements remains

$$|Q| = (k+n-1)(k-1)^{n-1}$$

□

Corollary 1. *The determinant of Q is not zero if and only if $k \neq 1$ and $k \neq 1-n$.*

Theorem 3. *The adjoint matrix (Q^*) of the matrix Q is*

$$Q^* = (k-1)^{n-2} \begin{pmatrix} k+n-2 & -1 & -1 & \cdots & -1 \\ -1 & k+n-2 & -1 & \cdots & -1 \\ -1 & -1 & k+n-2 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & k+n-2 \end{pmatrix} \tag{10}$$

Proof. *To show this we divide the problem into two parts: (i) calculation diagonal adjoints and (ii) calculation off-diagonal adjoints.*
(i) calculation diagonal adjoints
The adjoint of any element of the diagonal (all adjoints from any element of the diagonal are equal) will be a determinant of order $n-1$, applying the formula (9) is obtained

$$Q_{ii} = \begin{vmatrix} k & 1 & 1 & \cdots & 1 \\ 1 & k & 1 & \cdots & 1 \\ 1 & 1 & k & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & k \end{vmatrix} = (k+n-2) \cdot (k-1)^{n-2}$$

(ii) calculation off-diagonal adjoints
As seen in (11) to calcultae the adjoint Q_{ij} $(i \neq j)$ must be removed the row i and column j (solid line) of Q.

$$Q = \left(\begin{array}{ccccccc|c|ccc} k & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & k & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ \hline 1 & \cdots & 1 & k & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ \hline 1 & \cdots & 1 & 1 & k & \cdots & 1 & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & 1 & \cdots & k & 1 & 1 & \cdots & 1 \\ 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & k & 1 & \cdots & 1 \\ 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & k & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & k \end{array}\right) \tag{11}$$

Thereafter, it easy to see in (12) that appear only one row and only one column with all ones (solid line) in the adjoint Q_{ij} (with $i \neq j$)

$$C_{ij} = (-1)^{i+j} \left|\begin{array}{ccc|c|ccccccc} k & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & k & 1 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ 1 & \cdots & 1 & 1 & k & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & 1 & \cdots & k & 1 & \cdots & 1 \\ \hline 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ \hline 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & k & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & \cdots & & 1 & 1 & \cdots & k \end{array}\right| \tag{12}$$

This row will all about swapping around to its first row, similar to the column, for each permutation the determinant changes sign.
Suppose that $i < j$ then the column with all ones appearing at position i while the row with all ones appears at position $j-1$. Therefore the number of permutations (and consequent changes of sign) of the row and column with all ones is $j-2$ and $i-1$ respectively, being affected the determinant by $(-1)^{i+j-3}$. Analogous result is obtained assuming $j < i$.
In short, the coefficient that multiplies the determinant is $(-1)^{i+j}(-1)^{i+j-3}$ will always be worth -1 for the exponent always odd, indeed

$$(-1)^{i+j}(-1)^{i+j-3} = (-1)^{2i+2j-3} = (-1)^{2(i+j)-3} = -1$$

Then the adjoint is as follows ($i \neq j$)

$$Q_{ij} = -\begin{vmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & k & 1 & 1 & \cdots & 1 \\ 1 & 1 & k & 1 & \cdots & 1 \\ 1 & 1 & 1 & k & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & \cdots & k \end{vmatrix}$$

The calculation of this determinant is similar to that of Theorem 2, being for $i \neq j$.

$$C_{ij} = -(k-1)^{n-2}$$

Finally it has

$$Q^* = (k-1)^{n-2} \begin{pmatrix} k+n-2 & -1 & -1 & \cdots & -1 \\ -1 & k+n-2 & -1 & \cdots & -1 \\ -1 & -1 & k+n-2 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & k+n-2 \end{pmatrix}$$

□

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