Combining model predictive control with constraint-satisfaction formulation for the operative pumping control in water networks

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Abstract

This paper proposes a method to combine linear Model Predictive Control (MPC), a Constraint Satisfaction Problem (CSP) formulation and a Network Aggregation Method (NAM) for the predictive operational control of water pumping in DWNs. The proposed method can produce optimal pumping strategies for complex DWNs in short computation times, while avoiding the need for non-linear programming techniques to cater for non-linear flow-head equations. The proposed approach is simulated using Epanet to represent the hydraulic DWNs. The D-Town benchmark water network is used as a case study.

Keywords: linear MPC, CSP, DWNs, NAM, Epanet

1. Introduction

Limited supplies as well as the complex infrastructure and, possibly, parametric uncertainty make water management a challenging problem. The use of non-linear models in DWNs is essential for the operational control which involves manipulating not only flow but also pressure head models [1]. No general-solution exists for the non-linear programming problem when it is non-convex [3]. Plenty of optimization approaches rely on a simplified network hydraulic model [8, 10, 11, 13], which is often unacceptable in practice. Other authors employ discrete dynamic programming [12, 14–18], which is mathematically sound but only applicable to small networks. The MPC method has been successfully applied to control and optimize linear flow model of DWNs [21, 22] but the computation burden of MPC will increase with size of network when the non-linear pressure head model is considered.

Solving the non-linear problem of DWNs by the combined of linear MPC and CSP aims to maintain optimality and feasibility with the added linear constraints [4]. The real hydraulic behavior of the DWNs is simulated by means of Epanet [6]. As shown in Figure 1, the whole controlling methodology works in a three-layer structure as initially proposed in [7]: the original DWN is firstly simplified into a conceptual network using NAM in the first layer, CSP is the second step of this methodology and it is used for converting the non-linear pressure head constraints into linear MPC constraints. MPC is the lower layer producing optimal set-points for controlling actuators according to the defined objective functions. The continuous flow set-points in the lower layer are translated to ON-OFF pump

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operation using the Pump Scheduling Algorithm (PSA), which optimizes the difference between optimal pump flow $V_{\hat{c}}$ and the simulated pump flow V_t [2].

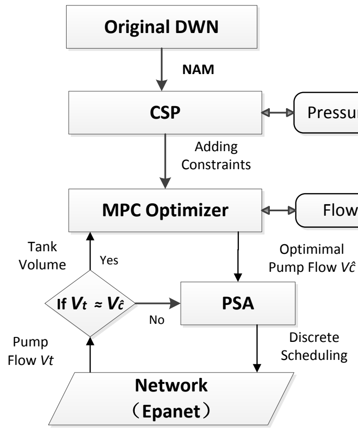


Figure 1: The multi-layer control scheme

2. Control-Oriented Modelling Methodology

2.1. Flow Model

2.1.1. Reservoirs and Tanks.

The mass balance expression of these storage elements relating the stored volume V, the manipulated inflows $q_{\text{in}}^{i,j}$ and outflows $q_{\text{out}}^{i,l}$ for the i^{th} storage element can be written as the discrete-time difference equation

$$V_i(k+1) = V_i(k) + \Delta t \left(\sum_j q_{\text{in}}^{i,j}(k) - \sum_l q_{\text{out}}^{i,l}(k) \right), \tag{1}$$

where Δt is the sampling time and k denotes the discrete-time instant. The physical constraint related to the range of admissible water in the i^{th} storage element is expressed as

$$\underline{V}_i \leq V_i(k) \leq \overline{V}_i, \quad \text{for all } k, \tag{2}$$

where $\underline{V}_i$ and $\overline{V}_i$ denote the minimum and the maximum admissible storage capacity, respectively.

2.1.2. Actuators.

The manipulated flows through the actuators (valves and pumps) represent the manipulated variables, denoted as q_u. Both pumps and valves have lower and upper physical bounds. As in (2), they are expressed as

$$\underline{q_u}_i \leq q_{ui}(k) \leq \overline{q_u}_i, \quad \text{for all } k, \tag{3}$$

where $\underline{q_u}_i$ and $\overline{q_u}_i$ denote the minimum and the maximum flow capacity, respectively.

2.1.3. Nodes.

The nodes are modelled as equality constraints related to inflows and outflows:

$$\sum_j q_{\text{in}}^{i,j}(k) = \sum_h q_{\text{out}}^{i,h}(k). \tag{4}$$

2.1.4. Demand Sectors.

A demand sector represents the water demand made by the network users of a certain physical area. The demand forecasting algorithm is typically explained in[23].

2.2. Pressure Head Model

When considering pressure heads, the flow model presented in the previous section should be extended using the non-linear relation between flow and pressure head, which appears at pipes, valves, pumps and tanks [24].

2.2.1. Pipes.

The Hazen-Williams model is one of the various widely used models to describe pressure head (h: sum of pressure and topographic elevation) loss between two nodes i and j linked by a pipe:

$$g(q) = h_i - h_j = g_{ij}(q_{ij}) = R_{ij}q_{ij}^{1.852} \tag{5}$$

where

$$R_{ij} = (10.654 \times L_{ij})/({C_{ij}}^{1.852} \times {D_{ij}}^{4.871}) \tag{6}$$

and L_{ij}, D_{ij} and C_{ij} denote the pipe length, diameter and Hazen-Williams coefficient.

2.2.2. Pumps.

Pumps introduce a increase of pressure head between the suction node s and the delivery node d. In the case that corresponds to variable speed pumps, the relation between the flow and the pressure head increase is given by:

$$g(q,u,s) = h_d - h_s = \begin{cases} Wq^2 + Mq + Ns^2, & \text{if } \ u \neq 0 \ \text{ and } \ s \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where s is the pump speed and u corresponds to the number of pumps that are turned on, W, M and N are pump specific coefficients.

2.2.3. Valves.

The valves can be modelled as a pipe with controlled conductivity, that is

$$g_{ij}(q_{ij}) = G_{ij}R_{ij}q_{ij}^2 \tag{8}$$

where R_{ij} is pipe conductivity and G_{ij} is control variable to manipulate the valve from 0 (closed) to 1 (open).

2.2.4. Tanks.

The pressure head established by the i^{th} tank is given by the following equation:

$$h_{ri}(t) = \frac{V_i(t)}{Sec_i} + E_i \tag{9}$$

where Sec_i is the cross-sectional area of the tank and E_i is the tank elevation.

3. Operational Control Problem Statement

3.1. MPC for Flow Control

Standard MPC problem based on the linear discrete-time prediction model is considered as in [25][5]:

$$x(k+1) = Ax(k) + Bu(k), \tag{10a}$$

$$y(k) = Cx(k), \tag{10b}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector representing the tank volume and $u(k) \in \mathbb{R}^{n_u}$ is vector of command variables of actuator flows at time step k, $y(k) \in \mathbb{R}^{n_y}$ is the vector of the measured outputs. Matrices A and B are obtained using flow modelling approach in Section 2.1.

Assume one network has n_c non-storage nodes and b branches. We define matrix Λ_c with n_c and b dimensions for junction nodes in order to write equation (4) in matrix form, where the definition is:

$$a_{ij} = \begin{cases} 1 & \text{if flow of branch } i \text{ enters node } j \\ 0 & \text{if branch } i \text{ and node } j \text{ are not connected} \\ -1 & \text{if flow of branch } i \text{ leaves node } j \end{cases} \tag{11}$$

A matrix form of equation (4) is as follows:

$$\Lambda_c q = d \tag{12}$$

where $q = (q_1, \ldots, q_b)^T$ is a vector of branch flows, d denotes an augmented demand vector by zero components corresponding to non-loaded nodes.

Thus, the following problem has to be solved when only the flow model for the DWN is considered:

Problem 1:

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{13a}$$

$$\begin{aligned} s.t.\quad & x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1, \cdots, H_p, \\ & x(0|k) = x_k, & (13b) \\ & \Lambda_c u(i|k) = d(i|k) & (13c) \\ & x_{min} \leq x(i|k) \leq x_{max}, \quad i = 1, \cdots, H_p, \\ & u_{min} \leq u(i|k) \leq u_{max}, \quad i = 0, \cdots, H_{p-1}, \end{aligned}$$

Problem 1 can be recast as a Quadratic Programming (QP) problem:

$$\mathcal{U}^*(k) \triangleq [u^*(0|k) \cdots u^*(H_p - 1|k)]^T \in \mathbb{R}^{H_p m \times 1} \tag{14}$$

At each sampling time k, *Problem 1* is solved for the given measured current state $x(k)$. Only the first optimal set-point $u^*(0|k)$ of the optimal sequence $\mathcal{U}^*(k)$ is applied to the process:

$$u_{MPC}(k) = u^*(0|k) \tag{15}$$

while the remaining optimal moves are discarded and the optimization is repeated at time $k + 1$.

3.2. Operational Goals for Flow Control

The main operational goals to be achieved in water distribution networks are:

- *Cost reduction* (J_{cost}): Minimize water cost during water supplying process.
- *Operational safety* (J_{safety}): Maintaining appropriate water storage levels in tanks for emergency-handling.
- *Control actions smoothness* ($J_{smoothness}$): Smooth flow set-point variations to present water in infrastructure.

3.3. Nodal Model for pressure head Control

When considering flow and pressure head in the DWN model, the dynamic equations of the tanks are:

$$\begin{cases} h_{ri}(t) = \frac{v_i(t)}{Sec_i} + E_i \\ V_i(k+1) = V_i(k) + \Delta t \left(\sum_j q_{\text{in}}^j(k) - \sum_h q_{\text{out}}^h(k) \right) \end{cases} \tag{16}$$

according to equation (1) and equation (9).

Considering a network with n nodes and b branches, the matrix Λ will have n rows and b columns. The value of element b_{ij} which appears in the i^{th} row and the j^{th} column of matrix Λ has the similar definition as a_{ij} of equation (11).

For the sake of convenience, we will place the rows corresponding to the tank/reservoir nodes on the first n_r position. The other rows correspond to the junction nodes. With the help of matrix Λ, we can write the flow-head equations as the following vector equation:

$$\Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(q) = 0 \tag{17}$$

where

- $h_r = (h_{r1}, \cdots, h_{r,n_r})^T$ pressure heads of tank nodes
- $h = (h_1, \cdots, h_{n_c})^T$ pressure heads of junction nodes
- $q = (q_1, \cdots, q_b)^T$ branch flows
- $G(q) = (g_1(q_1), \cdots, -g_i(q_i, u_i), \cdots, g_1(q_1, v_1), \cdots,)^T$
  functions defining flow-head relationships

This equation combined with equation (4) yields the nodal model:

$$\begin{cases} \Lambda_c q = d \\ \Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(q) = 0 \end{cases} \tag{18}$$

3.4. *MPC for Pressure Head Control*

In the case of pressure head control of DWNs, the MPC problem is defined as:

Problem 2

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{19a}$$

$$\begin{aligned} s.t. \quad & x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1, \cdots, H_p, \\ & x(0|k) = x_k, && (19b) \\ & \Lambda_c u = d && (19c) \\ & h_r(i|k) = \frac{x(i|k)}{Sec_i} + E_i && (19d) \end{aligned}$$

and:

$$\Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(u) = 0 \tag{20a}$$

where:

$$\begin{aligned} & x_{min} \leq x(i|k) \leq x_{max}, \quad i = 1, \cdots, H_p, \\ & u_{min} \leq u(i|k) \leq u_{max}, \quad i = 0, \cdots, H_{p-1}, \end{aligned} \tag{21}$$

As described above, MPC of pressure head control is highly non-linear because of added pressure head constrains (20a), which adds complexity to the optimization problem for the large scale DWNs.

4. Proposed Approach

4.1. Definition of CSP

As introduced in [9], a CSP on sets can be formulated as a 3-tuple $\mathcal{H} = (\mathcal{V}, \mathcal{D}, C)$, where

- $\mathcal{V} = \{v_1, \cdots, v_n\}$ is a finite set of variables.
- $\mathcal{D} = \{\mathcal{D}_1, \cdots, \mathcal{D}_n\}$ is the set of their domains.
- $C = \{c_1, \cdots, c_n\}$ is a finite set of constraints relating variables of $\mathcal{V}$.

Solving a CSP consists of finding all variable value assignments such that all constraints are satisfied. More detail definition and calculation process can be referenced in [2].

4.2. Network Aggregation Method (NAM)

Considering the definition and interval implementation characteristics of CSP, the building constraints C in DWNs can only be realized in one-directional network [7]. The bidirectional pipes in DWNs have added difficulties to build the required set for CSP. In order to apply successfully CSP-MPC to the bidirectional DWNs, NAM is used to simplify a complex water network into an equivalent simplified conceptual one [2, 26].

4.2.1. Simplification.

In order to obtain the conceptual model, the first step is aggregating the nodes in terminal branches with no control elements. We define the distance between nodes n_k and n_l by the pressure head and flow difference between them:

$$distance(n_k, n_l) = \Delta Ele(n_k, n_l) + \sum_{j=1}^{p} \Delta P_j \tag{22a}$$

$$flow(n_k, n_l) = \Delta flow(n_k, n_l) \tag{22b}$$

where ΔP_j means pressure head loss at arc j and p is the path between n_k and n_l, $\Delta Ele(n_k, n_l)$ and $flow(n_k, n_l)$ are the elevation and flow difference between node n_k and n_l.

Following all the nodes from the terminal branch upstream, the nodes whose upstream is also a demand node and connected by pipe, can be deleted after adding their pressure head and flow distances. As shown in Fig. 2, node n_6 can

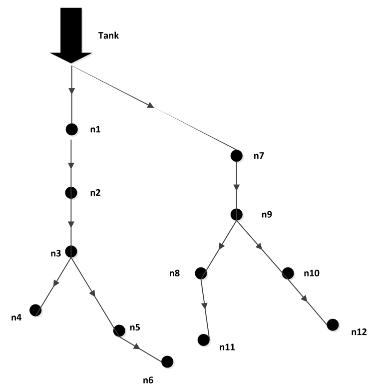


Figure 2: Node topology example used to illustrate NAM

be deleted after adding $distance(n_6, n_5)$ and $flow(n_6, n_5)$ to node n_5 and then continue to the upstream. Both node n_4 and node n_5 can be deleted after adding $max(distance(n_4, n_3), distance(n_5, n_3))$ and $sum(flow(n_4, n_3), flow(n_5, n_3))$ to node n_3. This process will continue until the branch meets pumps, valves or tanks.

4.2.2. Conceptualization

The main idea of this conceptual modelling approach is to assign demands to specific sources (tanks). Considering that water flows in pumps and valves are unidirectional, demand nodes located between pumps/valves and tanks can be considered as a demand allocated directly to a source (tank). This is illustrated in Fig. 3 and CSP will be used to guarantee the equivalence of both schemes.

It is worth noticing that the conceptual model is related to a specific network configuration. If the configuration is changed, the conceptual model must be revised to make sure it represents the network operation correctly.

Figure 3: Network conceptualization

4.3. Overview of the Integrated Control Scheme

4.3.1. Combining Linear MPC with CSP.

The main principle of the proposed approach is transferring the constraints of the non-linear pressure head model into safety volumes in the tanks, which may be tackled by the flow MPC model combined with the CSP algorithm. The linear constraints produced by CSP from the conceptual network will be combined with the initial constraints of linear MPC for the original DWNs as new constraints. With this scheme, *Problem 2* which is non-linear MPC will be updated into linear MPC problem:

Problem 3

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{23a}$$

$$\begin{aligned} s.t. \quad & x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1, \cdots, H_p, \\ & x(0|k) = x_k, \qquad (23b) \\ & \Lambda_c u = d \qquad (23c) \end{aligned}$$

where:

$$\begin{aligned} x'_{min} \leq x(i|k) \leq x'_{max}, \quad i = 1, \cdots, H_p, \qquad (24a) \\ u'_{min} \leq u(i|k) \leq u'_{max}, \quad i = 0, \cdots, H_{p-1}, \end{aligned}$$

The constraints x'_{min}, x'_{max}, u'_{min}, u'_{max} are obtained by CSP-MPC as Algorithm 1 presented in next section.

4.3.2. The Algorithm of Linear MPC with CSP.

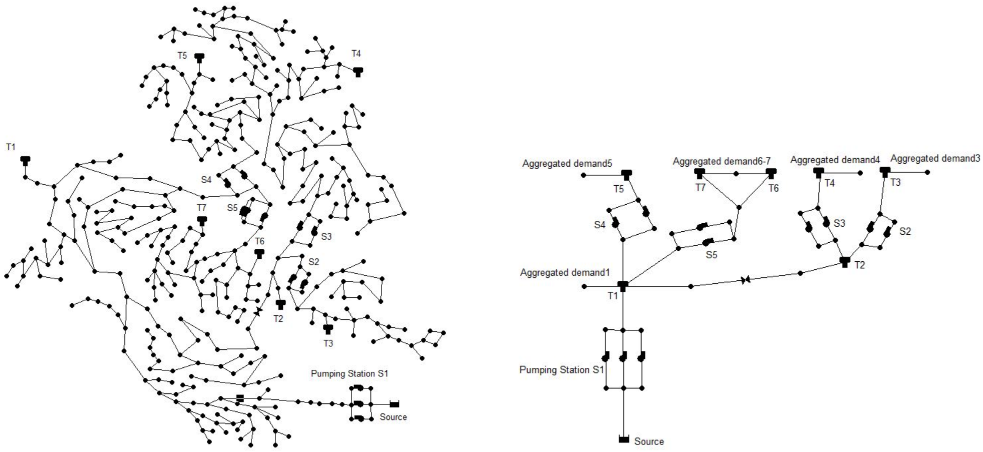
The algorithm combining Linear MPC with CSP is described as Algorithm 1. At each time iteration, this CSP algorithm will produce updated constraints equation (24) to *Problem 2* by means of propagating the effect of non-linear constraints equation (20a) into the operational constraints equation (21), which will be used for linear MPC to generate optimized control strategies.

5. Case Study and Results

The D-Town network is used as the case study, which is a complex DWN with 388 nodes, 405 actuators and 7 tanks and multiple bidirectional links as shown in sub-figure (*a*) of Fig. 4 [19, 20]. Sampling time used by the CSP-

Algorithm 1 CSP-MPC Algorithm

for $k := 1$ to H_p **do**
    $\mathcal{U}(k-1) \Leftarrow [u_{min}(k), u_{max}(k)]$
    $\mathcal{X}(k) \Leftarrow [x_{min}(k), x_{max}(k)]$
end for
$\mathcal{V} \Leftarrow \overbrace{x(1), x(2), ..., x(H_p)}^{X}, \overbrace{u(0), u(1), ...u(H_p-1)}^{U}$
$\mathcal{D} \Leftarrow \mathcal{X}(1), \mathcal{X}(2)...\mathcal{X}(H_p), \mathcal{U}(0), \mathcal{U}(1)...\mathcal{U}(H_p-1), D(0), D(1)...D(H_p-1)$
$C \Leftarrow \Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(u) = 0$
$\mathcal{H} \Leftarrow \mathcal{V}, \mathcal{D}, C$
$S = solve(\mathcal{H})$
Update limits for the linear MPC problem using the CSP solution



(a) Original D-Town network (b) Conceptual D-Town network

Figure 4: D-Town Networks

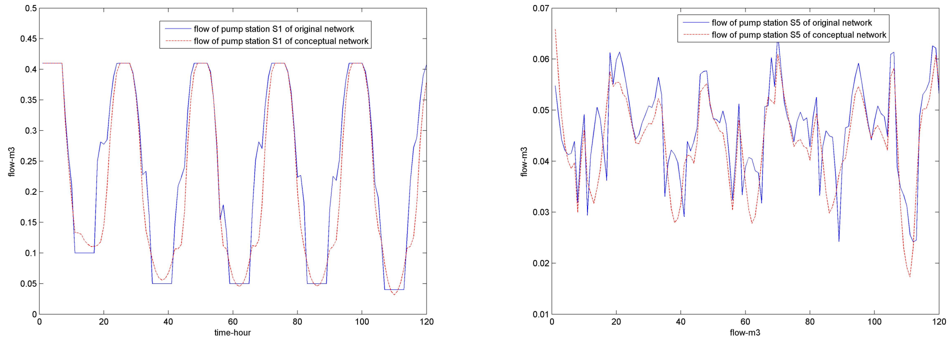
MPC controller is 1 hour, while the needed real-time computing time for every iteration of CSP-MPC is around 60 seconds which is mainly consumed by PSA.

5.1. Results of NAM

With the algorithm NAM, the original D-Town network is first simplified into a network with 88 nodes, 144 actuators and 7 tanks. Then, the conceptual network of D-Town is presented in sub-figure (b) of Fig. 4, where all the demand nodes have been aggregated inside one demand node connected with tanks. Fig. 5 shows the consistency of optimized pumping flow in the original and conceptual networks, which confirms the feasibility of NAM.

5.2. Results of Linear MPC and CSP

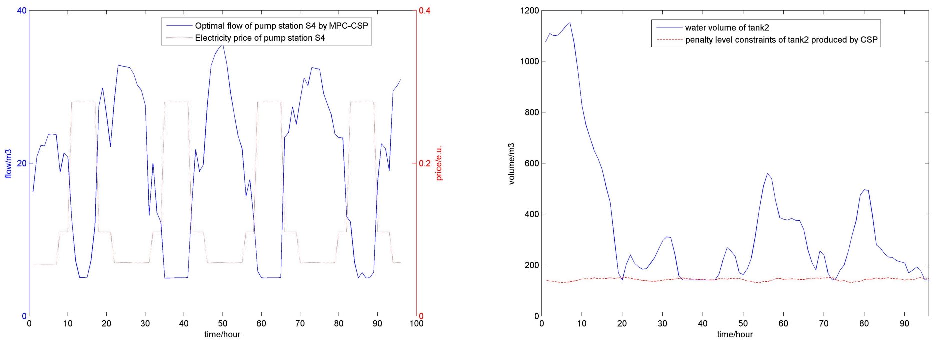
The objective function of MPC includes minimizing the economic water transportation cost. Sub-figure (a) of Fig. 6 shows in the same plot the pump flow after applying MPC with the electricity fee of pump station S_4. This figure shows that MPC decides to pump when the electricity price is lower to reduce the operational cost. By means of CSP, non-linear pressure head equations have been transferred into linear constraints that impose new limitations for tanks and actuators. Sub-figure (b) of Fig. 6 shows evolution of real tanks volume compared with its updated minimal safety volume, which has been produced by CSP in order to satisfy the required pressure head. As shown from the plot, the added constraints for tanks affect water volume to guarantee pressure head for the demand node.



(a) Flow comparisons of pump S1

(b) Flow comparisons of pump S5

Figure 5: Pump flow comparisons between original and conceptual networks



(a) Pump flow with electricity price

(b) Water evolution of tank2

Figure 6: Results of combined Linear MPC and CSP

5.3. Comparison with other Approaches

Operations of D-Town network were optimized previously by a Pseudo-Genetic Algorithm (PGA) [19] and a successive linear programming [20], whose optimal annual pump costs are 168, 118 and 117, 740 Euros. Up to the information in the referenced papers, a comparison of CSP-MPC is included with the calculation for annual operational cost of D-Town network. Considering gravitational acceleration as $9.8m/s^2$, e as pump efficiency (70% here), ρ as water density, ΔH as the pressure head gain by pumps, $\tilde{u}$ as pump flow and a_2 as pumping price, the operational annual cost by CSP-MPC is 137, 880 Euros, which is in order of the results obtained by [19, 20].

$$C_{ann.} = \frac{9.8 \sum_{j=1}^{7} \sum_{i=1}^{K_{365}} \rho \tilde{u}(i, j) \Delta H(i, j) a_2(i, j)}{e} \tag{25}$$

6. Conclusions

This paper proposes and validates the integrated control scheme of linear MPC for DWNs with the cooperation of NAM and CSP which has been used to avoid formulating the problem using non-linear MPC. D-Town network, a well known benchmark in the DWN community, has been used to as the case study, results produced by PGA and successive linear programming have been compared to validate feasibility of this approach.

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