Two-layer Scheduling Scheme for Pump Stations*

Cong Cong Sun[1], Vicenç Puig[1] and Gabriela Cembrano[1,2]

Abstract— In this paper, a two-layer scheduling scheme for pump stations in a water distribution network has been proposed. The upper layer, which works in one-hour sampling time, uses Model Predictive Control (MPC) to produce continuous flow set-points for the lower layer. While in the lower layer, a scheduling algorithm has been used to translate the continuous flow set-points to a discrete (ON-OFF) control operation sequence of the pump stations with the constraints that pump stations should draw the same amount of water as the continuous flow set-points provided by the upper layer. The tuning parameters of such algorithm are the lower layer control sampling period and the number of parallel pumps in the pump station. The proposed method has been tested in the Richmond case study.

I. INTRODUCTION

In conventional water distribution systems, pumping water comprises the major fraction of the total energy budget. In practice, the operation of a pump station is simply a set of rules or a schedule that indicates when a particular pump or group of pumps should be turned on or off. The optimal policy will result in the lowest operating cost and highest efficiency of pump station [1].

Optimal pump scheduling policies will indeed decrease economic consumption of the whole flow systems. But, the dynamical and mixed-integer nature associated to the optimization of scheduling pump stations increases the complexity of the optimal control problem of water networks [2]. Some typical mixed-integer linear programming or dynamic programming-based algorithms are not applicable because of high computation load, being infeasible or may be not being efficient enough for large water networks [2][3].

MPC is a well-established class of advanced control methods [4][5] and is used for the control of distribution water networks [6][7] as it can compute advanced control strategies (e.g. to minimize energy cost) while guaranteeing certain quality of service and satisfying the operating constraints of the plant. MPC can handle large scale systems with operational constraints in states and inputs, multiple objectives and is flexible to extensions. However, the inclusion of discrete ON-OFF operation of pumps would lead to an hybrid MPC problem that would probably involve computational problems in case of large scale water networks.

The main contribution of this paper is to propose a two-layer control scheme for optimizing the scheduling of pump stations, avoiding mixed-integer optimization problems. At the upper layer, MPC is used to optimize the continuous flow model and then, at the lower layer, scheduling algorithm translates the continuous flow set-points into ON-OFF control operation sequences of the pumps. MATLAB and EPANET have been used to simulate and validate the proposed approach in the Richmond network case study [8].

This paper is organized as follows. Section II provides the control oriented modelling methodology for water distribution systems. Section III presents the MPC mixed-integer optimization problem for the scheduling of pump stations. Section IV provides the two-layer control scheme and describes the two layers individually. Section V provides the scheduling algorithm details with its tuning parameters. In Section VI, complexity analysis of this algorithm is provided. Then, in Section VII, the considered case study (Richmond Water Network) and the application results of the proposed methods are described and finally, the main conclusions are presented in Section VIII.

II. CONTROL-ORIENTED MODELLING METHODOLOGY

A control-oriented modelling approach is outlined for the upper layer, which follows the principles presented in [9], [7] and [10]. The extension to include the pressure-model can be found in references provided by [2] and [11].

A. Tanks and Reservoirs.

Water tanks and reservoirs provide the entire network with the water storage capacity. The mass balance expression of these storage elements can be written as the discrete-time difference equation

$$v_i(k + 1) = v_i(k) + \Delta t \left(\sum_j q_{u_j}(k) - \sum_l q_{u_l}(k) \right), \quad (1)$$

where v is the stored volume, q_{u_j} are the manipulated inflows and q_{u_l} are the manipulated outflows for the ith storage element, Δt is the sampling time and k denotes the discrete-time instant. The physical constraint related to the range of admissible water in the ith storage element is expressed as

$$\underline{v}_i \leq v_i(k) \leq \bar{v}_i, \quad \text{for all } k, \quad (2)$$

where $\underline{v}_i$ and $\bar{v}_i$ denote the minimum and the maximum admissible storage capacity, respectively.

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B. Actuators.

Two types of control actuators are considered: valves and pumps (more precisely, complex pumping stations). The manipulated flows through the actuators represent the manipulated variables, denoted as q_u. Both pumps and valves have lower and upper physical limits as system constraints. In (2), they are expressed as bound constraints

$$\underline{q_{u_i}} \le q_{ui}(k) \le \overline{q_{ui}}, \quad \text{for all } k, \tag{3}$$

where $\underline{q_{u_i}}$ and $\overline{q_{ui}}$ denote the minimum and the maximum flow capacity, respectively.

C. Nodes.

The nodes represent mass balance relations of inflows and outflows (e.g., because of a pipe junction).

$$\sum_j q_{u_j}(k) = \sum_h q_{u_h}(k). \tag{4}$$

We define an incidence matrix Λ_c for junction nodes in order to write equation (4) in matrix form, where the element in the ith column and j^{th} row of junction nodes incidence matrix Λ_c is defined as:

$$a_{ij} = \begin{cases} 1 & \text{if flow of branch } i \text{ enters node } j \\ 0 & \text{if branch } i \text{ and node } j \text{ are not connected} \\ -1 & \text{if flow of branch } i \text{ leaves node } j \end{cases} \tag{5}$$

Assuming one network has n_c static nodes and n_b branches, thus, this matrix has n_c rows and n_b columns.

A matrix form of equation (4) is as follows:

$$\Lambda_c q_u(k) = d(k) \tag{6}$$

where $q_u = (q_{u_1}, \ldots, q_{u_{n_b}})^T$ is a vector of branch flows, d denotes an augmented demand vector where zero components to nodes without demand associated with.

D. Demand Sectors.

Demand and irrigation sector represents the water demand made by the network users of a certain physical area. The demand forecasting algorithm, that runs in parallel with the control algorithm, typically uses a two-level scheme composed by *(i)* a time-series model to represent the daily aggregate flow values, and *(ii)* a set of different daily flow demand patterns according to the day type to cater for different consumption during the weekends and holidays periods as provided by [12].

E. Network control oriented model

Finally, using the models of the different elements of the water network presented above, the following linear discrete-time control oriented model can be obtained taking into account the network topology and parameters:

$$x(k + 1) = Ax(k) + Bu(k) \tag{7a}$$
$$y(k) = Cx(k) \tag{7b}$$
$$\Lambda_c u(k) = d(k) \tag{7c}$$

where $x(k) \in \mathbb{R}^{n_x}$ is state vector that corresponds with the tank volumes and $u(k) \in \mathbb{R}^{n_u}$ is vector of command variables that correspond with the valve/pump flow set-points, while $y(k) \in \mathbb{R}^{n_y}$ is vector of the measured output that correspond with the tank levels.

III. PROBLEM PRESENTATION

Let us consider that the pump scheduling time horizon $[t_0, t_f]$ can be split into K time steps with Δt_k length each, where $\Delta t_k = t_k - t_{k-1}$; $k = 1, ..., K$, $t_f = t_k$. This results in timing of the scheduling problem which is determined by the intervention time instants $t_0, t_1, ..., t_k, ..., t_f$. Naturally, the system control vector, $p(k)$, represents status of pumps (ON-OFF) in each of these time stages [2].

The pump scheduling problem (PSP) for a given time horizon can be formulated as follows:

$$\min_{p, \Delta t_k} \sum_{k=1}^{K} \alpha(k)\,\tilde{u}(k)\,p(k)\,\Delta t_k \tag{8a}$$

$$s.t. \quad x(k) = Ax(k) + B\tilde{u}(k), \quad k = 1, \cdots, K$$
$$x(0) = x_0, \quad k = 1, \cdots, K \tag{8b}$$
$$\Lambda_c \tilde{u}(k) = d(k) \tag{8c}$$
$$x_{min} \le x(k) \le x_{max}, \quad k = 1, \cdots, K$$
$$\tilde{u}_{min} \le \tilde{u}(k) \le \tilde{u}_{max}, \quad k = 1, \cdots, K$$
$$p(k) \in \{0, 1\}, \quad k = 1, \cdots, K$$

where $\tilde{u}(k)$ is the nominal pump flows (when the pump is ON) and $\alpha(k)$ is the unitary electrical costs for the k time stage, $x(k)$ represents the continuous tank volumes, and the system operating cost associated to pumping.

The PSP is solved by selecting a proper Δt_k according to pump operatinal constraints and pump control sequence p that requires minimal economic pumping cost while satisfying flow or volume requirements induced by the demands. The small value of Δt_k and the complex topology and number pumps could consequently increase the computation load. The mixture of discrete control parameters (ON-OFF pump schedule) together with the continuous dynamics of tank volumes makes PSP problem a complex mixed-integer optimization problem [2].

For this complex mixed-integer problem, the method of conversion of the mixed-integer problem into the continuous one by *switching times as the control variables* is indeed useful, but the solution is obtained at the expense of an increased number of decision variables, whose applicability is limited to rather small networks due to poor robustness with respect to numerical errors [2]. This paper will derive an efficient alternative approach which separates the mixed-integer problem into individual continuous and discrete problems by decomposition of the scheduling problem into a two-layer control scheme, as Fig. 1 shows.

IV. PRESENTATION OF THE TWO-LAYER CONTROL SCHEME

As shown in Fig. 2, the proposed control scheme includes two layers. The upper layer is the continuous MPC model
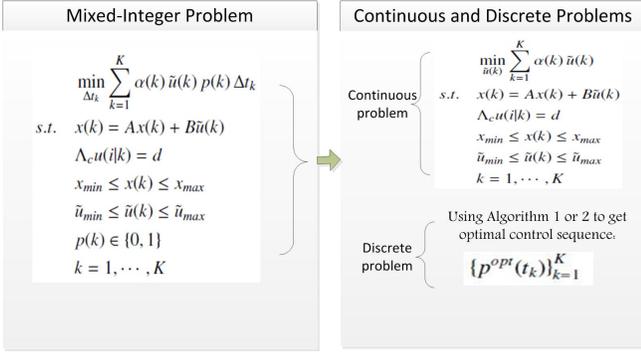
Fig. 1: Presentation of the proposed approach

that produces continuous pump flow set-points. Sampling time in the upper layer is one hour and every pump station is simplified into a controlled flow $u(k)$ and cost (electricity price) model $\alpha(k)$. The lower layer is the scheduling algorithm, who works in Δt_k (smaller than one hour) sampling time, is responsible of translating the continuous flow set-points into discrete ON-OFF actions to be executed by the pumps. The resulting pump schedule is simulated by EPANET before being sent to real pumps in the network.
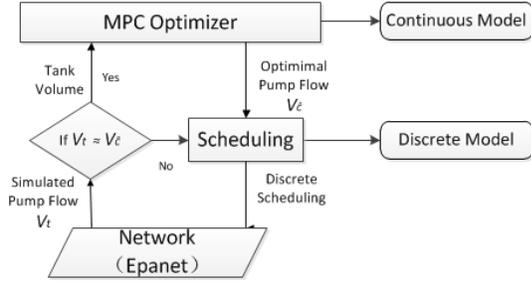

Fig. 2: Two-layer Control Scheme

A. Optimizing Flow at the Upper layer

MPC is used to produce optimal continuous set-points pump flows for being scheduled in the lower layer. The extension to include non-linear pressure model is researched in parallel in [13], which uses CSP (Constraints Satisfaction Problem) to transfer the non-linear MPC into linear ones with added constraints.

The upper layer MPC problem is based on a linear discrete-time prediction model obtained applying the control oriented methodology introduced in Section II considering the network topology and parameters:

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{9a}$$

$$s.t. \quad x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1,\cdots,H_p$$

$$x(0|k) = x_k \tag{9b}$$

$$\Lambda_c u(i|k) = d \tag{9c}$$

$$x_{min} \leq x(i|k) \leq x_{max}, \quad i = 1,\cdots,H_p$$

$$u_{min} \leq u(i|k) \leq u_{max}, \quad i = 0,\cdots,H_{p-1}$$

As described above J is a performance index, representing operational goals of the system, H_p is the prediction horizon, $x(0)$ is the initial condition of the state vector, x_{min} and x_{max} are minimal and maximal capacities of tanks, while u_{min} and u_{max} are known vectors defining the saturation constraints on inputs variables. *Problem* (9a) can be recast as a Quadratic Programming (QP) problem, whose solution:

$$\mathcal{U}^*(k) \triangleq [u^*(0|k)\cdots u^*(H_p-1|k)]^T \in \mathbb{R}^{H_p m \times 1} \tag{10}$$

is a sequence of optimal control inputs that generates an admissible state sequence. Only the first optimal move $u^*(0|k)$ of the optimal sequence $\mathcal{U}^*(k)$ is applied to the optimal process and the optimization is repeated at time $k+1$.

1) Operational Goals: The water distribution network is operated with a 24-hour horizon, at hourly time interval. The main operational goals to be achieved are:

- *Cost reduction* (J_{cost}): Minimize water cost during water supplying process, which includes operating cost of the pump station and also water source cost.
- *Operational safety* (J_{safety}): Maintain appropriate water storage levels in dams and reservoirs of the network for emergency-handling situations.
- *Control actions smoothness* ($J_{smoothness}$): Smooth flow set-point variations for sustainable process operation.

Above mentioned goals lead to the following function:

$$J = J_{safety} + J_{smothness} + J_{cost}$$
$$= \varepsilon_{\widetilde{x}}(k)^\top W_{\widetilde{x}}\varepsilon_{\widetilde{x}}(k) + \Delta\widetilde{u}(k)^\top W_{\widetilde{u}}\Delta\widetilde{u}(k) \tag{11}$$
$$+ W_a(a_1 + a_2(k))\widetilde{u}(k)$$

where

$$\varepsilon_{\widetilde{x}}(k) = \widetilde{x}(k) - \widetilde{x}_r$$
$$\widetilde{u} = \Theta\Delta\widetilde{u} + \Pi\widetilde{u}(k-1)$$
$$\Delta\widetilde{u}(k) = \widetilde{u}(k) - \widetilde{u}(k-1)$$

$$\Theta = \begin{pmatrix} I_{m_i} & 0 & \cdots & 0 \\ I_{m_i} & I_{m_i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_{m_i} & I_{m_i} & \cdots & I_{m_i} \end{pmatrix}, \quad \Pi = \begin{pmatrix} I_{m_i} \\ I_{m_i} \\ \vdots \\ I_{m_i} \end{pmatrix}.$$

and $W_{\widetilde{x}}$, $W_{\widetilde{u}}$, W_a are also related weights.

The vectors a_1 and a_2 contain the cost of water treatment and pumping price, respectively and $\widetilde{x}_r$ is the penalty level for emergency-handling in reservoirs of the network.

2) Formulation of the optimization problem: The objective functions (11) of the MPC problem can be formulated in the following way:

$$J = z^T \Phi z + \phi^T z + c \tag{12}$$

where

$$z = [\Delta\widetilde{u} \quad \varepsilon_{\widetilde{x}} \quad \varepsilon]^T \tag{13}$$

and c is a constant value.

This allows to determine optimal control actions at each instant k by solving a quadratic optimization problem by means of quadratic programming (QP) algorithm in form as:

$$\min_z z^\top \Phi x + \phi^\top z$$

$$A_1 z \le b_1$$

$$A_2 z = b_2$$

B. Pump scheduling of the Lower layer

Denoting $\hat{c}$ as the optimal flow set-points produced by the upper layer MPC controller during the time period $[t_0, t_f]$, the total water volume pumped during this period is

$$V_{\hat{c}} = \hat{c}\,(t_f - t_0) \tag{14}$$

As explained in Section II, the scheduling algorithm will split the time period $[t_0, t_f]$ into K time steps with Δt_k step length. Let us denote p as the vector which contains the discrete ON ($p(i) = 1$) and OFF ($p(i) = 0$) pump control actions [14] and $\tilde{u}(t)$ as its nominal pump flow (that is when the pump is ON). Then, the total water volume drawn by these pump control actions during $[t_0, t_f]$ is

$$V_t(p(1), \cdots, p(K)) = \sum_{i=1}^{K} p(i) \int_{t_{i-1}}^{t_i} \tilde{u}(t)\,dt \tag{15}$$

The goal of the scheduling algorithm is to minimize the difference between $V_{\hat{c}}$ and V_t in (14) and (15). Since this difference could not completely be eliminated, the scheduling algorithm should find a scheduling sequence such that the following control objective is minimized

$$J_{dis} = V_t - V_{\hat{c}} \tag{16}$$

V. FACTORS AFFECT SCHEDULING ALGORITHMS

There are two parameters which can affect accuracy of the scheduling algorithm as described in previous section
- time interval (Δt_k)
- number of units in the pump configuration

A. Time interval

In order to guarantee that the pump station configuration can meet the pump flow set-points provided by the upper layer, the sampling time can be selected to reduce the error introduced in (16).

Assuming Δt_k is small enough in order to accurately compute the term $\int_{t_{i-1}}^{t_i} \tilde{u}(t)\,dt$ in (15) as $u^*(k)\,\Delta t_k$, where $u^*(k)$ is the nominal pump flow in time stage $[t_{k-1}, t_k]$, the equation (15) can be rewritten as:

$$V_t(p(1), \cdots, p(K)) = \sum_{i=1}^{K} p(i) \int_{t_{i-1}}^{t_i} \tilde{u}(t)\,dt \cong \sum_{i=1}^{K} p(i) u^*(k)\,\Delta t_k \tag{17}$$

Consequently, the accuracy of scheduling algorithm according to (16) can be calculated as follows:

$$J_{dis} = \min(V_t - V_{\hat{c}}) \cong \min\left(\sum_{i=1}^{K} p(i) u^*(k)\,\Delta t_i - \hat{c}\,(t_f - t_0)\right) \tag{18}$$

In practice, J_{dis} is affected by Δt_k, the smaller Δt_k is, the smaller will be (18). However, the value of Δt_k is limited by technological constraints imposed by the pumps. Pump scheduling algorithm with Δt_k works as presented in Algorithm 1.

Algorithm 1 Scheduling algorithm for one pump

1: $p^{opt} = [p(1), p(2), ..., p(K)]$
2: $p(1) = 1$
3: **for** $i := 2$ to K **do**
4:     $p(i) = 0$
5: **end for**
6: **for** $i := 2$ to K **do**
7:     Get J_{dis} using Equation (18)
8:     **if** $J_{dis} < 0$ **then**
9:         $p(i) = 1$
10:     **end if**
11: **end for**

In this algorithm, p^{opt} is the optimal working schedule for the pump, and J_{dis} is the optimal scheduling accuracy.

B. Parallel pump configuration

If a single pump cannot meet the pump flow set-points determined in the upper layer, additional units in the pump station should be activated.

Assuming $n - 1$ supplementary units are available at the pump station in order to minimize (16):

$$V_{\hat{c}} \le V_{1t} + V_{2t} + ... + V_n \tag{19}$$

Then, scheduling accuracy of (18) could be evaluated as follows

$$J_{dis} = \min(V_{1t} + V_{2t} + ... + V_n - V_{\hat{c}})$$
$$\cong \min\left(\sum_{1}^{n} \sum_{i=1}^{K} p(i) u^*(k)\,\Delta t_k - \hat{c}\,(t_f - t_0)\right) \tag{20}$$

where n means the number of parallel units in the pump station which is another factor that could be used to increase the schedule accuracy: the bigger n is, the more degrees of freedom and the higher accuracy could be achieved by means of the scheduling algorithm.

Algorithm 2 presents the extension of Algorithm 1 to n parallel units of the pump station.

The values of p_n^{opt} are the optimal schedules of the parallel pumps, J_{dis} is the optimal scheduling accuracy.

VI. COMPLEXITY

Regarding complexity, computation load of the scheduling Algorithm 1 is K, where $K = \frac{t_f - f_0}{\Delta t_k}$. This means that, Δt_k can affect computation load of the algorithm since more computations will be added with a smaller Δt_k, and consequently decreased with a bigger Δt_k. The same reasoning can be used in case of Algorithm 2, where the computation load is K^n and the computation load is increased when the number of units n in parallel. Because of that, although smaller time interval and more parallel pumps can increase scheduling accuracy, more computation load is needed. Therefore, it is important to choose proper Δt_k and n even that establishes a trade-off between accuracy and computation load.

Algorithm 2 Scheduling algorithm for parallel pumps

1: $ms = 1$
2: $n = 1$
3: **while** $ms = 1$ **do**
4:     $ms = 0$
5:     $p_n{}^{opt} = [p_n(1), p_n(2), ..., p_n(K)]$
6:     $p_n(1) = 1$
7:     **for** $i := 2$ to K **do**
8:         $p_n(i) = 0$
9:     **end for**
10:     **for** $i_n := 2$ to K **do**
11:         Get J_{dis} using Equation (20)
12:         **if** $J_{dis} < 0$ **then**
13:             $p_n(i_n) = 1$
14:         **end if**
15:         **if** $i_n = K$ and $J_{dis} < 0$ **then**
16:             $n = n + 1$
17:             $ms = 1$
18:         **end if**
19:     **end for**
20: **end while**

VII. CASE STUDY

The case study used to test the proposed approaches is the Richmond water distribution system [15]. Fig. 3 shows the conceptual diagram of Richmond network. A MPC controller at the upper layer is used to produce the pump flow set-points, while the pump scheduling algorithm described in Section IV-B is used to transfer the continuous flow set-points into discrete ON-OFF operations of the pump.

The MPC controller and the pump scheduling algorithm are implemented into MATLAB, while the simulation of the Richmond network is realized using EPANET, which simulates the water network using a discretization time step Δt_k to realize operations of the scheduling algorithm.
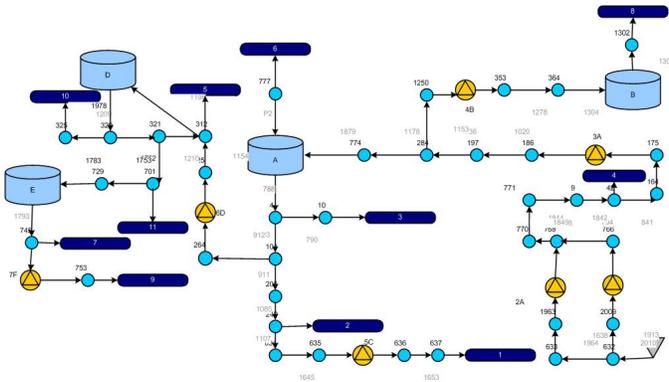

Fig. 3: The Richmond water distribution system

A. Results for the upper layer MPC controller

As described in Section IV-A, the objective function of the upper layer MPC controller leads to minimize the electrical pumping cost. Fig. 4 shows pump flow together with its electricity fee. From this figure, it is confirmed that pump send more water at its lower price period but less or no water at the higher price period, which is consistent with cost term in the objective function.
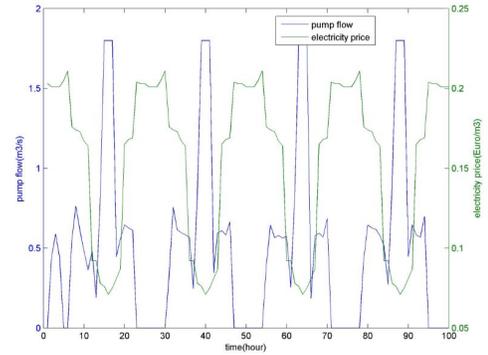

Fig. 4: Pump flow with electricity price

B. Results for the lower layer scheduling algorithm

After applying scheduling algorithm, continuous optimal flow will be scheduled into discrete pump actions. Figure 5 shows in detail the pump actions of *Pump*4*B* after using the scheduling algorithm.
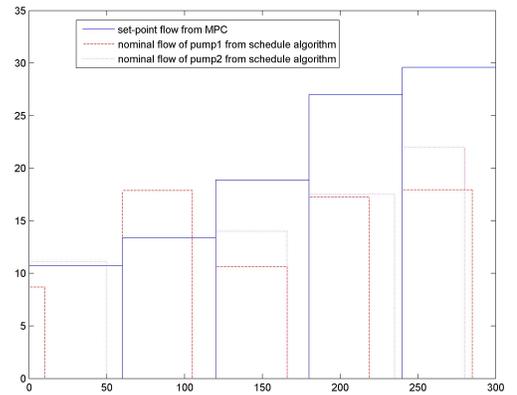

Fig. 5: Optimal Schedule for Pump4B with two pump branches

C. Scheduling Results using Different Δt_k

As analyzed in Section IV, time interval Δt_k can affect accuracy of scheduling algorithm. Take *pump4B* as an example. In this case, the sampling time at the upper layer that determines $t_f - t_0$ is equal with 1 hour while the sampling time at the lower layer will be changed from one minute to two minutes to see the effect in the scheduling algorithm result. Δt_k will use 1 minute and 2 minute two different values. Scheduling accuracies for these two different sampling times at lower layer are plotted in Fig. 6, which proves that, the smaller time interval can lead to higher scheduling accuracy.

Accuracy comparisons are provided in detail in Table I, which shows that, the scheduling accuracy when lower layer sampling time Δt_k is 1 minute results in 0.64%, which is much smaller than that of 1.71% when Δt_k is 2 minutes.
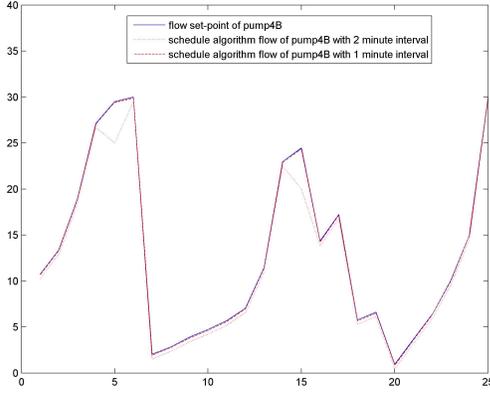
Fig. 6: Flow errors in different time intervals

TABLE I: Accuracy Comparison

Sc.	With 2-minute Time Interval			
Es.	Optimal flow	Simulated flow	Flow Errors	Errors in Prop.
T.	332.4045	326.7351	5.6694	1.71%
Sc.	With 1-minute Time Interval			
Es.	Optimal flow	Simulated flow	Flow Errors	Errors in Prop.
T.	329.6	327.4818	2.1182	0.64%

D. Scheduling Results for Different Pump Configurations

Number of parallel pump branches can also affect scheduling accuracy, since the bigger the number is, the higher the accuracy is. Considering $t_f - t_0$ of *pump4B* as 1 hour, Δt_k as 1 minute cases with single *pump4B* and two paralleled pump *pump4B* are simulated. Their accuracies are provided in Fig. 7.
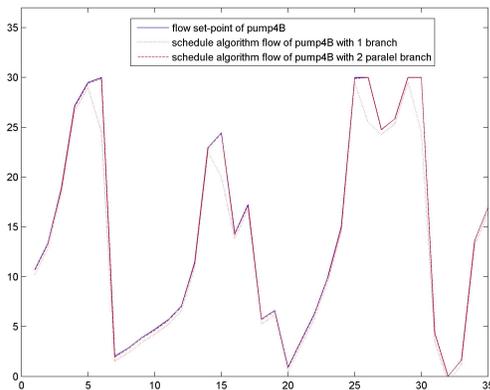

Fig. 7: Flow errors in different parallel when $\Delta t_k = 1$

Accuracy comparisons are provided in detail in Table II, which shows that, the scheduling accuracy at the paralleled pump station is nearly 100% and much higher than that at the single pump.

TABLE II: Accuracy Comparison

Sc.	With Single Pump			
Es.	Optimal flow	Simulated flow	Flow Errors	Errors in Prop.
T.	526.7603	521.3726	5.3877	1.02%
Sc.	With 2 Pumps in Parallel			
Es.	Optimal flow	Simulated flow	Flow Errors	Errors in Prop.
T.	518.1651	518.0742	0.0909	0.01%

VIII. CONCLUSION

This paper has proposed a two-layer control scheme for scheduling pump stations in water distribution networks in order to solve the mixed-integer optimization problem. The upper layer, who works in one-hour sampling time, uses a MPC strategy to optimize continuous flow model to produce set-point pump flows for the lower layer. While in the lower layer, a scheduling algorithm is used to translate the optimal continuous set-points flow into ON-OFF pump operations. The effect of tuning parameters (sampling time and pump configurations) of the scheduling algorithm is compromised chose in reality according to scheduling accuracy and computation load.

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