Coordinating Model Predictive Control of Transport and Supply Water Systems

C.C. Sun, V. Puig and G. Cembrano

Abstract

Transport and supply water networks are two types of systems which have received a significant amount of attention in the recent years. Issues on how to obtain the best performance for a given transport or supply water systems, or how to coordinate interactions between them are still open and need more research.

This chapter presents a hierarchical Model Predictive Control (MPC) scheme with a supervisor that coordinates transport and supply water systems. First, a two-level hierarchical control structure resulting from a functional decomposition of water network is briefly presented. Inside each hierarchy, a MPC controller is used. In the two-level hierarchy, a supervisory coordinating mechanism is used to generate control strategies which consider objectives at different time scales. The first level, in charge of managing the transport system, works in a daily scale in order to achieve the global management policies for the transport over water (e.g., navigation, vessels and barges) in different rivers and balance management of different reservoirs. The second level, in charge of managing the supply system, works in a hourly scale and manipulates actuator (pumps and valves) set-point to satisfy the local water supplying objectives (e.g.,minimizing economic cost, handling emergency storage and smoothing actuator operation). The results of the modelling will be applied to the Catalunya Regional Water Network and based on an aggregate model.

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1 Introduction

From a functional perspective, a regional water network can be structurally organized into two separate systems:

- Transport system, composed by water sources, open channels, large reservoirs and also natural aquifers.
- Supply system, which links water treatment and desalinization plants with reservoirs distributed all over the city.

Both of the partitions of a regional water network must be operated at different time scale because of the different dynamics they present according to their specified objectives. In general, these systems are normally separately operated. The coordinated operation of supply and transport systems in a regional network is one of the main motivations for the research reported in this chapter.

In recent literature, there is a renewed interest in hierarchical MPC either from industrial practice or from academia [19, 20]. This is specially the case when a system is composed of subsystems with multiple time scales as the case of the regional water networks. A straightforward task of designing and implementing a single centralized control unit is too difficult as discussed in [2], because the required long prediction horizon and short control time steps might lead to an optimization problem of high dimension and under large uncertainty radius. A way to cope with this problem is to apply a hierarchical control based on decomposing the original control task into a sequence of different, simpler and hierarchically structured subtasks, handled by dedicated control systems operating at different time scales [3].

This chapter proposes a temporal hierarchical MPC scheme for complex transport and supply water systems. The proposed strategy will coordinate the MPC controllers for the transport and supply systems by means of a temporal hierarchical sequence of optimizations and constraints going from one level of the hierarchy to the other. Case study and results are described in Section 2. Interdisciplinary discussion about the unified framework and some open topics are explained in Section 3 and 4. In Section 5, conclusions and future research are provided.

2 Case Study: Catalunya Regional Water Network

The Catalunya Regional Water Network in Fig. 1 lies within the Catalunya Inland Basins, from which the Metropolitan area of Barcelona is fed and where most of the population is concentrated. It composed by river *Llobregat*, *Ter* and the related components. According to definition of functional decomposition, the Catalunya Regional Water Network can be separated into two systems. The transport system, composed by rivers *Llobregat*, *Ter* and all the connected elements, lies at the two side of Fig. 1. The supply system, composed by metropolitan areas and also treatment, desalination plants inside them, lies in the center.

Fig. 1 Aggregate diagram of Catalunya Regional Water Network

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2.1 Operational Goals of the Transport System

The goal of a water transport system is basically to maintain the river water levels while providing water to the supply system and satisfying irrigation demands.

   The control problem of the transport system is typically operated within a 30-day horizon, at daily time scale and operational goals should be achieved are:

- *Operational safety* (J_{safety}): This criterion refers to maintain appropriate water storage levels in dams and reservoirs for emergency-handling.
- *Demand management* (J_{demand}): This is especially important in the transport system when urban and irrigation demands exist since urban demands must be fully satisfied while irrigation demands allow some degree of slackness.
- *Balance management* ($J_{balance}$): This is necessary for keeping rivers or reservoirs to be used in a balanced way and escaping water deficit problem in a longer time.
- *Minimizing waste* (J_{mwaste}): Taking into account that the river water eventually goes to the sea, this term ties to avoid unnecessary water release from reservoirs (that is release water that does not meet any demand and is eventually wasted).
- *Transport water levels* (J_{level}): Water sources such as boreholes, reservoirs and rivers are usually subject to operational constraints to maintain certain water levels which are needed for transporting over water and maintaining ecological flows.
- *Control actions smoothness* ($J_{smoothness}$): The operation of transport over water usually requires smooth flow set-point variations for best process operation.

2.2 Operational Goals of the Supply System

The immediate control goal of water supply system is to meet the demands at consumer sites with appropriate flows and optimized costs according to users' needs.

   The supply network is typically operated with a 24-hour horizon, at hourly time scale. The main operational goals to be achieved in the supply network are:

- *Cost reduction* (J_{cost}): Water cost is related to treatment, which have different prices at different sources, and due to different source elevations, which implies electrical pumping costs affected by power tariffs which may vary in a day.
- *Operational safety* (J_{safety}): This criterion refers to maintain appropriate water storage levels in dams and reservoirs of the network for emergency-handling.
- *Control actions smoothness* ($J_{smoothness}$): The operation of water treatment plants and main valves/pumps usually requires smooth flow set-point variations for best process operation.
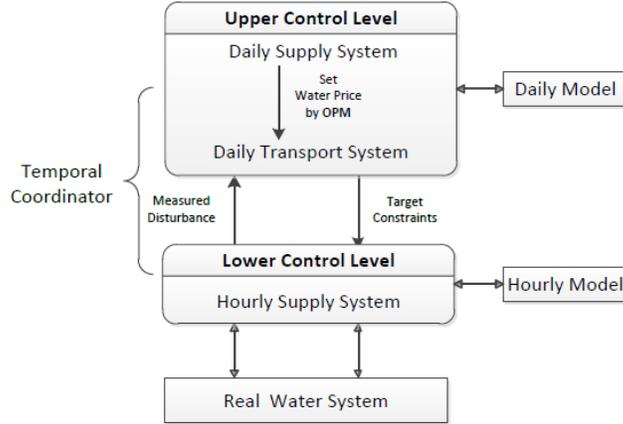
Fig. 2 Temporal hierarchical coordinating structure

2.3 Temporal Hierarchical Coordinating Technique

The general principle of a pure temporal hierarchical controller is that: decision of a higher level has a wider temporal extent than that of a lower level, and the higher level decision units process more aggregated information than the lower ones [1].

In this chapter, transport system could be assumed as the upper level, while supply systems could be considered as the lower level. This temporal hierarchical coordinating structure is proposed in Fig. 2. In the upper level, the daily model of the supply system is used in order to estimate the aggregated prices (which include both water and electricity costs) by means of the *optimal path method* (OPM) in [6][8]. Detailed algorithms for this temporal coordination mechanism will be provided in detail in the following section.

2.4 MPC of the Transport System

State Space Model

State space model of transport system includes two kind of states and control variables. First kind of state variables represent reservoirs and the managed variables correspond to actuator flows:

$$x(k+1) = A x(k) + B u(k) + B_p [d(k) - \varepsilon(k)], \quad k \in \mathbb{Z} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ are reservoir volumes, $u(k) \in \mathbb{R}^{n_u}$ are actuator flows, $d(k) \in \mathbb{R}^{n_d}$ represents disturbance demands, $\varepsilon(k) \in \mathbb{R}^{n_d}$ is slack variable for violated demands and $\varepsilon(k)$ is introduced to control the amount of demand which has not been satisfied.

The second kind of state and control variables represent river flows with delays. For simplicity and brevity, consider river reach model as a transport delay [9]:

$$q_{out_i} = q_{in_i}(k - \tau_d) \tag{2}$$

where τ_d represents delay values.

For time delays associated with flows within the network, the following auxiliary state equations are introduced:

$$x_{j,1}(k+1) = q_j(k) \tag{3}$$

$$x_{j,i+1}(k+1) = x_{j,i}(k), i = 1, \cdots, \tau_d \tag{4}$$

where $x_{j,i}(k) \in \mathbb{R}^{n'_x}$ represent flows, $q_j(k) \in \mathbb{R}^{n'_u}$ means flow as part of control variables and $\tau_d \in \mathbb{Z}$ is number of delays.

Combining (3), (4) with (1), state space representation (1) becomes

$$\widetilde{x}(k+1) = \widetilde{A}\widetilde{x}(k) + \widetilde{B}\widetilde{u}(k) + \widetilde{B}_p[d(k) - \varepsilon(k)], \quad k \in \mathbb{Z} \tag{5}$$

where

$$\widetilde{x}(k) = \begin{bmatrix} x(k) \\ x_{j,i}(k) \end{bmatrix}, \quad \widetilde{u}(k) = \begin{bmatrix} u(k) \\ q_j(k) \end{bmatrix}$$

and $\widetilde{x}(k) \in \mathbb{R}^{\widetilde{n}_x}$, $\widetilde{u}(k) \in \mathbb{R}^{\widetilde{n}_u}$.

All the variables are subject to the following inequality constraints:

$$\widetilde{x}_{min} \leq \widetilde{x}(k) \leq \widetilde{x}_{max} \tag{6}$$

$$\widetilde{u}_{min} \leq \widetilde{u}(k) \leq \widetilde{u}_{max} \tag{7}$$

$$\varepsilon_{min} \leq \varepsilon(k) \leq \varepsilon_{max} \tag{8}$$

where $\widetilde{x}_{min}, \widetilde{x}_{max}$ are physical limitations of reservoirs, $\widetilde{u}_{min}, \widetilde{u}_{max}$ are physical limitations of the river flows and ε_{min} lies between zero and the related demand.

Besides that, the balance at every node should be satisfied, where E, E_d, $E_{\widetilde{x}}$ are matrices which parameters can be obtained from topology of the water network:

$$E\widetilde{u} + E_d d - E_d \varepsilon + E_{\widetilde{x}}\widetilde{x} = 0$$

Control Objectives

Operational goals of the transport system lead to the following function:

$$
\begin{aligned}
J &= J_{safety} + J_{demand} + J_{mwaste} + J_{balance} + J_{level} + J_{smothness} \\
&= \varepsilon_{\widetilde{x}}(k)^\top W_{\widetilde{x}} \varepsilon_{\widetilde{x}}(k) + \varepsilon(k)^\top W_f \varepsilon(k) \\
&\quad + (\widetilde{u}_{i\ldots j}(k) - \widetilde{u}_s(k))^\top W_{\widetilde{w}} (\widetilde{u}_{i\ldots j}(k) - \widetilde{u}_s(k)) \\
&\quad + \left(\left(0 \ldots 0 \ \tfrac{1}{x_{i,max}} \ 0 \ldots 0 \ \tfrac{-1}{x_{j,max}} \ 0 \ldots 0 \right) \widetilde{x}(k) \right)^\top w_{\widetilde{m}} \\
&\quad \times \left(\left(0 \ldots 0 \ \tfrac{1}{x_{i,max}} \ 0 \ldots 0 \ \tfrac{-1}{x_{j,max}} \ 0 \ldots 0 \right) \widetilde{x}(k) \right) + \Delta \widetilde{u}(k)^\top W_{\widetilde{u}} \Delta \widetilde{u}(k)
\end{aligned}
\tag{9}
$$

where

$$
\begin{aligned}
\varepsilon_{\widetilde{x}}(k) &= \widetilde{x}(k) - \widetilde{x}_r \\
\widetilde{u} &= \Theta \Delta \widetilde{u} + \Pi \widetilde{u}(k-1) \\
\Delta \widetilde{u}(k) &= \widetilde{u}(k) - \widetilde{u}(k-1)
\end{aligned}
$$

and $W_{\widetilde{x}}$, W_f, $W_{\widetilde{w}}$, $W_{\widetilde{x}}$, $w_{\widetilde{m}}$, $W_{\widetilde{u}}$ are weights which decide the priorities (established by the water network authorities) for all the objective terms.

Water storage in reservoirs should be kept above a given level (named as water safety level) which is used as emergency supply for drought period. Any situation below the emergency level should be penalized using soft constraints:

$$
\widetilde{x} \geq \widetilde{x}_r - \varepsilon_{\widetilde{x}}
\tag{10}
$$

$$
\varepsilon_{\widetilde{x}} \geq 0
\tag{11}
$$

where $\widetilde{x}_r$ is the water safety level and $\varepsilon_{\widetilde{x}}$ is the slack to $\widetilde{x}_r$.

2.5 MPC of the Supply System

Basic state space model is used for the supply system [16].

Control Objectives

Operational goals of the supply system will lead to the following function:

$$
\begin{aligned}
J &= J_{safety} + J_{smothness} + J_{cost} \\
&= \varepsilon_{\widetilde{x}}(k)^\top W_{\widetilde{x}} \varepsilon_{\widetilde{x}}(k) + \Delta \widetilde{u}(k)^\top W_{\widetilde{u}} \Delta \widetilde{u}(k) + W_a(a_1 + a_2(k)) \widetilde{u}(k)
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
\varepsilon_{\widetilde{x}}(k) &= \widetilde{x}(k) - \widetilde{x}_r \\
\widetilde{u} &= \Theta \Delta \widetilde{u} + \Pi \widetilde{u}(k-1) \\
\Delta \widetilde{u}(k) &= \widetilde{u}(k) - \widetilde{u}(k-1)
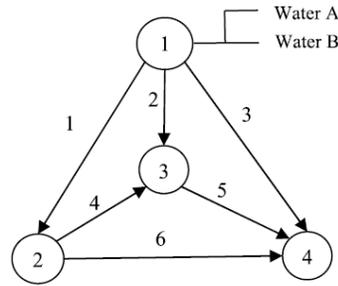\end{aligned}
$$

Fig. 3 A hypothetical network system

and $W_{\widetilde{x}}$, $W_{\widetilde{u}}$, W_a are weights which establish the priorities for the objective terms and the vectors a_1 and a_2 contain the cost of water treatment and pumping, respectively.

2.6 Temporal Hierarchical Coordinating Technique

2.6.1 Optimal Path Method

When optimizing the transport system, the whole supply system will be simplified into a virtual demand with unitary price after considering both the treatment and electricity costs. In order to determine this unitary price, OPM is used [6].

There are three steps for realizing OPM:

- Step 1. *Searching Exhaustive Paths*: Find all possible paths from sources to demands detecting closed cycles to avoid infinite loops.
- Step 2. *Choosing Optimal Path*: Find optimal path from the all paths set obtained in Step 1.
- Step 3. *Calculating the source price*: Calculate the source price by the total cost and the water consumption in the optimal path obtained in Step 2.

Searching Exhaustive Paths

In order to search optimal economical paths from sources to demands, it is necessary to determine all possible paths between them [7]. Before that, a node-arc representation method for a regional water network is provided, where a node represents a source, reservoir, demand or junction and an arc represents a transfer or trade [6].

In a regional water network, all flow paths can be obtained from node-arc incidence matrices because water always flows from upstream sources to downstream. In a node-arc incidence matrix, a node is represented by a row and an arc is represented by a column. In a row of the matrix, entry arcs are represented by $+1$ and

Table 1 Node-Arc Incidence Matrix for the Network of Fig. 2

Node	Arc 1	Arc 2	Arc 3	Arc 4	Arc 5	Arc 6
1	-1	-1	-1	0	0	0
2	+1	0	0	-1	0	-1
3	0	+1	0	+1	-1	0
4	0	0	+1	0	+1	+1

leaving arcs are represented by -1. In a column, an element of $+1$ and an element of -1 represent the ending and starting nodes, respectively, of this arc.

Table 1 shows the node-arc incidence matrix for the network in Fig. 3.

On the other hand, the node-arc incidence matrix that defines the relationship of the direction between nodes and arcs is transformed into a flow path matrix that defines all flow paths of the network. The flow path matrix A is a set of binary parameters $a_{s,r}$ that describe all flow paths in a water network:

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots & \dots & a_{1,p} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & \dots & a_{2,p} \\ a_{3,1} & a_{3,2} & a_{3,3} & \dots & \dots & a_{3,p} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \dots & \dots & a_{n,p} \end{pmatrix}$$

In this matrix A, which is the target matrix in this section, p denotes number of paths and n denotes total number of arcs (or flow actuators) in a water network. A column represents a flow path and a row represents an arc in the network. The connection parameters $a_{s,r}$ is binary $(0,1)$ and is used to describe the connection between source nodes and receiving nodes. The connection parameters are assigned equal to 1 for linking arcs s in a flow path r, while other arcs are assigned 0.

Choosing Optimal Path

The objective of this step is to find the optimal flow through each path. The optimal flow path problem can be formulated as a linear optimization problem as follows:

$$min_x c^T A x \qquad \text{subject to} \qquad \begin{aligned} Ax &\leq b \\ A_{eq}x &= b_{eq} \\ l_b &\leq x \leq u_b \end{aligned} \qquad (13)$$

where c, x, b, b_{eq}, l_b and u_b are vectors and A and A_{eq} are matrices. The meaning of these vectors and equations is described in the following:

Optimal path solution: x

The vector *x* contains the optimal flow through each path that minimize the total operational cost. This cost is measured by the operational cost of each actuator, and the actuators involved in each path according to the flow path matrix *A*. The cost function can be expressed as $c^T A x$, where *Ax* provides the total flow through each actuator.

Operational cost: c

The daily cost of each actuator is calculated as the mean cost value:

$$c(i) = \sum_{k=1}^{24} \frac{\text{cost}(i,k)}{24} \tag{14}$$

where *i* represents the actuator and index *k* represents the instant time.

Actuator constraints: $Ax \leq b$

Inequality constraints are related to actuator operational limits. One actuator can be involved in different paths, and each path can require a different constant flow through it. So, it is necessary to guarantee that the total flow for each actuator does not go beyond its upper limit.

As explained in the previous section, *A*, whose row dimension is the number of actuators and column dimension is the number of paths, is a matrix formed by ones and zeros that indicates which actuators are used in each path. The product of this matrix with the solution vector *x* gives as a result the flow that goes through each actuator. Vector *b* contains the maximum actuator flow.

Demand constraints: $A_{eq}x = b_{eq}$

The total volume of water from sources to each demand sector must be equal to its demand. This can be expressed by using equality constraints related to demands and by introducing matrix A_{eq} that indicates which demand sector is supplied from which path. The row dimension of matrix A_{eq} is number of demand sectors while column dimension is number of paths.

Path capacity constraints: l_b and u_b

They are used to restrict the flow in each path by establishing the interval of possible values due to operational limits of the actuators involved in the path. The upper limit

u_b is given by the minimal of the actuator upper bounds involved the path, while the lower limit l_b is the maximal of the actuator lower bounds in the path.

Calculating the source price

From the optimal flow path calculation, the source price for the transport layer (including both the production and transportation cost) can be obtained as indicated in Algorithm 1 in lines 23 and 24. The economical unitary costs for the sources, C_{s1} and C_{s2}, are calculated by weighted averaging the optimal flow paths linking each source with the supply demands. The detailed calculations for every step of OPM are described in Algorithm 1:

Algorithm 1 Optimal Path Method

1: $x := [x_1, x_2, ..., x_p]$;
   {optimization vector}
2: $l_b := [min_{x_1}, min_{x_2}, ..., min_{x_p}]$;
   {lower bounds of x}
3: $u_b := [max_{x_1}, max_{x_2}, ..., max_{x_p}]$;
   {upper bounds of x}
4: $Source := [s_1, s_2]$;
   {source matrix}
5: $b_{eq} := [d_1, d_2, ..., d_m]$;
   {demand node matrix}
6: $Actuator := [a_1, a_2, ..., a_n]$;
   {actuator matrix}
7: build $0 - 1$ exhaustive path matrix
8: $Path := [s_1, a_{11}, a_{21}, ..., d_1; ...; s_2, a_{12}, a_{22}, ..., d_m]$
   {number of row is p}
9: build $0 - 1$ actuator and path matrix
10: $A := [a_{11}, a_{21}, ..., a_{p1}; ...; a_{n1}, a_{n2}, ..., a_{pn}]$
11: $b := [max_{a_1}, max_{a_2}, ..., max_{a_n}]$
   {maximum flow column for all the actuators}
12: build $0 - 1$ demand and path matrix
13: $A_{eq} := [d_{11}, d_{21}, ..., d_{p1}; ...; d_{m1}, d_{m2}, ..., d_{mp}] (24)(3600)$
14: Build cost matrix contain electrical and water cost
15: $c := [c_1; c_2; ...; c_n]$;
16: Set objective function
17: $f_{obj} = c^T A x$;
18: Optimizing the problem
19: $x = linprog(f_{obj}, A, b, A_{eq}, b_{eq}, l_b, u_b, x_0, option)$
   {x_0 is provided}
20: Calculating the flow through each actuator;
21: $flow = Ax$
22: Calculating the source price by weighted averaging the optimal flow paths linking sources and demands;
23: $C_{s1} = c_{s1}^T (A[s1, :]x(s1))/flow[s1]$
24: $C_{s2} = c_{s2}^T (A[s2, :]x(s2))/flow[s2]$

2.6.2 Coordinating Mechanism

As shown in Fig. 2, the transport and supply systems are coordinated by interchanging the following information:

- Measured disturbance (d_s): which provides the daily demands to the transport layer by aggregating the hourly demands in the supply layer.
- Target constraint (T_d): which expresses management policies from the transport system to the supply system in the form of control constraints.

Measured Disturbance

In the topology of the transport system, the whole supply system is simplified as one aggregated demand. Measured disturbance for the transport system is obtained by aggregating the hourly demands in the supply layer.

$$d_s(k) = \sum_{m=1}^{24} d_t(k,m) \tag{15}$$

where d_t is the hourly demand vector at the supply system corresponding to the k-th day, while $d_s(k)$ could be considered as the demand for the transport system.

Target Constraints

The goal for the temporal coordination algorithm is transferring management policies from the transport to the supply system for overall controlling of transport over water network and supply water network. In order to achieve this coordination, the following constraint is added to the MPC controller in the supply system:

$$\sum_{m=1}^{24} u(k,m) \le T_d(k) \tag{16}$$

where u is the shared control vector between transport and supply systems.

   This constraint is introduced to enforce the amount of water decided to be transferred from the transport to the supply system is satisfying the management policies in the transport layer. Structure of the temporal coordinator is shown at Fig. 4.

2.6.3 Formulation of Temporal Coordination Problem

The goal for the temporal coordination algorithm is transferring management policies from the transport to the supply layer. In order to achieve this coordination, the constraint (16) is added to the the supply hierarchical MPC. Algorithm 2 shows how this constraint, that establishes a daily limitation, is generated and adapted at every
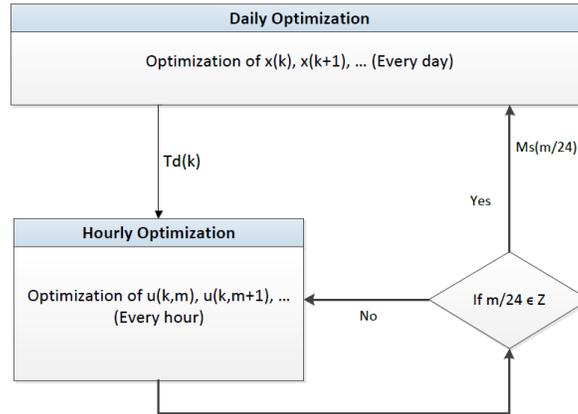
Fig. 4 Upper and Lower layer optimizations of multi-layer MPC

time iteration of the lower layer MPC that operates at a hourly scale. Algorithm 2 takes into account the following facts when generating the constraint (16):

- after the application of n hourly control actions $u_s(m)$ corresponding to the k-th day, the total remaining water for this day will be: $T_d(k) - \sum\limits_{m=1}^{n} u(m)$

- when limiting the control actions in the prediction horizon L, there is a part of control actions $u(m)$ that corresponds to hours of the current day k that should be limited by $T_d(k)$, while the control actions correspond to hours of the next day $k+1$ that should be limited by $T_d(k) - \sum\limits_{m=1}^{n} u(m)$.

- the generated constraints are added as additional constraints of the BOP problem associated to the supply MPC.

2.7 Results for the Transport System

There are three scenarios in practical use, which are:

- *Scenarios 1*: More initial water in Llobregat than in Ter.
- *Scenarios 2*: More initial water in Ter than in Llobregat.
- *Scenarios 3*: Initial water in both rivers are similar.

According to reality use, for the first two scenarios, when water in one river is adequate while in another river limited, management policies will be set to ask water from only one of the rivers. For the Scenario 3, when water is abundant in both of rivers, according to the balance management control objectives, water consumption in both of the rivers will be proportional to their supplying capacity.

Algorithm 2 Temporal multi-level coordinator

1: $L := 24$ hours
2: $I := 24N$ hours
3: $T_s := 1$ hour
   {start creating new constraints for lower-layer *BOP* }
4: **for** $i := 1$ to I **do**
5:     $d := floor(i/24)$
6:     $t := rem(i, 24)$
7:     **if** $t == 0$ **then**
8:         Update BOP by adding the following constraints:
9:         $u(1|k) \leq T_d(d) - \sum_{j=i-L+1}^{i-1} u_s(j|k);$
10:        $\sum_{j=2}^{L} u(j|k) \leq T_d(d+1);$
11:    **end if**
12:    **if** $t == 1$ **then**
13:        Update BOP by adding the following constraints:
14:        $\sum_{j=1}^{L} u(j|k) \leq T_d(d+1);$
15:    **end if**
16:    **if** $t == 2$ **then**
17:        Update BOP by adding the following constraints:
18:        $\sum_{j=1}^{L-1} u(j|k) \leq T_d(d+1);$
19:        $u(L|k) \leq T_d(d+2);$
20:    **end if**
21:    **if** $t \geq 3$ **then**
22:        Update BOP by adding the following constraints:
23:        $\sum_{j=1}^{L-t+1} u(j|k) \leq T_d(d+1) - \sum_{j=i-L+1}^{i-1} u_s(j|k);$
24:        $\sum_{j=L-t+2}^{L} u(j|k) \leq T_d(d+2);$
25:    **end if**
26:    Solve *BOP* to obtain $u(j|k), u(j+1|k), \ldots$ with the new constraints added
27:    $u_s(i|k) := u(1|k);$
28: **end for**
   {end of loop}

Table 2 provides detailed results and also the improvement of water usages in the two rivers achieved by the proposed multi-hierarchical MPC scheme. In this table, *Source* means outside sources flow into rivers, *Fixed Demand* means fixed demands which can not choose water source while *Variable Demand* is the demand which can receive water from more than one river. *BD*, abbreviation of *Balanced Demand*, is water volume that has been consumed from each of the reservoirs and *PB*, abbreviation of *Proportion of Balanced demand*, is the proportion of *BD* for the two reservoirs. *PR*, abbreviation of *Proportion of Reservoir capacity*, is the proportion of storage capacities of the two reservoirs. The similar values for *PB* and *PR* is what the multi-hierarchical scheme wants to reach. And *SA*, abbreviation of *Supplying*

Table 2 Balancing comparison of Scenarios 3

Sc.		**Multi-hierarchical MPC Control Scheme**					
Es.	Source	Fixed Demand	Variable Demand	BD	PB	PR	SA
L.	3008	2981	724	697	58.93%	53.48%	242 Days
T.	3532	3518	1196	1182			
Sc.		**Model Predictive Control**					
Es.	Source	Fixed Demand	Variable Demand	BD	PB	PR	SA
L.	3008	2981	7.6	-19.4	-1.02%	53.48%	177 Days
T.	3532	3518	1914	1900			

Ability, is the ability of water supply and transport over water in days of the whole water network before meeting a deficit problem at the hypothesis of no rain and no water flow in from outside. The comparisons prove that, after using the proposed MPC scheme, the proportion of water usage from two rivers (58.93%, which is ratio of Llobregat/Ter) is much closer to the proportion of their storage capacities (53.48%). And what is more, the Catalunya Regional Water Network can transport over water and supply water 65 days longer than that without balance management, which is a good improvement regarding the sustainable usage of water resource in the long term perspective.

Fig. 5 is one of the examples of one river reach. The plot shows that, after controlling transport water levels, water flow at this reach could meet the water level needed by transporting over water during the whole optimization process.

2.8 Results for Coordination

During the coordination process, management policies at the transport system are transferred to the supply system establishing the set-points for the shared actuators. Fig. 6 and 7 show the amount of water consumed by the supply system from different rivers for satisfying the same demands before and after coordination, respectively. The two figures prove that average levels of water consumptions from two rivers are much closer after balance management.

Fig. 8 shows source flow comparisons between multi-hierarchical MPC and centralized MPC, which proves the similarity between the two kinds of controllers.

Table 3 provides detailed numerical results and compares the obtained control results in terms of economical and computational performance over four days among the multi-hierarchical and centralized MPC control techniques:

- *Current Control*: Control the supply system of Catalunya Regional Water Network using heuristic strategies by human operators.
- *Multi-hierarchical Model Predictive Control Scheme*:
  Control transport-supply system of Catalunya Regional Water Network using Multi-hierarchical Model Predictive Control techniques with temporal hierarchical coordinator between the transport and supply systems.
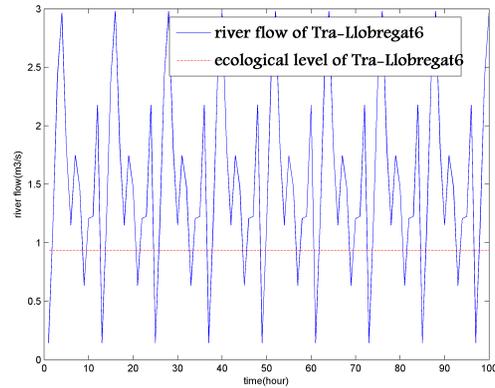
(a) before
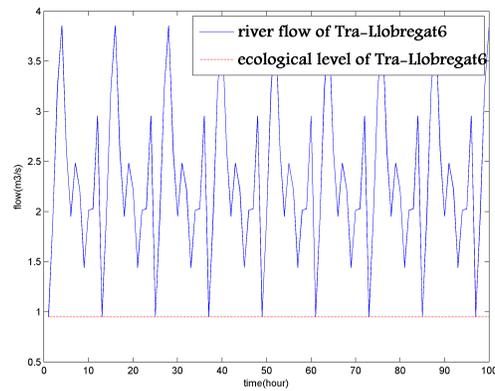
(b) after

Fig. 5 River flow with ecological level before and after control in river Llobregat

Table 3 Closed-loop performance results (all values in e.u.)

Define	**Current Control**				**Multi-hierarchical MPC**				**Centralized MPC**			
Day	Ele.	Wat.	Tot.	Comp.(s)	Ele.	Wat.	Tot.	Comp.(s)	Ele.	Wat.	Tot.	Comp.
1*st*	240	100	340	23	213	44	257	32	141	40	181	58
2*nd*	239	106	345	21	237	47	284	27	170	39	209	57
3*rd*	246	94	340	19	238	48	286	34	171	41	212	61
4*th*	264	110	374	21	253	66	319	29	168	42	210	62
Proportion					-5%	-50%	-18%	45%	-34%	-61%	-42%	183%

- *Centralized Model Predictive Control*:
  Control the transport-supply system of Catalunya Regional Water Network using centralized Model Predictive Control techniques without coordination.
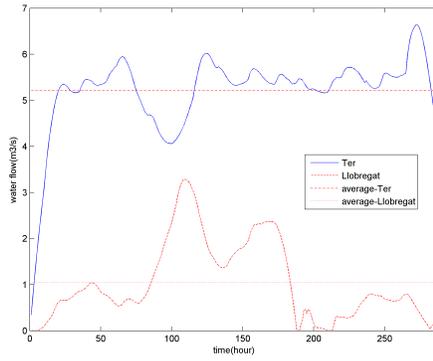
Fig. 6 Flows from the two rivers before using temporal coordination with x-time and y-flow axis
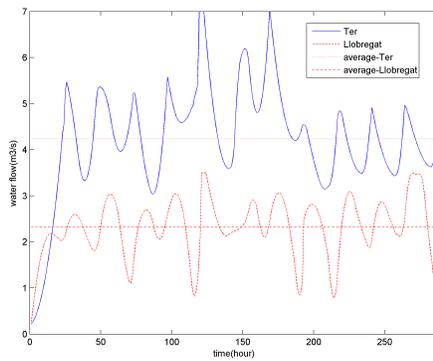

Fig. 7 Flows from the two rivers after using temporal coordination with x-time and y-flow axis
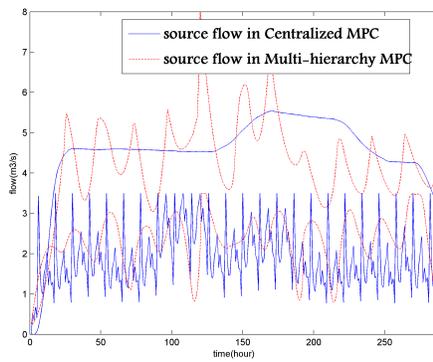

Fig. 8 Source flows comparison between Multi-hierarchical MPC and Centralized MPC

In the Table 3, *Wat.*, abbreviation of *Water*, means water cost during the day, while *Ele.*, abbreviation of *Electricity*, shows electricity cost, *Tot.*, abbreviation of *Total*, means the total cost which include both water and electricity, where the indices representing costs are given in economic units (e.u.) instead of Euro due to confidentiality restrictions, and *Comp.*, abbreviation of *Computation time*, which means the needed computing time for that optimizing process in seconds. The row of *Proportion* is the improved proportion to the current control. From this table, the result shows that, Multi-hierarchical MPC technique with temporal coordination is much better than the current controller but is a little worse than centralized MPC technique regarding economical cost, especially of electricity cost. The explanation is that while introducing multi-hierarchical control scheme, which simplifies the whole supply system into a virtual demand, the optimization of electricity price for the actuators becomes invalid. On the other hand, computation time is much improved after introducing multi-hierarchical control scheme, which is also necessary for the large scale water systems. Besides that, scalability is also improved after introducing the proposed multi-hierarchical MPC scheme.

3 Interdisciplinary Discussion into the Unified Framework

Transport of water is part of the master planning of communities, counties and municipalities. Limited water supplies, conservation and sustainable policies, as well as the infrastructure complexity for meeting consumer demands with appropriate flow pressure and quality levels make water management a challenging control problem. Decision support systems provide useful guidance for operators in complex networks, as reported in [15] and [22].

Transport over water, which has the advantages of less investment and low cost, has been developed as the main bearer of large, bulky freight. In reality, plenty of natural conditions, like seasonal fluctuations of water level, water forms complex, could impact easily transportation ability of inland waterways, which affect both the play of navigation and also economical interests. Stability of river flows and adequate water levels of open channels in transport water networks are critical control factors in the operational control models.

This chapter takes into account *transport of water* from the dams to the consumers while at the same time considers the *transport over water* by keeping some ecological flows in the rivers. The transport system works in a daily time scale in order to achieve the global management policies for the transport over water in different rivers and balance management of different reservoirs. The supply system works in a hourly time scale and manipulates actuator set-points to satisfy the local water supplying objectives. For each water system, a MPC controller is provided to produce optimized control sequence for different control objectives. A supervisor coordinating mechanism that coordinates transport and supply water systems for sustainable and ecological benefits is used to generate control strategies which consider objectives at different time scales. OPM is used to generate unitary source

prices for the transport water system. The results of this control scheme after being applied to the Catalunya Regional Water Network in Section 2 shows that, comparing with the separated control strategies, this unified framework which deals with both short term objectives and long term objectives provides sustainable and ecological results, which even has economical benefits in the long term perspective.

4 Open Topics

Besides advantages and benefits of the unified control scheme proposed in this chapter, there still exist topics which need more research and verification, which are:

T1: How to manage the priorities when long term objectives have conflicts with short term objectives?
T2: How to choose the control strategies between multi-hierarchical and centralized MPC with coordination when computation load is not a problem?
T3: In reality, how to deal with uncertainties for the complex water system, which affect both transport and supply systems?

5 Conclusions and future research

In this chapter, a multi-hierarchical MPC scheme with temporal coordination for complex transport and supply water systems is proposed. The need of multi-hierarchical scheme derives from the fact that different networks in the transport over water and supply systems are operated according to different management goals, with different time horizon. While the management of the transport network is mainly concerned with long term safe-yield and ecological issues, the supply hierarchical must achieve economic goals in the short term (hourly strategy), while meeting demands and operational constraints. The use of the multi-hierarchical modelling and the temporal hierarchical MPC coordination techniques proposed in this chapter makes it possible to realize communication and coordination between the two separated systems in order to let individual operational goals affect to each other, and finally, obtain short-term strategies which can effectively consider long-term objectives as well.

References

1. M. Brdys and B. Ulanicki. *Operational Control of Water Systems: Structures, algorithms and applications*. Prentice Hall International, UK, 1994.
2. M. A. Brdys, M. Grochowski, T. Gminski, K. Konarczak, and M. Drewa. Hierarchical predictive control of integrated wastewater treatment systems. *Control Engineering Practice*, 16(6):751–767, 2008.
3. Mietek A Brdys and Piotr Tatjewski. *Iterative Algorithms for Multilayer Optimizing Control*. Imperial College Press, London, 2005.
4. E. Caini. Sviluppo di un tool di simulazione per il controllo predittivo di reti idriche di trasporto di acqua potabile. *Master Thesis of Universita Degli Studi Di Siena*, 2008.
5. G. Cembrano, J. Quevedo, V. Puig, R. Perez, J. Figueras, G. Ramon, P. Rodryguez, G. Barnet, M. Casas, J. M. Verdejo, A. Gil, J. Marti, and H. Konig. First results of predictive control application on water supply and distribution in santiago - chile. *16th IFAC World Congress*, pages 25–30, Praga, República Txeca, 2005.
6. Wei-Chen Cheng, Nien-Sheng Hsu, Wen-Ming Cheng, and William W.-G. Yeh. A flow path model for regional water distribution optimization. *Water resources research*, 45:W09411, 2009.
7. G. H. Danielson. On finding simple paths and circuits in a graph. *IEEE Trans. Circuit Theory, CT-15*, pages 294–295.
8. Saul Montes de Oca Armeaga. Hierarchical multilayer and decentralized MPC control of the barcelona water network. *Master Thesis of Universitat Politecnica de Catalunya*, 2011.
9. R. Evans, L. Li, I. Mareels, N. Okello, M. Pham, W. Qiu, and S. K. Saleem. Real-time optimal control of river basin networks. *Preprints of the 18th IFAC World Congress*, pages 11459–11464, Milano, Italy, 2011.
10. X. Litrico and V. Fromion. Simplified modeling of irrigation canals for controller design. *Journal of Irrigation and Drainage Engineering*, 130(5):373–383, 2004.
11. J. M. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, Great Britain, 2002.
12. M. Marinaki. A non-linear optimal control approach to central sewer network flow control. *International Journal of Control*, 72(5):418–429, March 1999.
13. M. Marinaki and M. Papageorgiou. *Optimal Real-time Control of Sewer Networks*. Springer, London, 2005.
14. R. R. Negenborn. *Multi-Agent Model Predictive Control with Applications to Power Networks*. Delft University Press, Delft, The Netherlands, 2008.
15. V. Nitivattananon, E. C. Sadowski, and R. G. Quimpo. Optimization of water supply system operation. *Journal of Water Resources Planning and Management*, 122(5):374–384.
16. C. Ocampo-Martinez, V. Puig, G. Cembrano, and J. Quevedo. Application of predictive control strategies to the management of complex networks in the urban water cycle. *IEEE Control Systems Magazine*, 33(1):15–45, 2013.
17. P. J. Van Overloop. *Model Predictive Control on Open Water Systems*. Delft University Press, Delft, The Netherlands, 2006.
18. J. Quevedo, V. Puig, G. Cembrano, and J. Blanch. Validation and reconstruction of flow meter data in the Barcelona water distribution network. *Control Engineering Practice*, 11(6):640–651, June 2010.
19. R.Scattolini. Architectures for distributed and hierarchical model predictive control: A review. *Journal of Process Control*, 19(5):723–731, 2009.
20. P. Tatjewski. Advanced control and on-line process optimization in multilayer structures. *Annual Reviews of Control*, 32(1):71–85, 2008.
21. R. Toro, C. Ocampo-Martinez, F. Logist, J. Van Impe, and V. Puig. Tuning of predictive controllers for drinking water networked systems. *Proc. 18th IFAC World Congress*, Milano, Italy, 2011.
22. M. Y. Tu, T. C. T. Frank, and W. G. Y. William. Optimizaiton of water distribution and water quality by hybird genetic algorithm. *Journal of Water Resources Planning and Management*, 131(6):431–440, 2005.