Evolutionary-game-based dynamical tuning for multi-objective model predictive control

J. Barreiro-Gomez, C. Ocampo-Martinez and N. Quijano

Abstract Model predictive control (MPC) is one of the most used optimization-based control strategies for large-scale systems, since this strategy allows to consider a large number of states and multi-objective cost functions in a straightforward way. One of the main issues in the design of multi-objective MPC controllers, which is the tuning of the weights associated to each objective in the cost function, is treated in this work. All the possible combinations of weights within the cost function affect the optimal result in a given Pareto front. Furthermore, when the system has time-varying parameters, e.g., periodic disturbances, the appropriate weight tuning might also vary over time. Moreover, taking into account the computational burden and the selected sampling time in the MPC controller design, the computation time to find a suitable tuning is limited. In this regard, the development of strategies to perform a dynamical tuning in function of the system conditions potentially improves the closed-loop performance. In order to adapt in a dynamical way the weights in the MPC multi-objective cost function, an evolutionary-game approach is proposed. This approach allows to vary the prioritization weights in the proper direction taking as a reference a desired region within the Pareto front. The proper direction for the prioritization is computed by only using the current system values, i.e., the current optimal control action and the measurement of the current states, which establish the system cost function over a certain point in the Pareto front. Finally, some simulations of a multi-objective MPC for a real multi-variable case study show a comparison between the system performance obtained with static and dynamical tuning.

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1 Introduction

Model Predictive Control (MPC) is an optimization-based control strategy widely used in the solution of the control of large-scale systems since it can manage a large number of variables in an straightforward manner, it may consider several objectives, and it can consider a variety of physical and operational constraints. The versatility of the MPC controller is reflected on the amount of elements that can be adjusted, e.g., control-oriented model of the system, horizon for the states prediction, control horizon, or weights in the multi-objective cost function. In this regard, one of the main issues of the multi-objective MPC design is the selection of all these parameters. This work focuses particularly on the tuning issue given by the selection of the weights in the cost function. These weights assign a prioritization to each objective affecting the solution of the optimization problem that is solved at each iteration by the controller. Consequently, tuning these weights might improve considerably the control performance.

The design problem of tuning has been already treated by many researchers using different strategies. Most of the existing strategies to tune an MPC controller utilize an off-line approach, and sometimes it is a trial and error procedure. In [5], a review of some tuning strategies has been made, and some approaches such as off-line and on-line strategies have been classified. Since the conditions over the system might vary over time, it has become a relevant issue the development of strategies that allow to tune MPC controllers permanently in a dynamical manner. Moreover, it must be taken into account that an on-line tuning strategy necessarily implies an extra computational burden. For instance in [1], a tuning strategy is presented based on a linear approximation between the closed-loop predicted output, and the parameters that may be tuned in the MPC controller. Also, it has been highlighted its simplicity as an advantage for implementation. More approaches to solve this problem have been proposed after the review presented in [5]. In [4], a tuning methodology based on a matching to a desired reference controller has been proposed. This method allows to select the MPC weight matrices, making the MPC perform as a desired linear controller. Afterwards, this methodology has been generalized in [23]. The use of a linear controller as a reference has also been studied for multiple-input-multiple-output systems in [21]. Other alternatives to perform the tuning of an MPC controller has been explored. For instance, a self-tuning terminal cost approach is applied in [13] for an economic MPC controller. In [22], a normalization procedure and a computation of the minimum distance from the Pareto front to a management point have been proposed as a tuning strategy. Other approaches use learning systems. For example, a learning approach based on artificial neural networks and fuzzy logic has been studied for performing the tuning of a predictive controller in [6]. Then, it is concluded that other learning techniques might be implemented in order to solve the problem of dynamical tuning for predictive controllers.

On the other hand, game theory has been used as a learning approach for a large variety of control systems. In [12], the use of game theory applied to distributed control design is discussed. The game theoretical approach has been used for the design of multi-agent systems, and to solve optimization problems [10, 11, 25]. Other perspective is the evolutionary game approach [20,24]. For instance in [2,3,14,17–19], a population dynamics approach has been presented for control and/or optimization purposes. Motivated by all the applications of this game-theoretical approach in control systems, this work uses the evolutionary game theory as a learning approach to propose a dynamical tuning strategy for multi-objective MPC controllers.

The contribution of this chapter is a novel dynamical tuning strategy based on evolutionary game theory. This approach varies the prioritization weights trying to maintain the values of the objective functions within a pre-established *management region* over the Pareto front. The *management region* is selected according to a desired performance of the system, and determines the proper direction for the evolution of the prioritization weights when disturbances in the system make objective functions take undesired values over the Pareto front. Furthermore, the population dynamics approach only requires information about the current condition of the system, which determine a current value in the Pareto front. In this regard, this methodology demands less computational burden with respect to other tuning strategies that need to compute more than one value over the Pareto front. The proposed evolutionary-game-based dynamical tuning is tested for an MPC controller with a drinking water network (DWN) as a case study.

This chapter is organized as follows. Section 2 introduces a brief background of MPC and population dynamics. Section 3 presents the proposed evolutionary-game-based dynamical tuning for a multi-objective MPC. Section 4 describes the real case study that has been used to test the proposed dynamical tuning approach. In this section, the results are shown and the proposed tuning strategy performance is compared with the performance of an MPC tuned with the static strategy. Finally, some concluding remarks are made and further work is pointed out in Section 5.

Notation

All column vectors are denoted by bold style, e.g., $\mathbf{x}$. Matrices are denoted by bold upper case, e.g., $\mathbf{A}$. In contrast, scalars are denoted by non-bold style, e.g., N. The sets are denoted by calligraphic upper case, e.g., $\mathcal{S}$. The norm $||\mathbf{x}||$ of the vector $\mathbf{x} \in \mathbb{R}^{n_x}$ is defined as $||\mathbf{x}|| = \sqrt{\mathbf{x}^\top \mathbf{x}}$. Finally, real numbers are denoted by $\mathbb{R}$, all the non-negative numbers are denoted by $\mathbb{R}_+$, and all the non-zero positive real numbers are denoted by $\mathbb{R}_{++}$. Similarly, the integer numbers, and non-negative integer numbers are denoted by $\mathbb{Z}$, and $\mathbb{Z}_+$, respectively. Throughout this document, discrete-time and continuous-time systems are treated. Therefore, the sub-index $k \in \mathbb{Z}_+$ denotes that the system is described in discrete time, whereas the use of time t in the continuous-time expressions is mostly omitted in order to simplify the notation.

2 Background

This section introduces some preliminaries such as the problem statement regarding multi-objective MPC, and the basic concepts within the framework of population dynamics, particularly regarding the Smith dynamics. These preliminaries are used later on in the statement of the proposed novel dynamical tuning.

2.1 Model predictive control

Consider a system represented by the following discrete-time state-space model:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_l\mathbf{d}(k), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u} \in \mathbb{R}^{n_u}$ is the vector of manipulated variables, $\mathbf{d} \in \mathbb{R}^{n_d}$ denotes the vector of disturbances affecting the system, $k \in \mathbb{Z}_+$ denotes the discrete time, and A, B and, B_l are the state-space system matrices with suitable dimensions. The states and control actions are subject to bounds and physical constraints, which define feasible sets given by

$$\mathcal{X} \triangleq \left\{\mathbf{x} \in \mathbb{R}^{n_x} : \mathbf{G}\mathbf{x} \leq \mathbf{g}\right\}, \tag{2a}$$

$$\mathcal{U} \triangleq \left\{\mathbf{u} \in \mathbb{R}^{n_u} : \mathbf{H}\mathbf{u} \leq \mathbf{h}\right\}, \tag{2b}$$

where $\mathbf{G}, \mathbf{H}, \mathbf{g}$, and $\mathbf{h}$ are matrices of suitable dimensions. Let $\hat{\mathbf{u}}$ be the control action sequence for a fixed-time prediction horizon H_p, let $\hat{\mathbf{x}}$ be the state sequence resulting from applying the control action sequence over the system (1) from the initial state $\mathbf{x}(0|k) \triangleq \mathbf{x}(k)$, and let $\hat{\mathbf{d}}$ be the disturbances sequence along H_p, i.e.,

$$\hat{\mathbf{u}} \triangleq \{\mathbf{u}(0|k), \mathbf{u}(1|k), ..., \mathbf{u}(H_p-1|k)\}, \tag{3a}$$

$$\hat{\mathbf{x}} \triangleq \{\mathbf{x}(0|k), \mathbf{x}(1|k), ..., \mathbf{x}(H_p|k)\}, \tag{3b}$$

$$\hat{\mathbf{d}} \triangleq \{\mathbf{d}(0|k), \mathbf{d}(1|k), ..., \mathbf{d}(H_p-1|k)\}. \tag{3c}$$

The system (1) is controlled by a multi-objective MPC controller whose optimization problem is composed by a cost function with N objectives, each one with an associated weight $\gamma_i, \;\; i = 1, ..., N$ that assigns a prioritization, i.e.,

$$\min_{\hat{\mathbf{u}}} J(\mathbf{x}(0), \mathbf{u}) = \sum_{j=1}^{N} \gamma_j J_j(\mathbf{x}(0), \mathbf{u}), \tag{4a}$$

subject to:

$$\mathbf{x}(i+1|k) = \mathbf{A}\mathbf{x}(i|k) + \mathbf{B}\mathbf{u}(i|k) + \mathbf{B}_l\mathbf{d}(i|k), \;\; i \in [0, H_p - 1] \subset \mathbb{Z}_+, \quad \text{(4b)}$$
$$\mathbf{u}(i|k) \in \mathcal{U}, \;\; i \in [0, H_p - 1] \subset \mathbb{Z}_+, \quad \text{(4c)}$$
$$\mathbf{x}(i|k) \in \mathcal{X}, \;\; i \in [0, H_p] \subset \mathbb{Z}_+. \quad \text{(4d)}$$

The issue treated in this work has to do with the proper tuning for the optimization problem (4), i.e., how to find the proper values for the weight factors $\gamma_1, ..., \gamma_N$.

Assuming that the optimization problem (4) is feasible, there is an optimal input sequence given by

$$\hat{\mathbf{u}}^* \triangleq \{\mathbf{u}^*(0|k), \mathbf{u}^*(1|k), ..., \mathbf{u}^*(H_p - 1|k)\} \in \mathcal{U},$$

and due to the fact that only one control action of the sequence is applied to the system, then the final optimal control action is given by

$$\mathbf{u}^*(k) \triangleq \mathbf{u}^*(0|k).$$

Once the optimal control action $\mathbf{u}^*(k)$ is applied to the system, a new optimization problem of the form in (4) is solved to compute the next optimal control action. Then, a new optimal sequence is computed for the iteration $k+1$ repeating the mentioned procedure and using a new measurement of the system states as an initial condition in the prediction model.

2.2 Population dynamics

Consider a population composed by a large and finite number of rational agents involved in a strategic game. During the interaction, each agent chooses a strategy from the set of the N available strategies in the population, which is denoted by $\mathcal{S} = \{1, ..., N\}$. Each objective in the cost function of the MPC is associated to a strategy in the population game. The fact that agents are rational implies that they make decisions in order to improve their benefits known as payoffs, which are determined by a fitness function. Let $p_i \in \mathbb{R}_+$ be the portion of agents in the population choosing the strategy $i \in \mathcal{S}$. Thus, the vector $\mathbf{p} = [p_1 \quad \cdots \quad p_N]^\top$ corresponds to a strategic distribution of agents among all the strategies. The set of possible strategic distributions within the population is given by a simplex denoted by

$$\Delta = \left\{ \mathbf{p} \in \mathbb{R}_+^N : \sum_{i=1}^{N} p_i = 1 \right\}, \quad (5)$$

where the unit in the sum of proportions is associated to the total amount of agents in the population, and the interior of the simplex is defined as

$$\text{int}\Delta = \left\{ \mathbf{p} \in \mathbb{R}_{++}^N : \sum_{i=1}^{N} p_i = 1 \right\}. \quad (6)$$

Each fitness function is a mapping $f_i : \Delta \mapsto \mathbb{R}$ that takes a strategic distribution in the population and returns a real value corresponding to the payoff that the portion of agents p_i receives for selecting the strategy $i \in \mathcal{S}$. Similarly, the fitness function defined by the mapping $\mathbf{F} : \Delta \mapsto \mathbb{R}^N$ is the vector of all fitness functions, i.e., $\mathbf{F}(\mathbf{p}) = [f_1(\mathbf{p}) \quad f_2(\mathbf{p}) \quad \cdots \quad f_N(\mathbf{p})]^\top$.

The vector of fitness $\mathbf{F}$ determines the evolution of the game. For instance, the framework of the proposed dynamical tuning methodology is given by stable games. This condition establishes conditions over the fitness functions. The formal definition of stable games is stated next [8].

Definition 1. The population game $\mathbf{F} : \Delta \mapsto \mathbb{R}^N$ is a stable game if

$$(\mathbf{p} - \mathbf{q})^\top (\mathbf{F}(\mathbf{p}) - \mathbf{F}(\mathbf{q})) \leq 0, \text{ for all } \mathbf{p}, \mathbf{q} \in \Delta, \tag{7}$$

and this condition is equivalent to the condition that $D\mathbf{F}(\mathbf{p})$ is negative semidefinite, where $[D\mathbf{F}(\mathbf{p})]_{ij} = \frac{\partial f_i(\mathbf{p})}{\partial p_j}$. ♢

The process of selecting an agent and making decisions to change strategies in order to improve the payoffs is mathematically described by the population dynamics, e.g., replicator dynamics, projection dynamics, or Smith dynamics. In this chapter, the Smith dynamics have been selected and their features, as the previously mentioned property, are presented and explained below.

Smith dynamics

The Smith dynamics are one of the six fundamental population dynamics [20]. Any of these six fundamental population dynamics can be used for the proposed tuning strategy. However, in this chapter, the Smith dynamics have been chosen for the following reasons: *i)* they satisfy non-negativeness of proportion of agents, and *ii)* proportions do not extinct under the Smith dynamics (i.e., if a $p_i(t_1) = 0$ for any $t_1 \geq 0$ and $\mathbf{p}^* \in \text{int}\Delta$, then there exists a $t_2 > t_1$ such that $p_i(t_2) > 0$, and a $t_3 > t_2$ such that $\mathbf{p}(t_3) = \mathbf{p}^*$). The Smith dynamics are given by the following equation:

$$\dot{p}_i = \sum_{j=1}^{N} p_j \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+ - p_i \sum_{j=1}^{N} \left[f_j(\mathbf{p}) - f_i(\mathbf{p})\right]_+, \text{ for all } i \in \mathcal{S}, \tag{8}$$

where $[\cdot]_+ = \max(0, \cdot)$. Notice that the Smith dynamics can be re-written as

$$\dot{p}_i = \frac{1}{2} \sum_{j=1}^{N} \left((1 - \phi_{ij})p_i + (1 + \phi_{ij})p_j\right) \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right], \text{ for all } i \in \mathcal{S}, \tag{9}$$

where $\phi_{ij} = \text{sgn}\left(f_i(\mathbf{p}) - f_j(\mathbf{p})\right)$. If the equilibrium point of the Smith dynamics is $\mathbf{p}^* \in \text{int}\Delta$, then $p_i^*, p_j^* > 0$, for all $i, j \in \mathcal{S}$, and the equilibrium point in (9) implies that $f_i(\mathbf{p}^*) = f_j(\mathbf{p}^*)$, for all $i, j \in \mathcal{S}$.

Proposition 1. *The simplex Δ is an invariant set under the Smith dynamics (8), i.e., if the initial condition of the population state $\mathbf{p}(0) \in \Delta$, then $\mathbf{p}(t) \in \Delta$, for all $t \geq 0$.*

Proof. The simplex is determined by the set of proportions such that $\sum_{i=1}^{N} p_i = 1$, and $p_i \geq 0$, for all $i \in \mathcal{S}$. First, the proof consists of showing that $\sum_{i=1}^{N} \dot{p}_i = 0$, i.e.,

$$\begin{aligned}\sum_{i=1}^{N} \dot{p}_i &= \sum_{i=1}^{N} \left\{ \sum_{j=1}^{N} p_j \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+ - \sum_{j=1}^{N} p_i \left[f_j(\mathbf{p}) - f_i(\mathbf{p})\right]_+ \right\}, \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} p_j \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+ - \sum_{j=1}^{N} \sum_{i=1}^{N} p_j \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+, \\ &= 0.\end{aligned}$$

This shows the invariance of the set given by condition $\sum_{i=1}^{N} p_i = 1$. Now suppose that the population states are in the limit of the simplex Δ, i.e., a proportion of agents $p_i = 0$, then the Smith equation associated to $i \in \mathcal{S}$ is given by

$$\dot{p}_i = \sum_{j=1}^{N} p_j \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+,$$

and consequently, $\dot{p}_i \geq 0$, then the positiveness of proportion of agents is satisfied. This completes the proof. □

Once it has been shown that the simplex is an invariant set under the Smith dynamics, it is necessary to show the convergence to the equilibrium point $\mathbf{p}^* \in \Delta$ as it is stated in the following theorem.

Theorem 1. *Let $\mathbf{F}$ be a continuously differentiable stable game, then the equilibrium point $\mathbf{p}^* \in \Delta$ is asymptotically stable under the Smith dynamics (8).*

Proof. The proof of this theorem is reported in [20]. However, a sketch of the proof is presented. Consider the Lyapunov candidate $V(\mathbf{p})$ given by

$$V(\mathbf{p}) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} p_i \left[f_j(\mathbf{p}) - f_i(\mathbf{p})\right]_+^2,$$

where $V(\mathbf{p}^*) = 0$, and $V(\mathbf{p}) > 0$, for all $\mathbf{p} \neq \mathbf{p}^*$. Its derivative is

$$\begin{aligned}&\dot{V}(\mathbf{p}) = \dot{\mathbf{p}}^\top D\mathbf{F}(\mathbf{p})\dot{\mathbf{p}} + \\ &\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} x_j \left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+ \left(\sum_{k=1}^{N} \left[f_k(\mathbf{p}) - f_i(\mathbf{p})\right]_+^2 - \left[f_k(\mathbf{p}) - f_j(\mathbf{p})\right]_+^2 \right).\end{aligned}$$

Notice that the first element $\dot{\mathbf{p}}^\top D\mathbf{F}(\mathbf{p})\dot{\mathbf{p}} \leq 0$ since $\mathbf{F}$ is a stable game. In order to analyze the second term, suppose that $f_i(\mathbf{p}) > f_j(\mathbf{p})$, then $\left[f_i(\mathbf{p}) - f_j(\mathbf{p})\right]_+ > 0$.

Now $f_k(\mathbf{p}) - f_i(\mathbf{p}) < f_k(\mathbf{p}) - f_j(\mathbf{p})$, and due to the fact that $[\cdot]_+$ is non-decreasing, then $\sum_{k=1}^{N} [f_k(\mathbf{p}) - f_i(\mathbf{p})]_+^2 - [f_k(\mathbf{p}) - f_j(\mathbf{p})]_+^2 \leq 0$. Finally, if $f_i(\mathbf{p}) < f_j(\mathbf{p})$, then $[f_i(\mathbf{p}) - f_j(\mathbf{p})]_+ = 0$ making zero the second term. As conclusion, $\dot{V}(\mathbf{p}) \leq 0$. Moreover using the La Salle's invariance principle, the equality $\dot{V}(\mathbf{p}) = 0$ holds for $f_i(\mathbf{p}^*) = f_j(\mathbf{p}^*)$ for all $i, j = 1, \ldots, N$ completing the proof. □

Due to the fact that the MPC controller works in discrete time, and the population dynamics evolve in continuous time, a way to sample the population dynamics is presented, i.e., a sampled Smith dynamics are established by using the continuous evolution of proportions as introduced next. Consider a sampling time denoted by τ to sample the evolution of the population states $\mathbf{p}(t)$ under the Smith dynamics (8), i.e., every time τ, the population states $\mathbf{p}(\tau)$ evolve as a discrete evolution denoted by $\tilde{\mathbf{p}}(k)$ (Sampled Smith Dynamics). Notice that the population dynamics sampling time must be shorter than the MPC controller sampling time since the evolution of the population dynamics determine the prioritization weights for next iteration in the MPC controller, i.e., $\tau < \Delta t$. Suppose that the initial condition is given by $\tilde{\mathbf{p}}(0) = \mathbf{p}(0)$, then the evolution of the discrete population states is given by $\tilde{\mathbf{p}}(k+1) = \mathbf{p}(\tau)$, $\tilde{\mathbf{p}}(k+2) = \mathbf{p}(2\tau)$, and so on, i.e.,

$$\tilde{p}_i(k+b) = p_i(b\tau), \text{ where } b \in \mathbb{Z}, \text{ and for all } i \in \mathcal{S}.$$

Suppose an arbitrary evolution of the proportion of agents playing strategy $i \in \mathcal{S}$ in continuous time $p_i(t)$ as it is shown in Figure 1. Then, it is obtained the evolution of the same proportion of agents by saving its values every time τ. Figure 1 also shows the discrete evolution of the proportion of agents $\tilde{p}_i(k)$ under the sampled Smith dynamics for different values of τ, i.e., $\tau = 6$ s, $\tau = 8$ s, and $\tau = 10$ s. Furthermore, the dynamical prioritization weights are given by the discrete proportion of agents $\tilde{\mathbf{p}}(k)$.

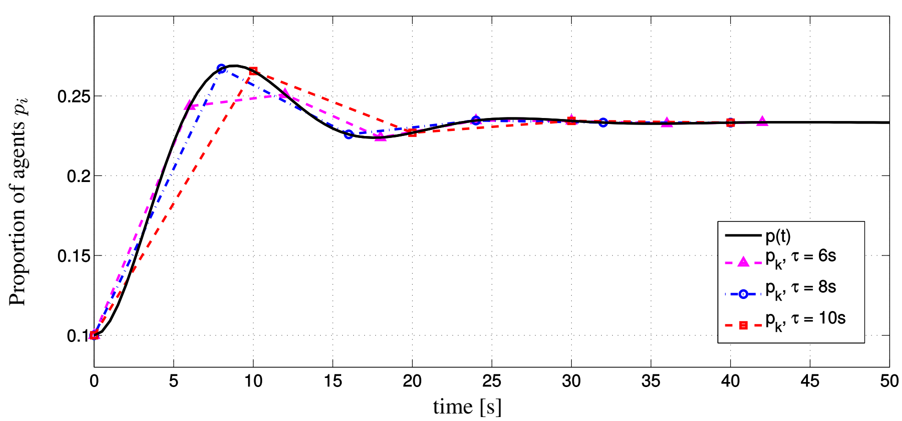


Fig. 1 Example of proportion of agents for different values τ.

3 Proposed dynamical tuning strategy

This section introduces the proposed evolutionary-game-based dynamical tuning for multi-objective MPC. The dynamical tuning strategy is divided in two parts. First, it is necessary to make a normalization procedure, and then a strategy to assign weights dynamically. Both stages are presented next.

3.1 Normalization

The cost function in (4a) involves multiple control objectives, making necessary to establish appropriate weights for each one. In general, each objective has a different nature and, as consequence of this, it is not a trivial issue to find the proper set of weights to obtain the desired control performance. In order to establish a proper distribution of weights in the objective functions, it is necessary first to normalize the cost function [9].

Let $\mathbf{x}_i^*, \mathbf{u}_i^*$ be the optimal solution for the single objective optimization of the i^{th} objective function J_i. The solution $\mathbf{x}_i^*, \mathbf{u}_i^*$ is obtained by solving the optimization problem of the MPC controller (4) with $\gamma_i = 1$ and $\gamma_j = 0$ for all $j \neq i$. Then, the utopia point, denoted by $\mathbf{J}^{\text{utopia}}$, is found as follows:

$$\mathbf{J}^{\text{utopia}} = \left[J_1(\mathbf{x}_1^*, \mathbf{u}_1^*) \quad J_2(\mathbf{x}_2^*, \mathbf{u}_2^*) \quad \cdots \quad J_N(\mathbf{x}_N^*, \mathbf{u}_N^*)\right]. \tag{10}$$

The i^{th} nadir value is denoted by

$$J_i^{\text{nadir}} = \max\left(J_i(\mathbf{x}_1^*, \mathbf{u}_1^*) \quad J_i(\mathbf{x}_2^*, \mathbf{u}_2^*) \quad \cdots \quad J_i(\mathbf{x}_N^*, \mathbf{u}_N^*)\right), \tag{11}$$

and the nadir point $\mathbf{J}^{\text{nadir}}$ is given by

$$\mathbf{J}^{\text{nadir}} = \left[J_1^{\text{nadir}} \quad J_2^{\text{nadir}} \quad \cdots \quad J_N^{\text{nadir}}\right]. \tag{12}$$

Finally, the normalized multi-objective cost function has the form

$$\tilde{J}(\mathbf{x}, \mathbf{u}) = \sum_{i=1}^{N} \tilde{J}_i(\mathbf{x}, \mathbf{u}),$$

where each normalized objective is

$$\tilde{J}_i(\mathbf{x}, \mathbf{u}) = \frac{J_i(\mathbf{x}, \mathbf{u}) - J_i^{\text{utopia}}}{J_i^{\text{nadir}} - J_i^{\text{utopia}}}.$$

After the normalization, weights in the cost function determine a prioritization without being affected by the order of magnitude of each objective.

3.2 Dynamical weighting procedure

Once the objective function has been normalized, it is adequate to establish weights $\tilde{\mathbf{p}}(k)$ to each one of the objectives at each discrete-time instant, i.e., the weight for the i^{th} objective at $k \in \mathbb{Z}_+$ is given by $\tilde{p}_i(k)$. Then, the optimization problem for the normalized MPC controller is stated as follows:

$$\min_{\hat{\mathbf{u}}} \sum_{j=1}^{N} \tilde{p}_j(k) \tilde{J}_j(\mathbf{x}(0), \mathbf{u}), \tag{13a}$$

subject to:

$$\mathbf{x}(i+1|k) = \mathbf{A}\mathbf{x}(i|k) + \mathbf{B}\mathbf{u}(i|k) + \mathbf{B}_l \mathbf{d}(i|k), \;\; i \in [0, H_p - 1] \subset \mathbb{Z}_+, \tag{13b}$$

$$\mathbf{u}(i|k) \in \mathcal{U}, \;\; i \in [0, H_p - 1] \subset \mathbb{Z}_+, \tag{13c}$$

$$\mathbf{x}(i|k) \in \mathcal{X}, \;\; i \in [0, H_p] \subset \mathbb{Z}_+, \tag{13d}$$

where $\tilde{\mathbf{p}}(k) = [\tilde{p}_1(k) \quad \tilde{p}_2(k) \quad \cdots \quad \tilde{p}_N(k)]^\top$, and $\sum_{i=1}^{N} \tilde{p}_i(k) = 1$. The unitary value in the equality constraint represents the total mass population according to (5). The proper prioritization of these objectives might vary over time as exogenous disturbances affecting the system also vary. In order to overcome this issue, it is proposed a dynamical tuning by using a population dynamics approach. Then, the fitness functions $f_i(p_i) \triangleq f_i(\tilde{p}_i(k))$ are selected to be function of each objective evaluated at the current optimal control action, i.e., $\tilde{J}_i(\hat{\mathbf{x}}^*(k), \hat{\mathbf{u}}^*(k))$. Note that this selection of fitness is appropriate since more priority tends to be assigned to those objectives with greater values.

Furthermore, it is desired to assign a prioritization over a region in the Pareto front known as management region (MR). The importance assigned over the MR is determined by a weight w_i in the i^{th} fitness function of the Smith dynamics, i.e.,

$$f_i(\tilde{p}_i(k)) = w_i \tilde{J}_i(\hat{\mathbf{x}}^*(k), \hat{\mathbf{u}}^*(k)). \tag{14}$$

A region is selected over the Pareto front instead of a point as reported in [22]. The selection of a management point as in [22] implies to have to compute several different prioritization weights at each iteration in order to find the proper combination of weights. This procedure must be made every iteration since conditions in the system vary over time as disturbances in the system also vary. Moreover, the disturbances behave in a stochastic manner, for which it is not possible to determine a strategy that uses a limited number of close values to the last one over the Pareto front. The selection of an MR helps to determine the proper direction for each weight only disposing of a single value over the Pareto front at each iteration. Notice that this proper direction can be computed despite the stochastic behavior of disturbances in the system since only the current condition is required. Furthermore, this relaxation of the point for a region allows to reduce the computational burden.

Remark 1. Note that the prioritization in (14) assigns an importance to a region in the Pareto front (i.e., at MR) for the population dynamics evolution, and the terms w_i, for $i = 1, ..., N$, do not appear in the optimization problem of the MPC, and should not be confused with the weights of the cost function in the MPC controller. $\diamondsuit$

The differences between the MR and the static weights in the multi-objective optimization problem are discussed. To do so, consider a simple and general optimization problem given by

$$\min_{\mathbf{z}} J(\mathbf{z}) = p_1 J_1(\mathbf{z}) + p_2 J_2(\mathbf{z}), \tag{15a}$$

subject to:

$$\mathbf{V}\mathbf{z} \leq \mathbf{v} + \mathbf{c}, \tag{15b}$$

where $\mathbf{z} \in \mathbb{R}^m$ is the decision variable, and $\mathbf{V} \in \mathbb{R}^{l \times m}$ is a constant matrix with suitable dimension. The values $p_1, p_2 \in \mathbb{R}$ establish a static prioritization for the objectives $J_1(\mathbf{z})$ and $J_2(\mathbf{z})$, respectively. The vector $\mathbf{v} \in \mathbb{R}^l$ is a constant component in the constraint, whereas the vector $\mathbf{c} \in \mathbb{R}^l$ is a time-varying component. For instance, the time-variant value of the vector $\mathbf{c} \in \mathbb{R}^l$ may be associated to a disturbance $\mathbf{d} \in \mathbb{R}^l$ involved in a constraint in the optimization problem of an MPC controller.

First, suppose that $\mathbf{c} = \mathbf{c}_1$ in (15b), being $\mathbf{c}_1 \in \mathbb{R}^l$ a vector of arbitrary entries. For this case, suppose that the obtained Pareto front is the one presented in Figure 2a), and its normalized Pareto front is the one presented in Figure 2b). This figure shows an example in which the management region is given by $w_1 = w_2 = 0.5$, and shows the solution for the optimization problem when static weights in the multi-objective functions are assigned as $p_1 = p_2 = 0.5$ to objectives $J_1(\mathbf{z})$, and $J_2(\mathbf{z})$, respectively. Notice the difference between the selection of the MR and the assignment of the weights in the cost function.

Now, suppose that $\mathbf{c}$ in (15b) varies, e.g., $\mathbf{c} = \mathbf{c}_2$, where the entries of $\mathbf{c}_1$, and $\mathbf{c}_2$ are near values, i.e., $\mathbf{c}_1 - \mathbf{c}_2 \approx \mathbf{0}$. In this case, the Pareto front varies. Suppose that the new Pareto front is the one obtained in Figure 2c), with its corresponding normalized front presented in Figure 2d). When making this modification over $\mathbf{c}$, the solution of the optimization problem for the weights $p_1 = p_2 = 0.5$ changes dramatically over the Pareto front (this fact illustrates the effect when the disturbances, denoted by $\mathbf{d}$, vary in the optimization problem (13)). However, notice that the MR is still defined as a region where the objective functions have a equitable value for the particular case $w_1 = w_2 = 0.5$.

When the MR is defined, the dynamical tuning strategy is in charge of finding the proper weights $\tilde{p}_1$, and $\tilde{p}_2$ in the normalized cost function, such that the solution lies inside the MR. This philosophy is different from the static tuning strategy where the weights are determined previously. The process to assign dynamically the tuning weights is performed by using the population dynamics, and then in order to guar-

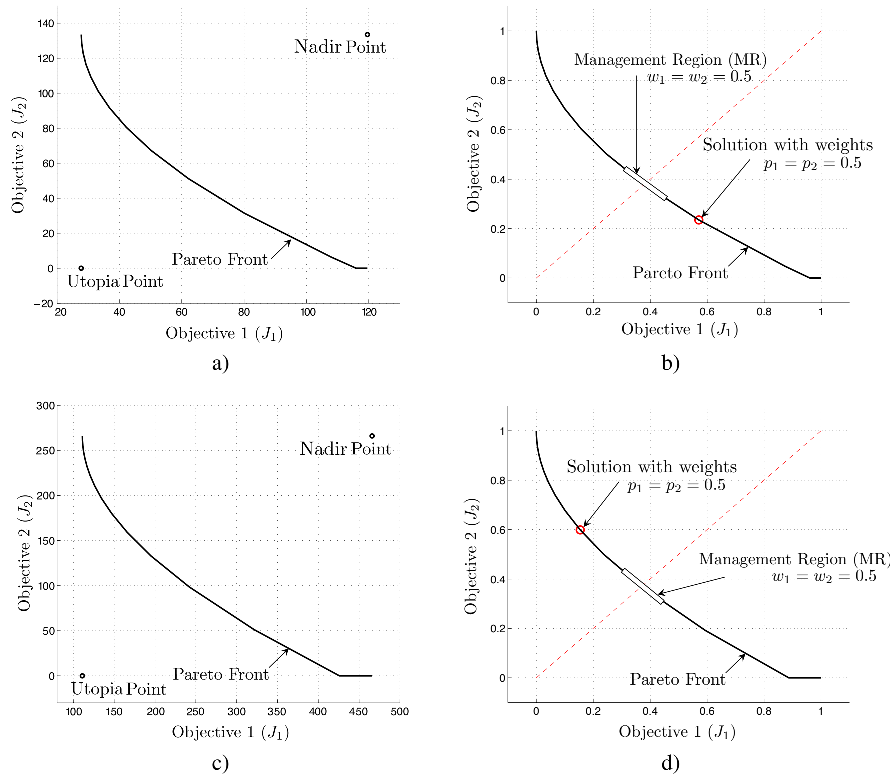


Fig. 2 Comparison between the MR and the optimization prioritizing weights.

antee that the Smith dynamics have a stable behavior, it is necessary the following assumption according to the definition of a stable game.

Assumption 1 *The fitness function* $f_i(p_i)$ *is a decreasing function with respect to* p_i*. Then the game* $\mathbf{F}$ *is a stable game, and stability of the population game is ensured according to Theorem 1. Note that it is expected that the value of the objective* $\tilde{J}_i(\hat{\mathbf{x}}^*(k), \hat{\mathbf{u}}^*(k))$ *decreases as bigger weight* $\tilde{p}_i(k)$ *is assigned to it when solving the corresponding optimization problem.* $\diamondsuit$

A detailed procedure to implement the evolutionary-game-based dynamical tuning for multi-objective model predictive control is presented in Algorithm 1.

4 Case study

The Barcelona Drinking Water Network (DWN) is a large-scale system composed by tanks, valves, pumps, drinking water sources, and water demands as reported

Algorithm 1 Evolutionary-game-based dynamical tuning for multi-objective MPC.

procedure INITIALIZATION
  $H_s \leftarrow$ simulation length
  $H_p \leftarrow$ prediction horizon
  $N \leftarrow$ number of objectives
  $\mathbf{x}(k) \leftarrow \mathbf{x}(0) \in \mathbb{R}^{n_x}$ states initial condition
  $\mathbf{p}(0) \leftarrow \mathbf{p} \in \mathbb{R}_+^N$ proportion initial condition for continuous Smith dynamics
  $\tilde{\mathbf{p}}(k) \leftarrow \mathbf{p}(0) \in \mathbb{R}_+^N$ discrete proportion initial condition
  $\tau \leftarrow$ time for population dynamics
end procedure
while $k \leq H_s$ **do**
  procedure NORMALIZATION
    $i \leftarrow 1$ initialization index for local objectives
    while $i \leq N$ **do**
      $\mathbf{u}_i^* \leftarrow \arg\min_{\hat{\mathbf{u}}} J_i(\mathbf{x}, \mathbf{u})$ with constraints
      $J_i^{\text{utopia}} \leftarrow J_i(\mathbf{x}_i^*, \mathbf{u}_i^*)$
      $i \leftarrow i + 1$
    end while
    $j \leftarrow 1$ initialization index for nadir points
    while $j \leq N$ **do**
      $J_j^{\text{nadir}} \leftarrow \max \left(J_j(\mathbf{x}_1^*, \mathbf{u}_1^*) \quad J_j(\mathbf{x}_2^*, \mathbf{u}_2^*) \quad \cdots \quad J_j(\mathbf{x}_N^*, \mathbf{u}_N^*) \right)$
      $j \leftarrow j + 1$
    end while
  end procedure
  procedure NORMALIZED MPC
    $\hat{\mathbf{x}}^*(k), \hat{\mathbf{u}}^*(k) \leftarrow \arg\min \sum_{i=1}^{N} \tilde{p}_i(k) \tilde{J}_i(\mathbf{x}, \mathbf{u})$ with constraints
    $\mathbf{u}^*(k) \leftarrow \mathbf{u}^*(0|k) \in \mathbb{R}^{n_u}$ optimal control action
  end procedure
  procedure COMPUTATION OF FITNESS FUNCTIONS
    $i \leftarrow 1$ initialization index for local objectives
    while $i \leq N$ **do**
      $f_i(p_i) \triangleq f_i(\tilde{p}_i(k)) \leftarrow \tilde{J}_i(\hat{\mathbf{x}}^*(k), \hat{\mathbf{u}}^*(k))$
      $i \leftarrow i + 1$
    end while
  end procedure
  procedure CONTINUOUS-TIME SMITH DYNAMICS $\forall\, i \in \mathcal{S}$
    $\dot{p}_i = \sum_{j=1}^{N} p_j \left[f_i(p_i) - f_j(p_j) \right]_+ - p_i \sum_{j=1}^{N} \left[f_j(p_j) - f_i(p_i) \right]_+$, for $0 \leq t \leq \tau$
    $\tilde{\mathbf{p}}(k) \leftarrow \mathbf{p}(\tau)$ update of discrete agent proportions
    $\mathbf{p}(0) \leftarrow \tilde{\mathbf{p}}(k)$ new initial condition for Smith dynamics
  end procedure
  procedure OPTIMAL CONTROL ACTION APPLIED TO THE SYSTEM
    $\mathbf{x}(k) \leftarrow \mathbf{A}\mathbf{x}(k) + \mathbf{B}\hat{\mathbf{u}}^*(k) + \mathbf{B}_l \mathbf{d}(k)$
    $k \leftarrow k + 1$
  end procedure
end while

in [16]. The volumes in tanks compose the state vector $\mathbf{x} \in \mathbb{R}^{n_x}$, the flows through the valves and pumps compose the vector of manipulated control actions $\mathbf{u} \in \mathbb{R}^{n_u}$, and the water-demanded flows are collected in vector $\mathbf{d} \in \mathbb{R}^{n_d}$. The corresponding discrete-time model is given by

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_l\mathbf{d}(k), \tag{16a}$$

$$\mathbf{0} = \mathbf{E}_u\mathbf{u}(k) + \mathbf{E}_d\mathbf{d}(k), \tag{16b}$$

where the difference equation in (16a) describes the dynamics of the storage tanks in the system, and the equation (16b) describes the static relations given by the mass balance at junction nodes within the network. Moreover, $\mathbf{0}$ is a column vector whose entries are null, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_l$, $\mathbf{E}_u$, and $\mathbf{E}_d$ are constant matrices with suitable dimensions determined by the DWN topology [7].

4.1 System management criteria

The cost function for the MPC controller is determined by operational objectives, which are established by the company in charge of the DWN. These objectives are usually determined by the following three aspects: *i)* economic operation, *ii)* smoothness operation, and *iii)* safety operation. For the economical aspect, there are two costs associated to the DWN operation. The first cost is related to water depending on the selected source to get water during the day, it is given by $\alpha_1 \in \mathbb{R}^{n_u}$ and whose units are economic units per flow unit ([e.u.] / [m^3/s]). The second cost is time variant during the day, associated to the energy required to operate the active elements in the DWN (i.e., valves and pumps), and it is given by $\alpha_2 \in \mathbb{R}^{n_u}$ in economic units per flow unit ([e.u.] / [m^3/s]). In general, the economic operation objective consists in minimizing the water production and transport costs given in economic units (e.u.), i.e.,

$$J_1(\mathbf{u}(k)) \triangleq \left| \left(\alpha_1 + \alpha_2(k)\right)^\top \mathbf{u}(k) \right|. \tag{17}$$

Regarding the smoothness operation, it is related to the variations of the control actions along the time, i.e., $\Delta\mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$. This objective consists in minimizing

$$J_2(\mathbf{u}(k)) \triangleq \left\| \Delta\mathbf{u}(k) \right\|^2. \tag{18}$$

Finally, the safety operation consists in guaranteeing that there is enough stored water to satisfy the demands during certain period of time. Due to the fact that demand is supposed to be obtained from a forecasting procedure, this operation objective is managed by the following soft constraint:

$$\mathbf{x}(k) \geq \mathbf{x}_s(k) - \boldsymbol{\xi}(k), \text{ for all } \ k, \tag{19}$$

where $\mathbf{x}_s \in \mathbb{R}^{n_x}$ is the vector of safety volumes for all the tanks. The variable $\boldsymbol{\xi} \in \mathbb{R}^{n_x}$ does not have a direct relationship with any element of the system, and it is introduced as a decision variable in the optimization problem to manage the safety volumes. Furthermore, it is desired that $\boldsymbol{\xi}$ tends to zero in order to avoid violations of the constraint (which implies the depletion of such safety volumes). Then, the third objective related to the system states is given by the minimization of

$$J_3(\boldsymbol{\xi}(k)) \triangleq \|\boldsymbol{\xi}(k)\|^2 . \tag{20}$$

It is worth to highlight that it is already known the importance in the prioritization of objectives, which is determined by the company in charge of the management of the network. This known importance among the objective functions is commonly used to determine a static tuning for the system. The most important objective is the economical aspect, and the second is to guarantee the safety volumes. This fact is going to be used below with the case study to determine different and possible cases to test the performance of the MPC controllers.

4.2 Optimization problem of the predictive controller

Once the system management criteria have been established with the objectives J_1, J_2 and J_3, it can be set the normalized optimization problem behind the MPC controller design, i.e.,

$$\begin{aligned}\min_{\hat{\mathbf{u}},\hat{\boldsymbol{\xi}}} J(\mathbf{u},\boldsymbol{\xi}) = & \sum_{j=0}^{H_p-1} \tilde{p}_1(k)\tilde{J}_1(\mathbf{u}(k+j)) + \sum_{j=0}^{H_p-1} \tilde{p}_2(k)\tilde{J}_2(\mathbf{u}(k+j)) + \\ & \sum_{j=0}^{H_p-1} \tilde{p}_3(k)\tilde{J}_3(\boldsymbol{\xi}(k+j)),\end{aligned}$$

subject to:

$$\begin{aligned}\mathbf{x}(i+1|k) &= \mathbf{A}\mathbf{x}(i|k) + \mathbf{B}\mathbf{u}(i|k) + \mathbf{B}_l\mathbf{d}(i|k), \;\; i \in [0, H_p-1] \subset \mathbb{Z}_+, \\ \mathbf{0} &= \mathbf{E}_u\mathbf{u}(i|k) + \mathbf{E}_d\mathbf{d}(i|k), \;\; i \in [0, H_p-1] \subset \mathbb{Z}_+, \\ &\mathbf{u}(i|k) \in \mathcal{U}, \;\; i \in [0, H_p-1] \subset \mathbb{Z}_+, \\ &\mathbf{x}(i|k) \in \mathcal{X}, \;\; i \in [0, H_p] \subset \mathbb{Z}_+, \\ &\mathbf{x}(i|k) \geq \mathbf{x}_s(k) - \boldsymbol{\xi}(i|k), \;\; i \in [0, H_p] \subset \mathbb{Z}_+,\end{aligned}$$

where the feasible sets for the control actions and the system states are given by $\mathcal{U} = \{\mathbf{u} \in \mathbb{R}^{n_u} | \mathbf{u}_{min} \leq \mathbf{u} \leq \mathbf{u}_{max}\}$, and $\mathcal{X} = \{\mathbf{x} \in \mathbb{R}^{n_x} | \mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max}\}$, where $\mathbf{u}_{min}$, and $\mathbf{u}_{max}$ are the minimum and maximum limits for the control actions, respectively. Similarly, $\mathbf{x}_{min}$, and $\mathbf{x}_{max}$ are the minimum and maximum limits for the system states. Finally, similarly as in (3), $\hat{\boldsymbol{\xi}}$ is a sequence during the horizon H_p.

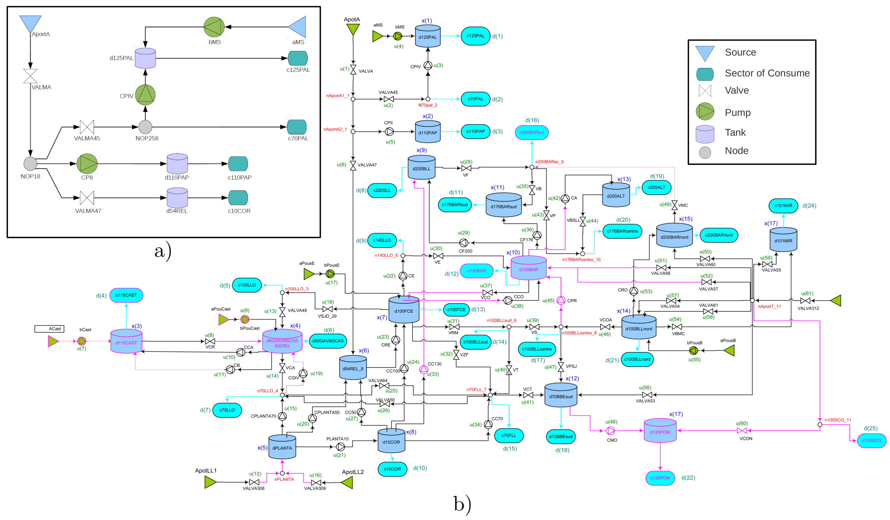


Fig. 3 Case study. a) Topology of the three-tanks DWN. b) Topology of the 17 tanks DWN.

Figure 3 shows two different significative portions of the Barcelona DWN. Figure 3a) is a portion of the DWN involving three tanks (states), three valves and three pumps (control actions), two drinking water sources, and four water demands (disturbances). Then, the matrices and limits for its discrete model are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \Delta t, \quad \mathbf{B}_l = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \Delta t,$$

$$\mathbf{E}_u = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{E}_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \mathbf{x}_{min} = [0 \quad 0 \quad 0]^\top,$$

$$\mathbf{x}_{max} = [470 \quad 960 \quad 3100]^\top, \quad \mathbf{u}_{min} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^\top,$$

$$\mathbf{u}_{max} = [1.2970 \quad 0.05 \quad 0.12 \quad 0.0150 \quad 0.0317 \quad 0.0220]^\top,$$

where the sampling time $\Delta t = 3600$ s. On the other hand, Figure 3b) shows a portion with 17 tanks (states), 61 manipulated flows (control actions), nine water sources, and 25 water demands (disturbances). Matrices and limits for this discrete model are not presented because of lack of space.

4.3 Scenarios

Two different scenarios are presented in order to analyze the performance of the proposed dynamical tuning strategy. In general, demand has a periodic behavior (seasonality), maintaining the same mean value and with a regular amplitude in time. However, it is considered the case in which the periodic demand varies unexpectedly during time, i.e., the case in which the demand profile varies its mean value and its regular amplitude. The purpose of these abrupt changes is to analyze how the prioritization weights are adapted when the system conditions suffer variations. Moreover, these scenarios can certainly occur because of unexpected situations such as public events, damages in the network as leaks, move of population, growth of the system, etc.

Consequently, it is possible to analyze both the performance when the demand decreases, and when demand increases unexpectedly as shown in Figure 4, i.e.,

- **Scenario 1:** decreasing in the demand profiles (see Figure 4a)).
- **Scenario 2:** increasing in the demand profiles (see Figure 4b)).

Both scenarios are analyzed to illustrate that the dynamical tuning strategy may adapt a proper combination of weights in the cost function of the optimization problem behind the MPC controller, for any change in the nominal system behavior.

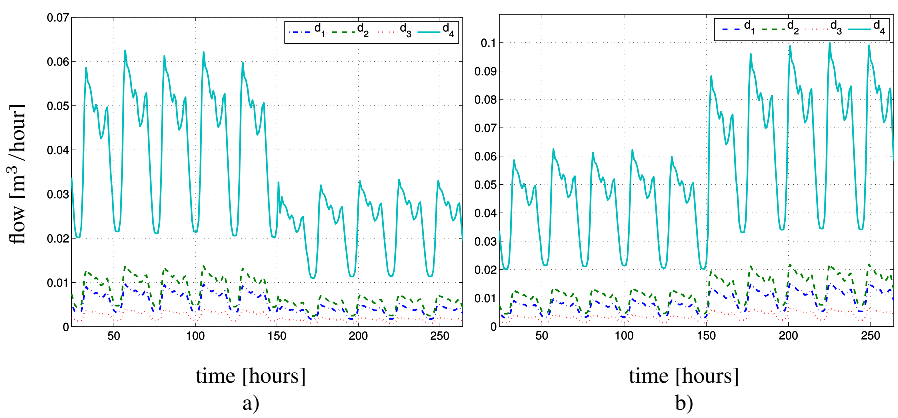


Fig. 4 Demand profile for: a) Scenario 1 with a decrease in the demands, and b) Scenario 2 with an increment in the demands.

In the proposed dynamical tuning methodology, the first step is to normalize the cost function by computing the nadir and the utopia points. After making this procedure, then the following step is to assign a prioritization to the MR where it is desired that different objectives evolve around. For this case study, the prioritization is given by w_1 for economic objective, w_2 for smoothness objective, and w_3 for the safety objective, where $\sum_{i=1}^{N} w_i = 1$.

In order to make a fair comparison between the performance of a multi-objective MPC with conventional static tuning and the performance of an multi-objective MPC with the proposed dynamical tuning strategy, there must be established a relationship between the weights for objective functions. The weights $\gamma_1, ..., \gamma_N$ for the cost function in Problem (4), and the weights for the MR $w_1, ..., w_N$ in (14), are selected to be the same as $w_i = \gamma_i$, for all $i \in \mathcal{S}$. This criterion establishes a fair comparison for a prioritization over the cost function without any tuning strategy, and a prioritization by using a dynamical tuning strategy.

4.4 Results and discussion

The performance of the controllers is determined by the economical costs C given in economic units (e.u.) during the total number of simulation days (in this case 11 days), i.e.,

$$C = \sum_{k=0}^{264} (\alpha_1 + \alpha_2(k))^\top \mathbf{u}(k), \tag{22}$$

where the costs are denoted by C_D for the dynamical tuning case, and by C_S for the static tuning case. For each scenario, four different cases corresponding to four MRs are tested:

- Tuning case 1: $[\gamma_1 \quad \gamma_2 \quad \gamma_3]^\top = [0.7 \quad 0.1 \quad 0.2]^\top$,
- Tuning case 2: $[\gamma_1 \quad \gamma_2 \quad \gamma_3]^\top = [0.6 \quad 0.15 \quad 0.25]^\top$,
- Tuning case 3: $[\gamma_1 \quad \gamma_2 \quad \gamma_3]^\top = [0.5 \quad 0.2 \quad 0.3]^\top$,
- Tuning case 4: $[\gamma_1 \quad \gamma_2 \quad \gamma_3]^\top = [0.4 \quad 0.25 \quad 0.35]^\top$.

Notice that these different cases for tuning satisfy the prioritization explained in Sub-section 4.1, i.e., $w_1 > w_3 > w_2$.

Table 1 Economic results for Scenario 1 and Scenario 2 in the case study of three states in Figure 3a). Notice that for the comparison of data the management region corresponds to the prioritization of the MPC controller with static tuning, i.e., $[w_1 \quad w_2 \quad w_3]^\top = [\gamma_1 \quad \gamma_2 \quad \gamma_3]^\top$.

	Tuning case	**Dynamical tuning costs C_D 11 days (e.u.)**	**Static tuning costs C_S 11 days (e.u.)**	**Reduction of costs $C_S - C_D$ (e.u.)**
Scenario 1	1	5649.45	5660.61	11.16
	2	5656.89	5666.13	9.24
	3	5657.53	5677.65	20.12
	4	5657.01	5738.12	81.11
Scenario 2	1	8983.86	8984.57	0.71
	2	8986.35	8993.64	7.29
	3	8985.69	9011.49	25.80
	4	8986.33	9075.49	89.16

Table 1 shows the comparison between the costs of the MPC with the proposed dynamical tuning and with a static prioritization for the four cases, and for Scenarios 1 and 2, for the case study of three states presented in Figure 3a). Table 1 also shows the difference of costs, i.e., a reduction of costs when changing the static tuning for the dynamical tuning strategy given by $C_S - C_D$. More specifically, results for Scenario 1 show a reduction of costs with the dynamical tuning for all the tested cases. During the 11 days, reduction of costs between 9.24 e.u. and 81.11 e.u. can be obtained.

For Scenario 2, it can be seen that for all the cases a reduction of costs is obtained if the dynamical tuning strategy is adopted. These reductions during 11 days oscillate between 0.70 e.u. and 89.16 e.u. depending on the management region case.

Regarding the dynamical behavior of the proposed strategy, and the evolution of weights, Figure 5 shows the performance of the MPC controller with the dynamical tuning strategy for a management point given by $\mathbf{w} = [0.6 \quad 0.15 \quad 0.25]^\top$, and for the Scenario 1. Similarly, Figure 6 shows the performance of the MPC controller with the dynamical tuning strategy for a management point given by $\mathbf{w} = [0.7 \quad 0.1 \quad 0.2]^\top$, and for the Scenario 2. In the performance of the dynamical weights, it can be seen that they oscillate with the same period as the disturbances in the system. Moreover, it can be seen that the mean value of each weight varies when the behavior of the demands changes at the the seventh day.

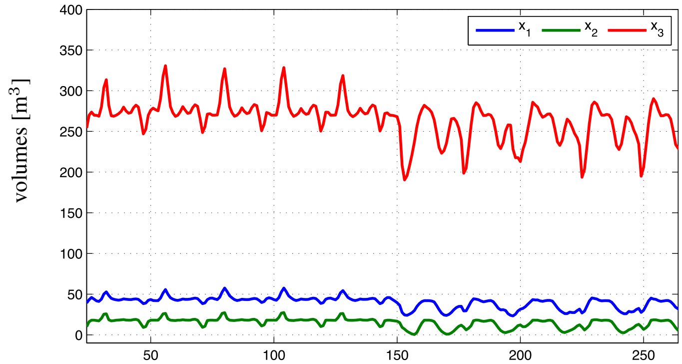
The previously presented results are a proof of concept to see how tuning is adapted dynamically as conditions over the system vary. The case study shown in Figure 3a) does not contain redundant paths to satisfy water demand, and involves a reduced number of states and control actions. Consequently, there is less freedom in order to adjust the proper prioritization values to potentially improve the performance.

Then, the dynamical tuning strategy is implemented in a bigger case study shown in Figure 3b) for Scenario 1, and the tuning case 2. This implementation is made in order to check the improvement of the performance for a larger-scale system with the proposed tuning approach.

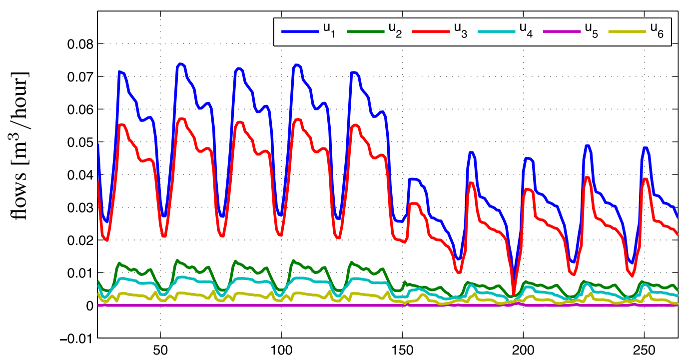
Table 2 Economic results for Scenario 1 in the case study of three states in Figure 3b). Notice that for the comparison of data the management region corresponds to the prioritization of the MPC controller with static tuning, and for the tuning case 2, i.e., $[w_1 \quad w_2 \quad w_3]^\top = [\gamma_1 \quad \gamma_2 \quad \gamma_3]^\top = [0.6 \quad 0.15 \quad 0.25]^\top$.

Dynamical tuning costs C_D 11 **days (e.u.)**	**Static tuning costs** C_S 11 **days (e.u.)**	**Reduction of costs** $C_S - C_D$ **(e.u.)**
398645.19	420894.99	22249.80

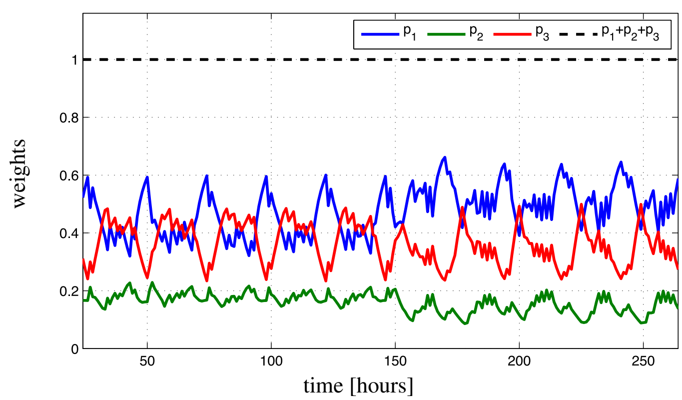
Table 2 shows the results for the case study shown in Figure 3b). It can be seen a higher reduction of costs when adopting the dynamical tuning strategy in a larger case study. The considerable reduction is obtained since the case study in Figure 3b) has redundant paths to satisfy the water demand.



a)



b)



c)

Fig. 5 MPC controller with evolutionary game-based dynamical tuning for the Scenario 1 with a decrease in the demands and a management region given by $\mathbf{w} = [0.6 \quad 0.15 \quad 0.25]^\top$. Sub-figures corresponds to: a) system states $\mathbf{x}$, b) control actions $\mathbf{u}$, and c) dynamical tuning $\tilde{\mathbf{p}}(k)$.

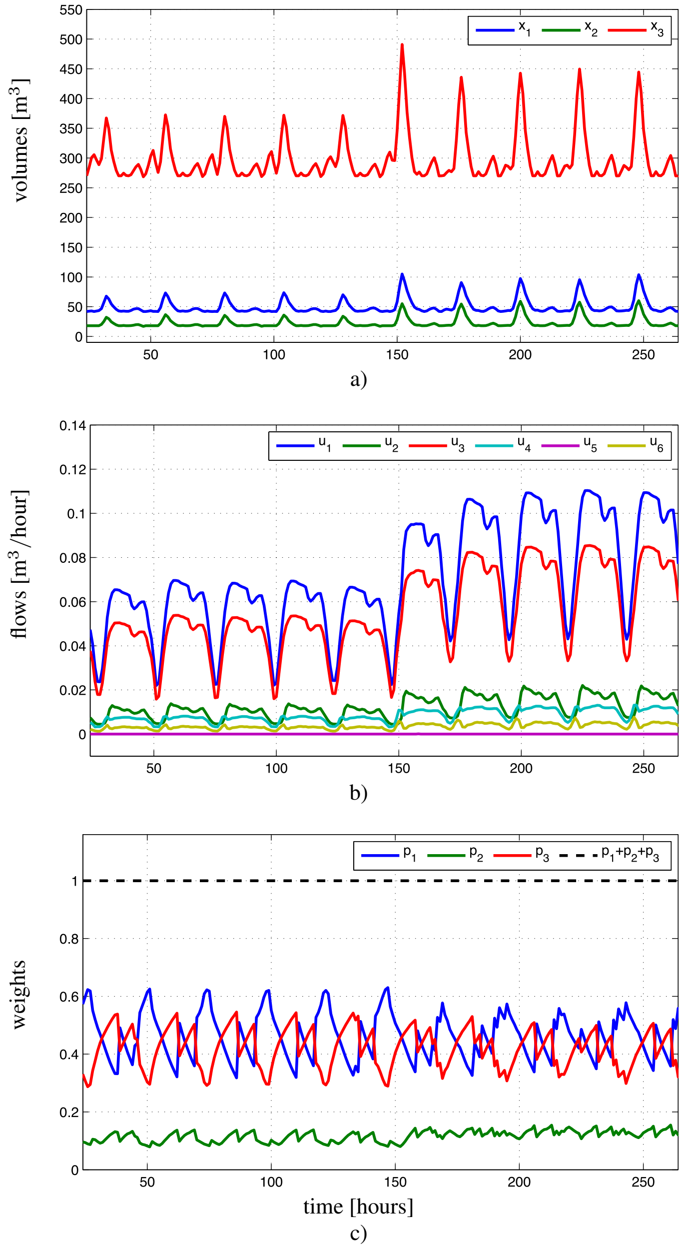


Fig. 6 MPC controller with evolutionary game-based dynamical tuning for the Scenario 2 with an increment in the demands and a management region given by $\mathbf{w} = [0.7 \quad 0.1 \quad 0.2]^\top$. Sub-figures corresponds to: a) system states $\mathbf{x}$, b) control actions $\mathbf{u}$, and c) dynamical tuning $\tilde{\mathbf{p}}(k)$.

5 Conclusions and further work

A novel dynamical tuning strategy based on evolutionary game theory has been proposed. Similarly as other tuning strategies suggest, in the proposed tuning strategy it is necessary to normalize the cost function. In this regard, the proposed strategy does not imply to have higher computation burden with respect to other on-line tuning strategies. Once the problem is normalized, it is not required to generate several points in the Pareto front. This is an advantage of the proposed strategy in comparison to other tuning strategies that require the computation of several points in the Pareto front to establish a proper tuning. However, it should be satisfied that the Pareto front satisfy an assumption clearly defined in this work.

The results obtained in this chapter reflect an improvement in the reduction of economical costs. Moreover, a higher reduction of costs is obtained with the 17 variable states network, than with the smaller system of three variable states. For future work, it is necessary to test the dynamical tuning in a larger system, whose topology includes redundancy paths, and more actuators and constraints (e.g., the whole Barcelona network that is composed by 63 states that can be found in [26]). Then, a more considerable improvement between the performance of an MPC with static tuning and the performance of an MPC with the proposed dynamical tuning might be obtained. Finally, the prediction horizon is considered within the MPC parameters that compose the issue of tuning, and it can be included in the dynamical tuning strategy.

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References

1. A. Al-Ghazzawi, E. Ali, A. Nouh, and E. Zafiriou. On-line tuning strategy for model predictive controllers. *Journal of Process Control*, 11:265–284, 2001.
2. J. Barreiro-Gomez, N. Quijano, and C. Ocampo-Martinez. Distributed constrained optimization based on population dynamics. In *Proceedings of the IEEE Conference on Decision and Control (CDC)*, pages 4260–4265, Los Angeles, USA, 2014.
3. J. Barreiro-Gomez, N. Quijano, and C. Ocampo-Martinez. Distributed control of drinking water networks using population dynamics: Barcelona case study. In *Proceedings of the IEEE Conference on Decision and Control (CDC)*, pages 3216–3221, Los Angeles, USA, 2014.
4. S. Di Cairano and A. Bemporad. Model predictive control tuning by controller matching. *IEEE Transactions on Automatic Control*, 55:185–190, 2010.
5. J.L. Garriga and M. Soroush. Model predictive control tuning methods: A review. *Industrial & Engineering Chemistry Research (I&EC)*, 49:3505–3515, 2010.
6. J.M. Grosso, C. Ocampo-Martinez, and V. Puig. Learning-based tuning of supervisory model predictive control for drinking water networks. *Engineering Applications of Artificial Intelligence*, 26:1741–1750, 2013.

7. J.M. Grosso, C. Ocampo-Martinez, V. Puig, and B. Joseph. Chance-constrained model predictive control for drinking water networks. *Journal of Process Control*, 24:504–516, 2014.
8. J. Hofbauer and W. H. Sandholm. Stable games and their dynamics. *Journal of Economic Theory*, 144(4):1665–1693, 2009.
9. I.Y. Kim and O. L. de Weck. Adaptive weighted sum method for multiobjective optimization: a new method for pareto front generation. *Structural and Multidisciplinary Optimization*, 31:105–116, 2006.
10. N. Li and J. Marden. Designing games for distributed optimization with a time varying communication graph. In *Proceedings of the IEEE Conference on Decision and Control (CDC)*, pages 7764–7769, Maui, Hawaii, 2012.
11. N. Li and J. Marden. Decoupling coupled constraints through utility design. *IEEE Transactions on Automatic Control*, 59:2289–2294, 2014.
12. J. Marden and J. Shamma. Game theory and distributed control. In H.P. Young and S. Zamir, editors, *Handbook of Game Theory Vol. 4*. Elsevier Science, 2013.
13. M.A. Müller, D. Angeli, and F. Allgöwer. On the performance of economic model predictive control with self-tuning terminal cost. *Journal of Process Control*, 24:1179–1186, 2014.
14. G. Obando, A. Pantoja, and N. Quijano. Building temperature control based on population dynamics. *IEEE Transactions on Control Systems Technology.*, 22(1):404–412, 2014.
15. C. Ocampo-Martinez, D. Barcelli, V. Puig, and Bemporad A. Hierarchical and decentralised model predictive control of drinking water networks: application to barcelona case study. *IET Control Theory and Applications*, 6(1):62–71, 2012.
16. C. Ocampo-Martinez, V. Puig, G. Cembrano, and J. Quevedo. Application of predictive control strategies to the management of complex networks in the urban water cycle. *IEEE Control Systems Magazine*, 33(1):15 – 41, 2013.
17. A. Pantoja and N. Quijano. A population dynamics approach for the dispatch of distributed generators. *IEEE Transactions on Industrial Electronics*, 58(10):4559–4567, 2011.
18. A. Pantoja and N. Quijano. Distributed optimization using population dynamics with a local replicator equation. In *Proceedings of the IEEE Conference on Decision and Control (CDC)*, pages 3790–3795, Maui, Hawaii, 2012.
19. E. Ramirez-Llanos and N. Quijano. A population dynamics approach for the water distribution problem. *International Journal of Control*, 83(9):1947–1964, 2010.
20. W. H. Sandholm. *Population games and evolutionary dynamics*. Cambridge, Mass. MIT Press, 2010.
21. G. Shah and S. Engell. Tuning MPC for desired closed-loop performance for MIMO systems. In *Proceedings of the American Control Conference (ACC)*, pages 4404–4409, San Francisco, USA, 2011.
22. R. Toro, C. Ocampo-Martinez, F. Logist, J.V. Impe, and V. Puig. Tuning of predictive controllers for drinking water networked systems. In *Proceedings of the 18th IFAC World Congress*, pages 14507–14512, Milan, Italy, 2011.
23. Q.N. Tran, R. Octaviano, L. Özkan, and A.C.P.M. Backx. Generalized predictive control tuning by controller matching. In *Proceedings of the American Control Conference (ACC)*, pages 4889–4894, Portland, USA, 2014.
24. J. W. Weibull. *Evolutionary Game Theory*. The MIT Press, London, England, 1997.
25. J. Zhang, D. Qi, and G. Zhao. A game theoretical formulation for distributed optimization problems. In *Proceeding of the IEEE International Conference on Control and Automation (ICCA)*, pages 1939–1944, 2013.
26. Y. Wang, C. Ocampo-Martinez, V. Puig, and J. Quevedo. Gaussian-process-based demand forecasting for predictive control of drinking water networks. In *9th International Conference on Critical Information Infrastructures Security*, pages 13-15, Limassol, Cyprus, 2014.