

ON THE MODELING AND REAL-TIME CONTROL OF URBAN DRAINAGE SYSTEMS: A SURVEY

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Drainage networks are complex systems composed by several processes including recollection, transport, storing, treatment, and releasing the water to a receiving environment. The way Urban Drainage Systems (UDS) manage wastewater is through the convenient handling of active elements such as gates (redirection and/or retention), storing tanks, and pumping stations, when needed. Therefore, modeling and control of UDS basically consists in knowing and representing the (dynamical) behavior of these elements and managing them properly in order to achieve a given set of control objectives, such as minimization of flooding in streets or maximization of treated wastewater in the system. Given the large number of elements composing an UDS and the interaction between them, management and control strategies may depend on highly complex system models, which implies the explicit difficulty for designing real-time control (RTC) strategies. This paper makes a review of the models used to describe, simulate, and control UDS, proposes a revision of the techniques and strategies commonly used for the control UDS, and finally compares several control strategies based on a case study.

INTRODUCTION

UDS have a considerable social, economic and environmental impact, so a proper and efficient urban drainage management to prevent flooding and polluting discharges to the environment are extremely important [1]. Depending on how wastewater and rainwater are managed, UDS can be either combined or separate. Combined sewage systems (CSS) carry all water into a single pipe, while separate systems transport them using different networks. During rainstorm, wastewater flows can easily overload CSS [2], producing flow discharges to the environment known as combined sewer overflows (CSO).

Over the last decades, the disproportionate growth of cities and urban areas has had a considerable impact on UDS. On one hand, population in cities has grown much faster than their infrastructures. On the other hand, population growth in cities has required an increase in the construction of buildings, roads, and other civil works. As a result, the soil in these areas has lost rainwater absorption capacity, making cities more vulnerable to flooding in the presence of rain events. Additionally, weather phenomena such as global warming have increased the frequency, intensity, and duration of rain events in many areas.

These circumstances have caused considerable increments in both wastewater and rainwater within cities, thereby increasing the risk of CSO and flooding events. Minimizing this risk becomes paramount. To attain this objective, two main alternatives can be considered. The most evident solution consists of enlarging the infrastructure of the sewer system (either by adding

more channels, pipelines and storage tanks or by expanding the capacity of the existing ones), in order to transport water and sewage away from cities in a faster and safely way.

However, this option generally involves costs and implementation times that may be too high, which is not feasible in many cases. This leads to the second alternative, which consists in the reduction of the amount and magnitude of overflows in UDS through an efficient management of the sewer system, requiring none or minimal volumetric extension of the system. This objective can be achieved by applying control theory to the handling of UDS. Control of these kind of systems can be applied either off-line (static rules) or online (real-time varying control actions). Due to the dynamic nature and complexity of drainage systems, as well as the dynamic loading conditions under which UDS operate, off-line control may not be the most appropriate option to consider. Therefore, RTC appears as a suitable alternative to operate and manage UDS [3]. The application of RTC to drainage systems has been studied by several researchers over the last years. Studies have shown RTC as a reliable and cost effective solution that improves performance of UDS and helps them to achieve operational objectives in a better way [4], [5].

For the correct design, tuning and implementation of RTC techniques to UDS, a suitable knowledge about the behavior of the system and dynamics is required. Thereby, proper modeling of UDS plays a very important role. The complex nature of UDS might include nonlinear dynamics, delays, and dead times. Additionally, RTC of these systems includes several features such as multi-variable and multi-objective control problems, combination of both continuous elements and discrete control devices, operational constraints, stochastic disturbances (*e.g., rain*), and distributed large-scale architectures with many sensors, actuators, and controllers [2]. This article presents a brief and compressed survey of the main modeling and RTC techniques applied to UDS. A review of the main modeling approaches adopted for drainage systems is shown, and a classification criterion is proposed. Furthermore, an introduction, analysis and comparison of the principal RTC techniques is made. The rest of the paper is organized as follows. Section MODELING deals with the most relevant models and approximations used to describe, simulate and control drainage systems. Section REAL-TIME CONTROL OF URBAN DRAINAGE SYSTEM describes the characteristics of RTC when applied to UDS, and introduces some of the most used RTC techniques in these kind of systems. Section SOFTWARE TOOLS presents different software tools to simulate these kind of hydraulic systems. In Section CASE STUDY some performance results when considering the described RTC techniques are presented and briefly discussed for a case study inspired on the Barcelona UDS. Finally, a discussion based on the literature review and the case study is included in Section CONCLUSION.

MODELING

Modeling of UDS basically consists in knowing and representing the (dynamical) behavior of the interaction between different elements (active and passive) in the system. The models of UDS studied here describe the wastewater transport in the systems, and these can be directed or oriented to simulation and/or to control. The Saint-Venant Equations (SVE) are two coupled nonlinear partial differential equations based on the physical principles of mass and energy conservation, which allow to describe accurately the flow in open-flow channels, such as sewage pipes within an urban drainage network [6], [7]. Modeling of drainage systems can be distinguished between two groups of models [8]. The first group consists of the physically based models, which come from the SVE and are used for hydraulic simulation. The second group consists of control-oriented models and it is not directly obtained from the SVE. Instead, these models might use conceptual relations, while satisfying conservation of mass and demanding a smaller computational effort. The main models of each group are presented in Figure 1. A brief description of these models is given next.

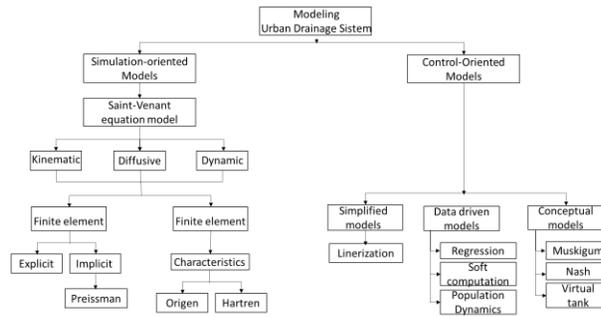


Figure 1 Proposed taxonomy of models for UDS.

Simulation-oriented Models

These type of models provide information on how the UDS will behave without actually testing it in reality. This can be done using SVE, but due to the complexity in obtaining the solution, it is necessary to simplify. These simplifications depend on the flow considered, *i.e.*, the flow can be considered as wave dynamics, diffusive wave, and wave kinematics. There are two numerical methods to solve SVE: finite element methods and finite difference methods. A brief description of these methods is given next.

- *Finite Element Methods* have been used only to a limited extent for open-channel analysis. They do not offer any significant advantage as compared to the other methods for one-dimensional flow problems, and several difficulties have to be overcome if a shock or bore is formed in the solution. The most used of these methods is known as the Preissman [25].
- *Finite Difference Methods*: In the implicit finite-difference schemes, the spatial partial derivatives and/or the coefficients are replaced in terms of the values at the unknown time level. The unknown variables, therefore, appear implicitly in the algebraic equations. The algebraic equations for the entire system have to be solved simultaneously in these methods. Several implicit finite-difference schemes have been used for the analysis of unsteady open-channel flows. The most used of these methods is the characteristic method, which can be subdivided into the original method and the Hartree method [26].

Control-oriented Models

The SVE have a very high level of detail, which might not be useful for RTC implementation, due to the complexity involved in obtaining the solution and the associated high computational burden associated with it. Therefore, some alternative models are presented below.

Linearization-based models: The SVE can be linearized around a steady-state equilibrium defined by flow and depth. It can be well approximated but it cannot capture dynamic features such as shocks. Some linearization-based models can be found in [7].

Data driven models: They are based on the analysis of the data from USD. The model can be defined on the basis of connections between the system state variables with a small number of physical assumptions about the network [9]. The models are updated from available information of the system, which can be either simulated or obtained from real data. The most important methods used are fuzzy logic, neural computing, evolutionary computation, machine learning, and probabilistic reasoning, among others.

Conceptual Models: consist of models made from the composition of simpler concepts that are used to facilitate the understanding and simulation of the subject matter they represent. The parameters in these models are usually estimated and their calibration is not always a simple task. The most common conceptual models are:

- *Muskingum model*: Muskingum routing is based on an assumed linear relationship between a channel of storage and inflow and outflow discharge. Therefore, it accounts for prism and wedge storage [10].

- *Nash Model*: In this nonlinear model, the network pipes are divided into n reference sections that form a retention cascade. Each of these sections is considered as a tank in the cascade, and the output of each tank corresponds to the input of the following tank. This results in a nonlinear model [11].
- *Virtual tanks model*: These tanks are storage elements that represent the total volume of wastewater inside the sewer mains associated with a determined subcatchment of a given sewer network. The sewage volume is computed via the mass balance of the stored volume, the inflows and the outflows related to the sewage mains, and the equivalent inflow associated with rainwater [1].

REAL-TIME CONTROL OF URBAN DRAINAGE SYSTEMS

An UDS is controlled in real time if process variables are monitored in the system and continuously used to operate actuators. Historically, the main objective in the application of RTC to UDS has been the reduction of the volume and/or the number of CSO in the system, and other objectives commonly taken into account such as the prevention of urban flooding and minimization of operation costs. Furthermore, RTC algorithms may pursue more than one of these objectives simultaneously through the use of multi-objective control strategies.

It has been shown that the application of RTC techniques is a reliable, adaptable and cost effective solution that allows significant reduction of CSO volumes, amongst other benefits that improve the performance of UDS [4]. However, RTC implementation on existing drainage systems usually requires considerable investments resources and tools, such as instrumentation, remote monitoring, process control, software development, mathematical modeling, and forecasting of rainfall. For this reason, RTC potential and benefits must be identified in a drainage network before any implementation to justify the related investments. There is no single set of criteria for determining if a particular drainage system is suitable for RTC implementation.

Efforts have been made to establish basic standard aspects to be taken into account when considering RTC implementation. There are very different kinds of RTC strategies, and there are many ways to classify them (some of them can be found in [5]). Depending on the type of RTC strategy chosen, different components are required for its implementation. A detailed description of the measurement and control components used for applying RTC to UDS can be found in [12]. Next, a classification between heuristic and optimization-based RTC techniques is made, and some of the most commonly used RTC techniques are briefly discussed.

Optimization-Based Algorithms

Optimization-based control algorithms involve an optimization problem that determines the desired behavior of the system. Based on the optimization problem and the measure (or estimation) of the current system variables, these algorithms look for the best possible (i.e., the “optimal”) control action that minimizes or maximizes certain criteria. These criteria are usually expressed mathematically as a scalar function $J(x)$ known as the objective or cost function.

It is possible for some algorithms to take into account two or more control objectives. This is known as multi-objective control, and can be done in several ways. An extensive review of several multi-objective optimization methods can be found in [13]. Some of the main optimization-based RTC algorithms are described next.

- *Model Predictive Control (MPC)*: is a model-based control strategy that uses a prediction of the system response to establish appropriate control actions $u(t)$ in order to minimize a given cost function. MPC uses a receding horizon philosophy, where decisions are made based on the prediction of the system future behavior, within a finite prediction horizon [20]. An MPC controller is compounded by four main elements: a mathematical model of the system, the cost function to be optimized, the restrictions of the system, and a dynamic optimizer that solves the optimization problem in real time [20]. In the case of UDS, the operational constraints are given by the volumetric capacity of tanks and pipes, and by flow restrictions in channels and actuators. The characteristics of MPC controllers have certain benefits in its application to UDS. Some of them are the ability to explicitly express constraints in the system, the possibility to anticipate

the response of the system to future rain events, and the capacity to consider non-ideal elements in the system such as delays and disturbances. MPC strategies have been successfully applied in an increasing number of industries during the last decades. Some examples of this can be found in [1]. MPC theory has been developed into a quite matured stage. However, some problems and subjects remain open in this field, such as adaptive and robust MPC [21], and distributed MPC [22].

- *Population Dynamics*: these techniques are inspired by natural selection, using a simple population dynamics to show how the proportion of individuals (players) in a habitat (game strategy) is affected according to the suitability perceived by each of the agent [27]. Wastewater in UDS may be seen as a dynamic resource allocation problem, which can be solved using techniques of population dynamics as replicator dynamics. In the UDS control problem, the habitats correspond to each tank and the proportion of individuals that is allocated is related to the sharing of the total available wastewater within the network.
- *Evolutionary Strategies (EA)*: evolutionary principles are aimed at searching optimal solutions. Unlike classical methods, EA use a population of solutions at each iteration instead of evaluating just one, and therefore reach a population of optimal solutions. This feature makes evolutionary algorithms to be particularly suited for solving multi-objective optimization problems [16]. In addition, EA allow the consideration of linear and non-linear constraints and the handling of complex optimization problems. One of the EA that has been studied and applied in the context of UDS is fuzzy decision making (FDM) [17]. Other EA applied to UDS include genetic algorithms, which mimic the natural genetic processes of evolution, and are usually used for solving complex and/or nonlinear optimization problems [18]. Applications of EA to UDS can be found in [17] [19].
- *Linear Quadratic Regulator (LQR)*: is an optimal controller whose purpose is to minimize a quadratic cost function and produces a linear control law given by $u = -Kx$, where x is a vector with the state variables of the system, u is a vector with the control actions (system inputs), and K is a gain matrix that is calculated by solving a quadratic, ordinary differential equation known as the Riccati equation, based on a space-state representation of the system. Some applications of LQR to UDS can be found in [14] [15].

Heuristic Algorithms

Heuristic algorithms are knowledge based techniques, usually developed to have low complexity and used for problems that are complex or cannot be easily solved. The heuristic nature of these algorithms causes that any solution found is not guaranteed to be optimal. The most broadly RTC heuristic algorithm used in drainage systems over the last decades is rule-based control (RBC).

Conventional RBC can be applied using different representations. Examples of these representations include “if-then” rules, scenarios, and decision matrices. Despite of being one of the simplest RTC algorithms to implement, understand, and operate, RBC has some disadvantages such as the lack of a conventional methodology to establish the control rules for RBC and the fact that rules are usually set using the expert knowledge available about the characteristics of the system, so the quality and performance of the rules and the controller highly depend on this expertise. Additionally, for large and complex systems the strategy may demand a very large number of rules and scenarios.

A particular RBC known as fuzzy logic control (FLC), has gained popularity in its application to UDS over the last two decades. FLC is a control technique based on fuzzy logic, which combines the simple rules of an expert system with a flexible specification of output parameters.

The way in which these controllers produce control actions can be summarized in three steps: (i) the scalar inputs are transformed into memberships of fuzzy sets by *fuzzifying* functions; (ii) this information is then given to the inference engine; and (iii) the membership values are transformed into required scalar output variables by a *defuzzification* step [23]. This process requires the fuzzy functions to be already defined, in order to establish the membership degrees of the inputs. Examples of applications of RBC and FLC to UDS can be found in [24].

Table 1 shows a comparison between the RTC techniques described in this article. Aspects such as the ability to deal with constraints and non-linear dynamics in the system were taken into account for the comparison. The configuration in which the control techniques can be implemented (centralized (C) and/or non-centralized (NC)) was considered too, as well as the degree of implementation of these techniques in applications related to UDS.

Table 1 Comparison between real-time controllers

Type of Algorithm	Type of Controller	Optimization Based	System non-linearities	Consideration of Constraints	Centralized or Non-centralized	Model Free	Degree of Implementation
Optimization	MPC	Yes	Yes	Yes	C/NC	No	High
	PD	Yes	Yes	Partially	C/NC	Yes	Low
	EA	Yes	Yes	Partially	C/NC	Yes	Low
	LQR	Yes	No	No	C	No	Medium
Heuristic	RBC	No	Yes	No	C/NC	Yes	Medium

SOFTWARE TOOLS

There are numerous software packages available to assist in the design and analysis of hydrologic and hydraulic models of UDS, and these tools can be also oriented to simulation and/or to control of UDS. Table 2 shows a comparison between some of the most important software packages. Several characteristics such as the kind of model used by the software, the type of solution method used, the ability of applying control, and the type of license are taken into account.

Table 2 Comparison between software tools for UDS.

Programs	Model	Solution Method	Control elements	License
Citydrain	Muskingum	Difference numerical	Yes	Open Source
Epaswmm	SVE	Finite difference	Yes	Open Source
Hec-Ras	SVE	Implicit finite difference	No	Purchase
Mike11	SVE	Finite difference	No	Purchase
Mikeswmm	SVE	Explicit finite differences	No	Purchase
Mouse	SVE	Implicit finite difference	Yes	Purchase
Simba	SVE	Finite difference	Yes	Purchase
Stormcad	SVE	Difference numerical	No	Purchase
Synopsis	Nash	N/A	No	Purchase

The software tools commonly used to design and apply RTC on UDS are Matlab® and Labview®. Figure 3 presents a schematic diagram about the software involved in the RTC of UDS. A schematic diagram of the hierarchical RTC for large-scale UDS is also presented.

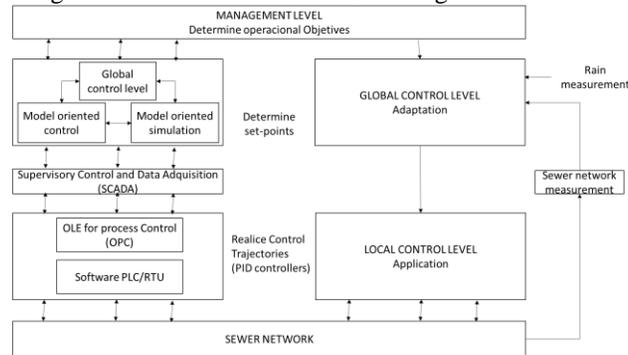


Figure 3 Software and RTC. Adapted from [2]

CASE STUDY

This section shows the control and decision-making design for a part of a real sewer network system by the UDS of Barcelona. More information and further details about this system can be found in [2]. For this system, the control strategies mentioned in this section are tested. The controllers designed are global in the sense that they manage set-points for local controllers see Figure 3. It is relevant to point out that more detailed characteristics related to the model (non-linearities, backward waves, etc.) are considered for the design and tuning of those local controllers. The objective is to reduce the volume of wastewater through the street. The comparative results of reduction of pollution and CPU time are shown in Table 3.

Table 3 Comparison between real-time controllers

Algorithms	Control Strategy	Reduction of pollution [%]	CPU Time (S)
Optimization	MPC	71.99	230.5000
	PD	71.98	205.6200
	EA	71.51	1.5972
	LQR	70.53	0.3065
Heuristic	RBC	49.98	0.6106
Without Control		0	0

The difference of performance strategies depends on the properties of the controllers, see (Table 1). MPC and PD show a favorable cost-benefit ratio as for the ability to deal with constraints and non-linear dynamics in the system with respect to both the reduction of pollution and CPU time. The application of RTC strategies improve the behavior and performance of the studied drainage system. Even in the case of RBC, the technique that shows less improvement, there is a reduction of almost 50% of volumes released to the street and the environment near in the system, compared to the uncontrolled case. The strategies of control presented as MPC, EA, LQR, and RBC have been used previously on UDS.

CONCLUSIONS

This paper has made a revision of most relevant modelling approaches commonly used for UDS, proposing a taxonomy of UDS models (control-oriented, simulation-oriented). Moreover, RTC strategies applied to UDS are also presented and briefly discussed, which can be divided into optimization-based and heuristic-based algorithms. The most relevant software tools used for to simulate and to control UDS are also presented. Finally, five control strategies, namely MPC, PD, EA, LQR, and RBC, have been compared based on a case study. The effectiveness and main advantages of such RTC strategies have been highlighted.

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