Combining CSP and MPC for the Operational Control of Water Networks

Cong Cong Sun[a], Vicenç Puig[a,*], Gabriela Cembrano[a,b]

[a]*Advanced Control Systems Group at the Institut de Robòtica i Informàtica Industrial (CSIC-UPC), Llorens i Artigas, 4-6, 08028 Barcelona, (Spain). Tel: +34 68 804 18 55*
[b]*CETaqua, Water Technology Centre at Esplugues 75, 08940, Cornellà de Llobregat, Barcelona*

Abstract

This paper presents a control scheme which uses a combination of linear Model Predictive Control (MPC) and a Constraint Satisfaction Problem (CSP) to solve the non-linear operational optimal control of Drinking Water Networks (DWNs). The methodology has been divided into two functional layers: First, a CSP algorithm is used to transfer non-linear DWNs pressure equations into linear constraints on flows and tank volumes, which can enclose the feasible solution set of the hydraulic non-linear problem during the optimization process. Then, a linear MPC with tightened constraints produced in the CSP layer is solved to generate control strategies which optimize the control objectives. The proposed approach is simulated using Epanet to represent the real DWNs. Non-linear MPC is used for validation. To illustrate the performance of the proposed approach, a case study based on the Richmond water network is used and a realistic example, D-Town benchmark network, is added as a supplementary case study.

Keywords: MPC, CSP, Epanet, water networks.

*Corresponding author
Email address: vicenc.puig@upc.edu (Vicenç Puig)

1. Introduction

Water is always a critical resource for supporting human activities and ecosystem conservation. Recently, the population and users' requirements are increasing while water resources are limited. This situation indicates the need for an optimal operation of water distribution networks, especially during shortage events as discussed in Miao et al. (2014) and Soltanjalili et al. (2013). Management of Drinking Water Networks (DWNs) involves objectives such as minimizing operational cost of pumps, which represents a significant fraction of the total expenditure of a water utility, as discussed in López-lbañez et al. (2008)), or minimizing risks of service failure (as explained in Kurek & A. Ostfeld (2014)).

The optimization problems associated to the operational control of DWNs are complex because of their large-scale, multiple-input, multiple-output nature, as well as the various sources of additive and, possibly, parametric uncertainty in DWNs. Additionally, DWNs models include both deterministic and stochastic components and involve linear (flow model) as well as non-linear (pressure model) equations. The use of non-linear models in DWNs is essential for the operational control which involves manipulating not only flows but also pressures.

Non-linear optimization refers to optimization problems where the objective or constraint functions are nonlinear, and possibly non-convex. No universally-applicable methods exist for solving a non-linear optimization problem when it is non-convex. Even simple-looking problems with a small number of variables can be extremely challenging, while problems with important number of variables can be intractable. Non-linear optimization may be addressed with several different approaches; each of which involving some compromise. Local optimization methods can be fast and can also handle large-scale problems although they do not guarantee finding the global optimum. Alternatively, global optimization is limited to be used in small problems (networks), where computational time is not critical, because usually the global solution search is time consuming, as discussed in Boyd & Vandenberghe (2004).

Early optimization approaches for DWNs typically rely on a substantially simplified network hydraulic model (by dropping all nonlinearities, for instance) as described in Coulbeck et al. (1988); Diba et al. (1995); Sun et al. (1995) and Papageorgiou (1983), which is often unacceptable in practice. Other authors employ discrete dynamic programming as presented in Can & Houck (1984); Carpentier & Cohen (1993); Cembrowicz (1990); Murray & Yakowitz (1979); Orr et al. (1990) and Zessler & Shamir (1989), which is mathematically sound but only applicable to small networks unless specific properties can be exploited to increase efficiency.

Model Predictive Control (MPC) is a well-established class of advanced control methods for complex large scale systems, as explained in Rawlings & Mayne (2009) and Mayne et al. (2000). In Ocampo-Martínez et al. (2013) and Fiorelli et al. (2014), MPC has been successfully applied to control and optimize linear flow model of DWNs. When the pressure model is considered, the non-linear functions involved will increase the computational burden of MPC especially when the size of the network increases. Besides, convergence to the global minimum cannot be easily guaranteed using non-linear MPC if non-linear programming algorithms are used. As described in Boyd & Vandenberghe (2004), for a non-convex problem, an approximate, but convex formulation is needed. By solving the approximate problem, which can be done easily and without an initial guess, the exact solution to the approximate convex problem is obtained. Many methods for global optimization require a cheaply computable lower bound on the optimal value of the non-convex problem. In the relaxed problem, each non-convex constraint is replaced with a looser, but convex constraint. In Mayne et al. (2011), a similar approach based on tube-based MPC is proposed. In this case, the way of circumventing the complexity problem is based on replacing the non-linear MPC by an approximation about a nominal trajectory. Trajectories are bounded by a level set of a value function that varies in a complex way with *(x, t)*.

This paper mainly provides a methodology for solving large scale complex non-linear DWNs problem using a convex approximation of the problem. The solution is compared to that of a nonlinear MPC implementation, obtained with a tool named PLIO (Cembrano et al. (2011)). Simulation results are compared using the Richmond case study introduced in van Zyl et al. (2004). Finally, the D-Town benchmark network, which is much more realistic as presented in Price & Ostfeld (2014) and P.L.Iglesias-Rey et al. (2014), is used as a supplementary case study for validation.

The aim of the proposed approach is to avoid the non-linear optimization problem of DWNs by the combined use of linear MPC and CSP while maintaining optimality and also feasibility with the tightened linear constraints provided by the CSP in Streif et al. (2014). To assess the proposed approach, the real hydraulic behavior of the DWNs is simulated by means of Epanet (Rossman (2000)), which simulates DWNs using the input optimal solution provided by MPC. As shown in Figure 1, the whole controlling methodology works in a two-layer structure as initially proposed in Sun et al. (2014a): CSP is the first step of this methodology and it constitutes the upper layer used for converting the non-linear hydraulic pressure constraints into the linear MPC constraints. MPC is the lower layer producing optimal set-points for controlling actuators (pumps and valves), according to the defined objective functions including minimizing operational costs of pumps, risks and safety goals.

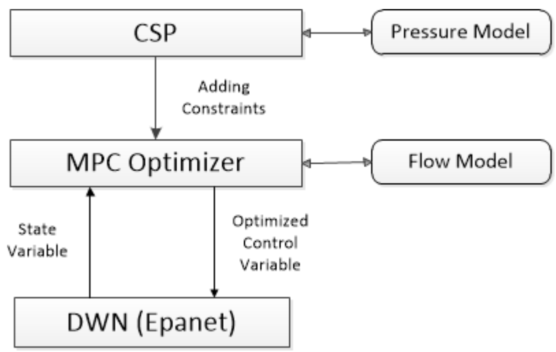


Figure 1: The multi-layer control scheme

The remainder of the paper is organized as follows: The control-oriented modelling methodology considering both flow and pressure dynamics is presented in Section 2. Then, in Section 3, the operational control problem is introduced in the context of non-linear MPC. In Section 4, the definition of CSP and also the proposed CSP-MPC control scheme are explained in detail. Section 5 summarizes the results and validations using the Richmond case study. Section 6 provides a supplementary application based on a more complex example, a benchmark network called D-Town. Finally, Section 7 contains the conclusions and future research plans.

2. Control-Oriented Modelling Methodology

Drinking Water Networks (DWNs) generally contain tanks, which store the drinking water at appropriate head level (elevation and pressure) to supply demand, a network of pipes and a number of demands. Valves and/or pumping stations are the elements that allow to manipulate the water flow according to a specific policy and to supply water requested by the network users at appropriate service pressures.

The DWNs can be considered as composed of a set of constitutive elements, which are presented below including first the flow model and then the pressure model.

2.1. Flow Model

2.1.1. Reservoirs and Tanks.

Water reservoirs and tanks play an important role in DWNs since they enable demand management, ensure water supply (e.g., in case of unexpected demand changes or in case of emergencies) and allow for the modulation of pump flow rate as discussed in Batchabani & Fuamba (2014) and Lee et al. (2013). Moreover, they provide the entire network with the water storage capacity. The mass balance expression of these storage elements relates with the stored volume V, the manipulated inflows q_{in}^{j} and outflows q_{out}^{h} (including the demand flows as outflows). The i^{th} storage element can be described by the discrete-time difference equation

$$V_i(k+1) = V_i(k) + \Delta t \left(\sum_j q_{\text{in}}^{j}(k) - \sum_h q_{\text{out}}^{h}(k) \right), \tag{1}$$

where Δt is the sampling time and k denotes the discrete-time instant. The physical constraint related to the admissible range of water levels in the i^{th} storage element is expressed as

$$\underline{V}_i \le V_i(k) \le \overline{V}_i, \quad \text{for all } k, \tag{2}$$

where $\underline{V}_i$ and $\overline{V}_i$ denote the minimum and the maximum admissible storage capacity, respectively. Although $\underline{V}_i$ might correspond to an empty storage element, in practice this value is normally set as nonzero in order to maintain an emergency stored volume for extreme circumstances.

For simplicity purposes, the dynamic behavior of these elements is described as function of volume. However, in most cases, the measured variable is the storage water level (by using level sensors), which implies the computation of the water volume taking into account the tank geometry.

2.1.2. Actuators.

Two types of control actuators are considered: valves and pumps (more precisely, complex pumping stations). In the flow model, valves and pumps are simplified and considered as similar control elements, and their flows are taken as the manipulated variables in the MPC problem, denoted as q_u. Both pumps and valves have lower and upper physical limits, which are taken into account as system constraints. As in (2), they are expressed as

$$\underline{q_u}_i \leq q_{u_i}(k) \leq \overline{q_{u_i}}, \quad \text{for all } k, \tag{3}$$

where $\underline{q_u}_i$ and $\overline{q_{u_i}}$ denote the minimum and the maximum flow capacity, respectively.

2.1.3. Nodes.

These elements correspond to the points in the water network where water flows are merged or split. Thus, the nodes represent mass balance relations, modelled as equality constraints related to inflows (from tanks through valves or pumps) and outflows, which may be manipulated flows or demand flows. The expression of the mass conservation in these nodes can be written as

$$\sum_{j} q_{\text{in}}^{j}(k) \; = \; \sum_{h} q_{\text{out}}^{h}(k). \tag{4}$$

where node inflows and outflows are denoted by q_{in}^{j} and q_{out}^{h}, respectively. Some manipulated flows could also be denoted by q_u, as required.

2.1.4. Demand Sectors.

Demand sectors represent the water consumed by the network users in a certain physical area. Water demands are considered as a measured disturbance of the system at a given time instant. The demand in urban areas can be anticipated by a forecasting algorithm which can predict the future demand using historical data, integrated within the MPC framework. The demand forecasting algorithm typically uses a two-level scheme composed of *(i)* a time-series model representing the daily aggregate flow values, and *(ii)* a set of different daily flow demand patterns according to the day type to cater for different consumption habits during the weekend and holiday periods (for more details see Quevedo et al. (2010)). Every pattern consists of 24 hourly values for each daily pattern. The daily time series of hourly-flow predictions are computed as a product of the daily aggregate flow value and the appropriate hourly demand pattern.

2.2. Pressure Model

The pressure model contains the flow model presented in the previous section and it is extended using the non-linear relationship between flow and head loss, which exists at pipes, valves, pumps and tanks as described in Brdys & Ulanicki (1994).

2.2.1. Pipes.

Pipes are links which convey water from one point in the network to another. During the transport, water pressure decreases because of friction.

The Chezy-Manning model as presented in Rossman (2000) is one of the various widely used models to describe head loss between two nodes h_i and h_j linked by a pipe:

$$g^c(q) = h_i - h_j = R_{ij} q_{ij}^2 \tag{5}$$

where

$$R_{ij} = (10.29 \times L_{ij})/({C_{ij}}^2 \times {D_{ij}}^{5.33}) \tag{6}$$

and L_{ij}, D_{ij}, C_{ij}, q_{ij} denote the pipe length, diameter, roughness and flow.

2.2.2. Pumps.

Pumps introduce an increase of head between the suction node s and the delivery node d. The estimate function that relates the pump flow with the head change depends on the technical characteristics of the pump (e.g., if the pump can be controlled for example with fixed or variable speed). In the more general case that corresponds to variable speed pumps, the relation between the flow and the head increase is given by:

$$g^f(q, n, s) = h_d - h_s = \begin{cases} Wq^2 + Mq + Ns^2, & \text{if } n \neq 0 \text{ and } s \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where s is the pump speed and n corresponds to the number of pumps that are turned on, W, M and N are pump specific coefficients.

2.2.3. *Valves.*

There are many types of valves which perform different functions, e.g. pressure reduction or flow regulation. In this paper for illustrative purposes, the valves are modelled as a pipe with controlled conductivity, that is

$$g^v(q, G) = h_i - h_j = G_{ij} R_{ij} q_{ij}^2 \tag{8}$$

where R_{ij} is the pipe conductivity and G_{ij} is the control variable that manipulates the valve from 0 (closed) to 1 (open).

2.2.4. *Tanks.*

The head established by the i^{th} tank is given by the following equation:

$$h_{ri}(t) = \frac{V_i(t)}{Sec_i} + E_i \tag{9}$$

where Sec_i is the cross-sectional area of the tank and E_i is the tank elevation.

3. Operational Control Problem Statement

The type of control used in DWNs can be mainly separated into two categories: flow control, mainly useful for transport networks and flow/pressure control, in case of distribution networks (see Brdys & Ulanicki (1994)).

3.1. *MPC for Flow Control*

In the case of the flow control problem, the MPC problem is based on the linear discrete-time prediction model that is obtained using the flow modelling approach introduced in Section 2. Linear MPC, as described in Maciejowski (2002) is based on representing the system to be controlled in discrete-time state space form:

$$x(k+1) = Ax(k) + Bu(k), \tag{10a}$$

$$y(k) = Cx(k), \tag{10b}$$

where $x(k) \in \mathbb{R}^{n_x}$ is state vector and $u(k) \in \mathbb{R}^{n_u}$ is vector of command variables at time step k, and $y(k) \in \mathbb{R}^{n_y}$ is the vector of the measured outputs. In the case of DWNs, states $x(k)$ are the volume of tanks/reservoirs while $u(k)$ are flow set-points for actuators (pumps and valves). Matrices A and B are obtained taking into account the DWNs topology and the control oriented modelling approach presented in Section 2.

An incidence matrix Λ_c is defined for junction nodes in order to write equation (4) in matrix form, where the element in the i^{th} column and j^{th} row of junction nodes incidence matrix Λ_c is defined as:

$$a_{ij} = \begin{cases} 1 & \text{if flow of branch } i \text{ enters node } j \\ 0 & \text{if branch } i \text{ and node } j \text{ are not connected} \\ -1 & \text{if flow of branch } i \text{ leaves node } j \end{cases} \tag{11}$$

Notice that the incidence matrix rows correspond to the non-storage nodes, while its columns are related to the network branches. Assuming one network has n_c non-storage nodes and b branches, this incidence matrix are n_c rows and b columns.

Thus, the matrix form of equation (4) is as follows:

$$\Lambda_c q(k) = d(k) \tag{12}$$

where $q = (q_1, \ldots, q_b)^T$ is a vector of branch flows, d denotes an augmented demand vector by zero components corresponding to non-loaded nodes.

Following Maciejowski (2002) for the basic formulation of a predictive control, the cost function is assumed to be quadratic and the constraints are in the form of linear inequalities. Thus, the following basic optimization problem (BOP) has to be solved:

Problem 1

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{13a}$$

$$\begin{aligned} s.t. \quad & x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1, \cdots, H_p, \\ & x(0|k) = x(k), && (13b) \\ & \Lambda_c u(i|k) = d(k), \quad i = 0, \cdots, H_p - 1, && (13c) \\ & x_{min} \leq x(i|k) \leq x_{max}, \quad i = 1, \cdots, H_p, && (13d) \\ & u_{min} \leq u(i|k) \leq u_{max}, \quad i = 0, \cdots, H_{p-1}, && (13e) \end{aligned}$$

Here, J is a performance index, representing the operational goals of the DWNs, H_p is the prediction horizon, $x(0|k)$ is the initial condition of the state vector obtained from the measurement (or estimation) of the DWNs state (tank volumes) at time k, x_{min}, x_{max}, u_{min} and u_{max} are known vectors defining the operational limits of state and input variables. The BOP can be recast as a Quadratic Programming (QP) problem, whose solution:

$$\mathcal{U}^*(k) \triangleq [u^*(0|k), \cdots, u^*(H_p - 1|k)]^T \in \mathbb{R}^{H_p m \times 1} \tag{14}$$

is a sequence of optimal control inputs that generates an admissible state sequence. At each sampling time k, BOP is solved for the given measured (or estimated) current state $x(k)$. Only the first optimal move $u^*(0|k)$ of the optimal sequence $\mathcal{U}^*(k)$ is applied to the system:

$$u_{MPC}(k) = u^*(0|k) \tag{15}$$

while the remaining optimal moves are discarded and the optimization is repeated at time $k + 1$ using the state $x(k + 1)$ as initial condition.

3.2. Operational Goals for Flow Control

The main operational goals to be achieved in DWNs are:

- *Cost reduction* (J_{cost}): To minimize water cost during water supplying process by selecting the less costly source and optimizing pump schedule according to electric tariff that varies with time of the day.

- *Operational safety* (J_{safety}): To maintain appropriate water storage levels in tanks of the network for emergency-handling and to consider unexpected demand changes.

- *Control actions smoothness* ($J_{smoothness}$): To produce smooth flow set-point variations in order to avoid pipe over-pressures, and also the sustainable operation of actuators.

The above-mentioned goals lead to the following function:

$$\begin{aligned} J &= J_{safety} + J_{smothness} + J_{cost} \\ &= \varepsilon_{\widetilde{x}}(k)^\top W_{\widetilde{x}} \varepsilon_{\widetilde{x}}(k) + \Delta\widetilde{u}(k)^\top W_{\widetilde{u}} \Delta\widetilde{u}(k) \\ &+ W_a(a_1 + a_2(k))\widetilde{u}(k) \end{aligned} \tag{16}$$

where

$$\begin{aligned} \varepsilon_{\widetilde{x}}(k) &= \widetilde{x}(k) - \widetilde{x}_r \\ \widetilde{u} &= \Theta\Delta\widetilde{u} + \Pi\widetilde{u}(k-1) \\ \Delta\widetilde{u}(k) &= \widetilde{u}(k) - \widetilde{u}(k-1) \end{aligned}$$

and $W_{\widetilde{x}}$, $W_{\widetilde{u}}$, W_a are the weights which decide the priorities (established by the water network managers) for all the terms appearing in the objective function. Some multi-criteria decision-making methods recommend converting multiple objectives into a single criterion using a weighting approach as in Woodward et al. (2014). The weight tuning method proposed in Toro et al. (2011), based on computing the Pareto front of the multi-objective optimization problem presented in (16), is used in this paper. The initial step of this tuning approach relies on finding what are known as the anchor points that correspond to the best possible value for each objective obtained by optimizing a single criterion at a time. Then, a normalization procedure is applied, a Management Point (MP) is defined by establishing objective priorities, and the optimal weights are determined by computing those that minimize the distance from the solutions of the Pareto front and the MP.

The vectors a_1 and a_2 contain the cost of water treatment and pumping, respectively, and a_2 is time varying taking into account the variation of electricity price during the day.

The objective equation (16) of the MPC problem can be formulated in the following way:

$$J = z^T \Phi z + \phi^T z + c \tag{17}$$

where

$$z = [\Delta\widetilde{u} \quad \varepsilon_{\widetilde{x}} \quad \varepsilon]^T \tag{18}$$

and c is a constant value.

This allows to determine the optimal control actions at each instant k by solving a Quadratic Programming (QP) algorithm in the form:

$$\min_{z} z^\top \Phi z + \phi^\top z$$
$$A_1 z \leq b_1$$
$$A_2 z = b_2$$

3.3. Nodal Model for Pressure Management

As described in the previous section, in the flow model of DWNs, pipes, valves and pumps constitute a static part of the DWNs. The system dynamics are associated with tanks. In equation (1), the mass balance in the i^{th} tank is provided, while equation (9) describes the relation between the tank volume and its head.

After combining equation (9) with equation (1), tank dynamics both considering flow and pressure will be presented as:

$$\begin{cases} h_{ri}(t) = \frac{V_i(t)}{Sec_i} + E_i \\ V_i(k+1) = V_i(k) + \Delta t \left(\sum\limits_j q_{\text{in}}^j(k) - \sum\limits_h q_{\text{out}}^h(k) \right) \end{cases} \tag{19}$$

For every junction node j, as equation (4) shows, the sum of inflows and outflows is equal to zero for every non-storage node.

Considering a network with n nodes and b branches, the node-branch matrix Λ will have n rows and b columns. Consider element b_{ij} in the i^{th} row in the j^{th} column as equation (11) holds. Therefore, the i^{th} row contains branch to node information, as opposed to the incidence matrix, where the i^{th} row contains node to branch information. For the sake of convenience, we will place the rows corresponding to the tank/reservoir nodes on the first n_r position. The other rows correspond to the junction nodes. With the help of matrix Λ, we can write the flow-head equations as the following vector equation:

$$\Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(q) = 0 \tag{20}$$

where

- $h_r = (h_{r1}, \cdots, h_{r,n_r})^T$ heads of reservoir/tank storage nodes
- $h = (h_1, \cdots, h_{n_c})^T$ heads of junction non-storage nodes
- $q = (q_1, \cdots, q_b)^T$ branch flows
- $G(q) = (g^c{}_1(q_1), \cdots, -g^f{}_i(q_i, n_i, s_i), \cdots, g^v{}_1(q_1, G_1), \cdots,)^T$ functions defining flow-head relationships

Combining this equation with equation (4) yields the nodal model:

$$\begin{cases} \Lambda_c q = d \\ \Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(q) = 0 \end{cases} \tag{21}$$

3.4. MPC for Pressure Management

The MPC for flow and pressure management may be defined in a similar way as MPC for flow control but including non-linear constraints. Thus, the MPC for DWN pressure control is defined as

Problem 2

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{22a}$$

$$s.t. \quad x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1, \cdots, H_p,$$

$$x(0|k) = x_k, \tag{22b}$$

$$\Lambda_c u(i|k) = d(k), \quad i = 0, \cdots, H_p - 1, \tag{22c}$$

$$h_r(i|k) = \frac{x(i|k)}{Sec_i} + E_i, \quad i = 1, \cdots, H_p, \tag{22d}$$

$$\Lambda^T \begin{bmatrix} h_r(i|k) \\ h(i|k) \end{bmatrix} + G(u(i|k)) = 0, \tag{22e}$$

$$x_{min} \leq x(i|k) \leq x_{max}, \quad i = 1, \cdots, H_p, \tag{22f}$$

$$u_{min} \leq u(i|k) \leq u_{max}, \quad i = 0, \cdots, H_{p-1}, \tag{22g}$$

As described above, MPC for flow and pressure management is non-linear because of added pressure constrains in equation (22e), which adds complexity to the optimization problem for the large scale DWNs, as already discussed.

There have been several attempts in recent years to develop optimal control algorithms to optimize the operation of DWNs including pressure control. As already presented in the introduction, many algorithms were oriented towards determining the optimum pump policies to achieve the minimum operating cost, and were based on the use of non-linear programming, dynamic programming, enumeration techniques, and general heuristics as described in Savic et al. (1997), Gupta et al. (1999) and Li et al. (2009). However, the success of these algorithms and methods have been very limited when actually being used in practice because of the complexity associated with solving the non-linear optimization problem for large scale DWNs in real-time as required when using MPC.

4. Proposed Approach

4.1. Overview of Scheme CSP-MPC

The scheme integrating CSP and MPC for DWNs is presented in Figure 2, which shows that the main principle of this proposed control scheme is translating the equations of the non-linear pressure model into linear constraints, which may be tackled by MPC using only the flow model with constraints updated by CSP. The linear constraints produced by CSP will be combined together with the initial constraints of the linear MPC for flow control.

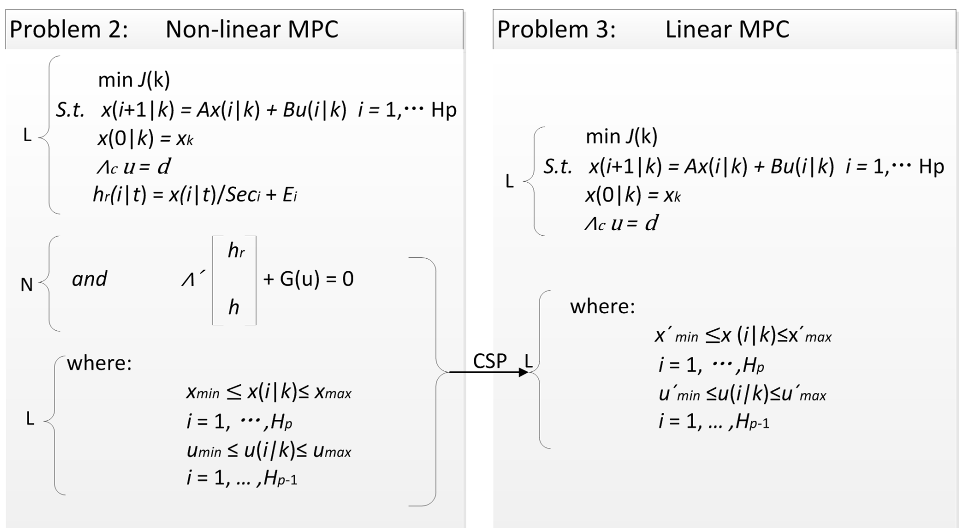


Figure 2: Working principle of CSP-MPC

With this scheme, the non-linear (represented by N in the scheme) MPC described in *Problem* 1, will be translated into a linear (represented by L in the scheme) MPC problem with updated constraints

Problem 3

$$\min_{(u(0|k),\cdots,u(H_{p-1}|k))} J(k) \tag{23a}$$

$$s.t. \quad x(i+1|k) = Ax(i|k) + Bu(i|k), \quad i = 1, \cdots, H_p,$$

$$x(0|k) = x_k, \tag{23b}$$

$$\Lambda_c u(i|k) = d(k) \tag{23c}$$

$$x'_{min} \leq x(i|k) \leq x'_{max}, \quad i = 1, \cdots, H_p, \tag{23d}$$

$$u'_{min} \leq u(i|k) \leq u'_{max}, \quad i = 0, \cdots, H_{p-1}, \tag{23e}$$

where equation (23d) and equation (23e) are the updated constraints resulting from solving the CSP associated to the pressure equations.

4.2. Definition of CSP

4.2.1. Introduction

As introduced in Jaulin et al. (2001), a CSP on sets can be formulated as a 3-tuple $\mathcal{H} = (\mathcal{V}, \mathcal{D}, C)$, where

- $\mathcal{V} = \{v_1, \cdots, v_n\}$ is a finite set of variables.
- $\mathcal{D} = \{\mathcal{D}_1, \cdots, \mathcal{D}_n\}$ is the set of their domains.
- $C = \{c_1, \cdots, c_n\}$ is a finite set of constraints relating variables of $\mathcal{V}$.

Solving a CSP consists of finding all variable value assignments such that all constraints are satisfied. The variable value assignment $(\hat{z}_1, \cdots, \hat{z}_n) \in \mathcal{D}$ is a solution of $\mathcal{H}$ if all constraints in C are satisfied. The set of all solution points of $\mathcal{H}$ is called the global solution set and denoted by $\mathcal{S}(\mathcal{H})$. The variable $v_i \in \mathcal{V}$

is consistent in $\mathcal{H}$ if and only if $\forall \hat{z}_i \in \mathcal{D}_i$, $\exists(\hat{z}_1 \in \mathcal{D}_1, \cdots, \hat{z}_n \in \mathcal{D}_n)$, such as $(\hat{z}_i, \cdots, \hat{z}_n) \in \mathcal{S}(\mathcal{H})$ as presented in Tornil-Sin et al. (2014).

The solution of a CSP is said to be globally consistent if and only if every variable is consistent. A variable is locally consistent if and only if it is consistent with respect to all directly connected constraints. Thus, the solution of the CSP is said to be locally consistent if all variables are locally consistent. An algorithm for finding an approximation of the solution set of a CSP can be found in Jaulin et al. (2001).

4.2.2. Implementation using Intervals

It is well known that the solution of CSPs involving sets has a high complexity as explained in Jaulin et al. (2001). However, a first relaxation consists of approximating the variable domains by means of intervals and finding the solution through solving an interval CSP. The determination of the intervals that approximate in a more fitted form the sets that define the variable domains requires global consistency, what demands a high computational cost as in Hyvonen (1992). A second relaxation consists solving the interval CSP by means of local consistency techniques, and deriving of conservative intervals. Interval constraint satisfaction algorithms have a polynomial-time worst case complexity since they implement local reasonings on constraints to remove inconsistent values from variable domains. In this paper, the interval CSP is solved using a tool based on interval constraints propagation, known as Interval Peeler. This tool has been designed and developed in Baguenard (2005). The goal of this software is to determine the solution of interval CSP in the case that domains are represented by closed real intervals. The solution provides refined interval domains consistent with the set of interval CSP constraints as provided in Puig et al. (2009).

4.3. CSP-MPC Algorithm

The CSP-MPC approach is described in Algorithm 1 where the non-linear constraints of the MPC considering the pressure model in *Problem* 3 are formulated as a CSP:

Algorithm 1 CSP-MPC Algorithm

for $k := 1$ to H_p **do**
  $\mathcal{U}(k-1) \Leftarrow [u_{min}(k), u_{max}(k)]$
  $\mathcal{X}(k) \Leftarrow [x_{min}(k), x_{max}(k)]$
  $D(k) \Leftarrow [d_{min}(k), d_{max}(k)]$
end for
$\mathcal{V} \Leftarrow \overbrace{x(1), x(2), ..., x(H_p)}^{X}, \overbrace{u(0), u(1), ...u(H_p-1)}^{U}, \overbrace{d(0), d(1), ...d(H_p-1)}^{D}$
$\mathcal{D} \Leftarrow \mathcal{X}(1), \mathcal{X}(2)...\mathcal{X}(H_p), \mathcal{U}(0), \mathcal{U}(1)...\mathcal{U}(H_p-1), D(0), D(1)...D(H_p-1)$
$C \Leftarrow \Lambda^T \begin{bmatrix} h_r \\ h \end{bmatrix} + G(u) = 0$
$\mathcal{H} \Leftarrow \mathcal{V}, \mathcal{D}, C$
$S = solve(\mathcal{H})$
Update limits for the linear MPC problem using the CSP solution

4.4. Modelling Uncertainty

Some of the functional elements in DWNs involve uncertainties. This is the case of demand forecasts during the MPC problem horizon. Combining MPC and Gaussian Process to solve the uncertainty problem, was first proposed by Maciejowski & Xiaoke (2013). It was suggested that Gaussian process could be an approach to model and forecast system disturbances and to implement MPC for a real system. In order to solve the difficulty of multiple-step-ahead forecasts, Wang et al. (2014) and Wang et al. (2015) propose a new algorithm scheme called Double-Seasonal Holt-Winters Gaussian Process (DSHW-GP) for multi-step ahead forecasting and robust MPC to take into account the influence of disturbances on state trajectories.

In this work, the demand uncertainty could also be included in the variable set $\mathcal{V}$ with the domains defined in equation (24) in order to incorporate it into CSP-MPC approach:

$$d_0(k) - \Delta e \leq d(k) \leq d_0(k) + \Delta e, \quad k = 1, \cdots, H_p, \tag{24}$$

where d is the real demand, d_0 is the nominal demand forecast, and Δe represents the demand uncertainty that can be obtained, e.g, using the method proposed by Wang et al. (2014) and Wang et al. (2015).

At each time interval, this CSP algorithm will produce updated constraints (23d) and (23e) to *Problem 1* by means of propagating the effect of non-linear constraints equation (22e) into the operational constraints equation (22f) and equation (22g), which will be used for linear MPC to generate optimized control strategies.

4.5. Simulation of the proposed approach

Hydraulic network models are widely used as tools to simulate water distribution systems, not only in academic research, but also by water companies in their daily operation, see Jun & Guoping (2013). There are many simulation software packages, such as Epanet, which is designed to be a research tool for improving and understanding the behavior of DWNs dynamics. This simulator has been used for many different applications in water distribution systems analysis: sampling program design, hydraulic model calibration, residual chlorine analysis and consumer exposure assessment are some examples, see Rossman (2000). In this paper, Epanet is used for simulating hydraulic behavior with the optimal actuator set-points obtained from the CSP-MPC optimizer.

The way of simulating CSP-MPC using Epanet is exchanging flow set-points and tanks/reservoirs dynamic behaviors at each time step, following the work flow shown in Figure 3. The continuous flow set-points are translated to ON-OFF pump operation using the Pump Scheduling Algorithm (PSA), which optimizes the difference between optimal pump flow $V_{\hat{c}}$ and the simulated pump flow V_t as proposed in Sun et al. (2014b).

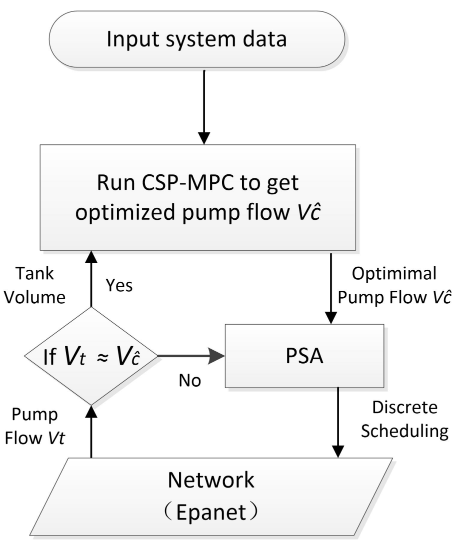


Figure 3: Simulating CSP-MPC using Epanet

5. Illustrative Example: Richmond Water Network

5.1. Description of Richmond Water Network

To validate the proposed CSP-MPC approach, the Richmond water distribution system which is available from the Centre of Water Systems of Exeter University, and also the object of study in van Zyl et al. (2004) is used. The Richmond case study includes one reservoir, four tanks, seven pumps and some one-directional pipes and valves, as Figure 4 shows, using Epanet.

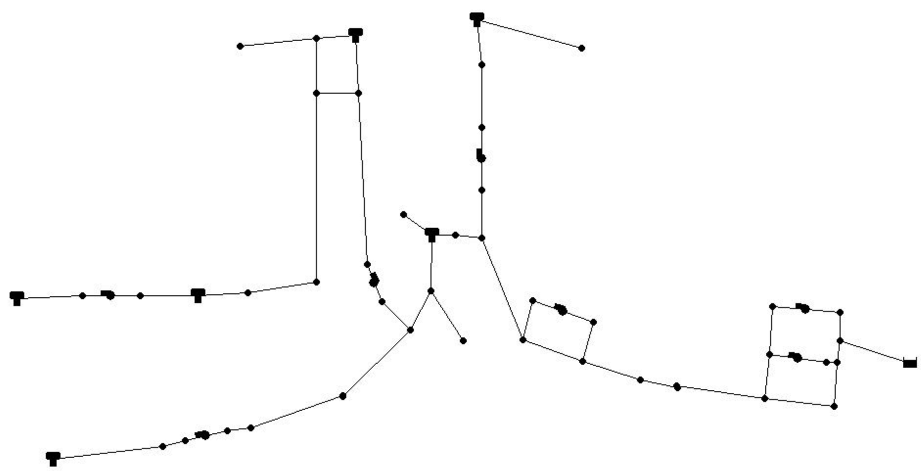


Figure 4: The Richmond water distribution system in Epanet

5.2. CSP for different configurations

In the Richmond distribution water network, there are mainly three different configurations, which lead to non-linear constraints in the MPC problem:

Case 1 *Valve Demand*: demand connected to one tank by means of a valve.

Case 2 *Pump Demand*: demand connected to one tank by means of a pump.

Case 3 *Complex Node Demand*: demand connected to a node, which has direct or indirect connection with more than one tank.

5.2.1. Case 1: CSP for a valve connected to a demand.

As shown in Figure 5, a tank is connected to a demand by means of a valve. In this case, the valve flow is always equal to the demand. Just as an example, assuming cross-section area of the tank Sec is $1m^3$, elevation difference between tank and demand ΔE is $1.65m$, L, D and C are length, diameter and friction coefficients of the connecting pipe, which are constant, demand flow d is 6.3375, R is the valve friction, g_p^c is the head loss for the pipe, g_v^v is the head loss for the valve, G is the valve control variable, which is between 0 and 1. The CSP in Algorithm 1 can be formulated considering that:

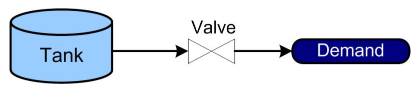


Figure 5: Valve Demand configuration

- $\mathcal{D}$: Variable domains coming from the physical limits

$$x \in [0, 50], \quad u \in [0, 6.3375]$$

- C: Mass conservation constraints

$$x/Sec = g_p^c + g_v^v + \Delta E.$$

$$g_p^c = (10.29 \times L)/(C^2 \times D^{5.33})d^2$$

$$g_v^v = GRd^2$$

After solving the CSP using Interval Peeler, it is found that:

- $\mathcal{H}$: The solution of the CSP provides the updated variable bounds to be used in the linear MPC as follows

$$x \in [10.66, 50], \quad u \in [0, 6.3375]$$

5.2.2. Case 2: CSP for a pump connected to a demand.

As shown in Figure 6, assume that A, B, C are constants for pump head loss equation, s is the speed, g_b^f is the head gain provided by the pump. The CSP in Algorithm 1 can be formulated considering that:

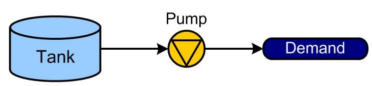


Figure 6: Pump Demand configuration

- $\mathcal{D}$: Variable domains coming from the the physical limits

$$x \in [0, 35], \quad u \in [0, 1.65]$$

- C: Mass conservation constraints

$$x/Sec = g_p^c - g_b^f + \Delta E$$

$$g_b^f = A(d)^2 + B(d)s + Cs^2$$

$$g_p^c = (10.29 \times L)/(C^2 \times D^{5.33})d^2$$

After solving the CSP using the Interval Peeler:

- $\mathcal{H}$: The solution of the CSP provides the updated variable bounds to be used in the linear MPC as follows

$$x \in [3.5, 35], \quad u \in [0, 1.65]$$

5.2.3. Case 3: Node connected to a complex demand.

One example for the configuration of a complex demand node, where demand may be supplied in more than one way, is shown in Figure 7. Node 249 is indirectly connected with more than one tank through a valve and a pump. In this

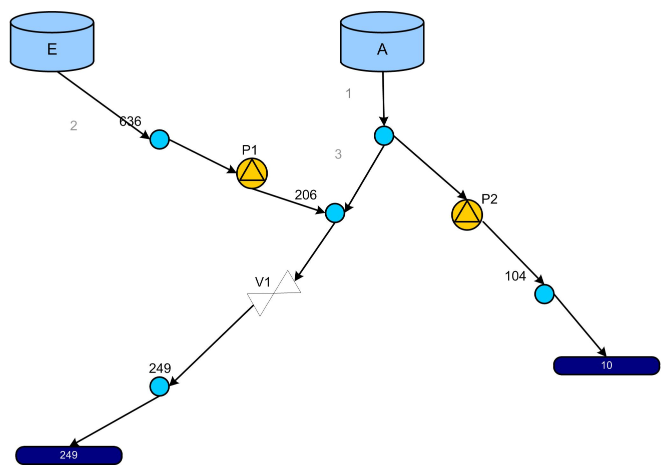


Figure 7: Node connected to a complex demand

case, the CSP problem in Algorithm 1 will be formulated by taking into account:

- $\mathcal{D}$: Variable domains coming from the the physical limits

$$u_1 \in [0, 10], \quad u_2 \in [0, 50]$$

$$u_3 \in [0, 30], \quad x_i/Sec_i \in [0, X_{max_i}]$$

- $\mathcal{C}$: Mass conservation constraints

$$\begin{aligned}
x_{E_1}/Sec_E &= g^f_{b_2} + \Delta E_1. \\
x_{E_2}/Sec_E &= g^f_{b_2} - g^c_{p_1} + \Delta E_2. \\
x_{E_3}/Sec_E &= g^f_{b_2} - g^c_{p_1} + g^v_{v_1} + \Delta E_3. \\
x_E/Sec_E &= \max(x_{E_1}, x_{E_2}, x_{E_3}) \\
x_{A_1}/Sec_A &= g^f_{b_1} + \Delta E_4. \\
x_{A_2}/Sec_A &= g^f_{b_1} + g^f_{b_3} + \Delta E_5. \\
x_{A_3}/Sec_A &= g^f_{b_1} + g^f_{b_3} + g^v_{v_1} + \Delta E_6. \\
x_{A_4}/Sec_A &= g^f_{b_1} - g^c_{p_2} + \Delta E_7. \\
x_A/Sec_A &= \max([x_{A_1}, x_{A_2}, x_{A_3}, x_{A_4}])
\end{aligned}$$

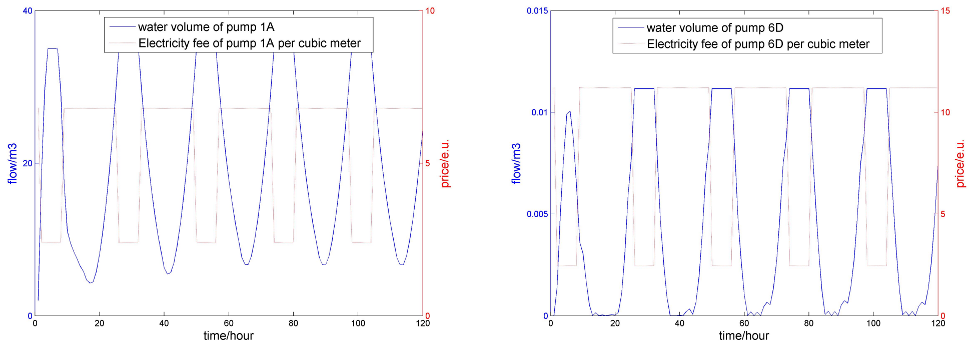
After solving the CSP in Algorithm 1, the updated variable bounds to be used in the linear MPC are:

$$\begin{aligned}
&u_1 \in [3.4, 10], \quad u_2 \in [2.3, 50] \\
&u_3 \in [1.5, 30], \quad x_A \in [10, 43] \\
&x_E \in [4.5, 30]
\end{aligned}$$

5.3. Results

5.3.1. Results of CSP-MPC.

According to the *Cost reduction* operational goal of MPC, the cost associated with water elevation should be minimized. The cases of *pump* $1A$ and *pump* $6D$

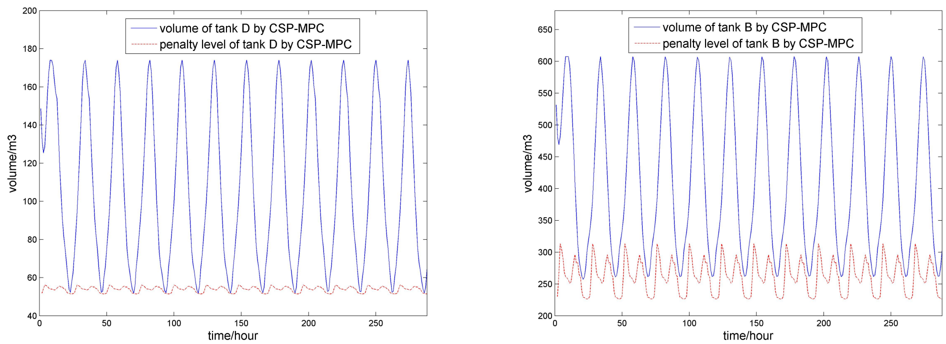


(a) *Pump 1A* (b) *Pump 6D*

Figure 8: Comparison between pump flow and its electricity price

are used as illustrative examples in order to show results of optimal electrical cost. Figure 8 provides evolutions of pump flow and electricity tariff in the same plot, in order to show that pumps send more water at the lower-price period but less or no water when the electricity is expensive, which confirms the completion of the economical objective.

As presented above, CSP is used to convert the nonlinear equations of flow-and-pressure MPC into additional linear constraints for a flow-only MPC in order to optimize the nonlinear model of a complex water network in an efficient way. By means of Algorithm 1, non-linear *Problem 1* has been transformed into linear *Problem 2* with tightened constraints for both tanks and actuators produced by CSP. *Tank B* and *Tank D* are used as illustrative examples in Figure 9, where the evolution of tank volumes are always above the new penalty level constraints, which guarantees the required pressure for appropriate service and confirms the effectiveness of CSP.



(a) *Tank D* (b) *Tank B*

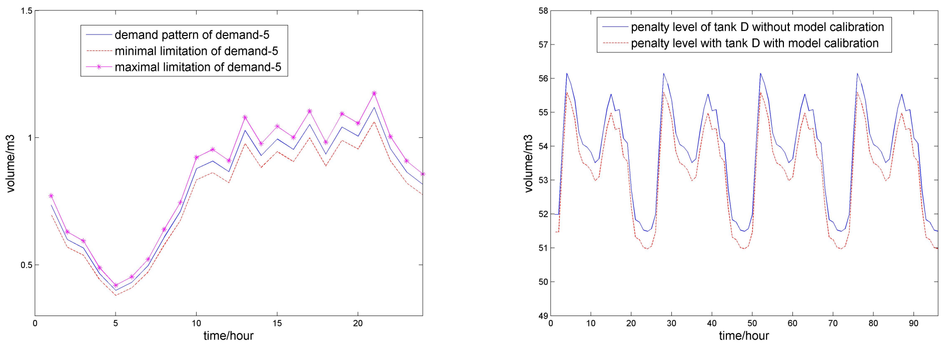
Figure 9: Comparison between tank penalty by CSP-MPC and its volume evolution

5.3.2. Results of Modelling Uncertainty

In the Richmond case study, leakages are not considered and the consumer demand is modelled by means of a deterministic pattern. In order to illustrate management of the demand uncertainty, 5% of the nominal demand value has been used as uncertainty as it has been described in Section 4.4. As shown in Figure 10 *(a)*, the evolution of *demand-5* has been changed from demand pattern into demand domains according to equation (24). Consequently, this affects the minimal safety volume produced by CSP-MPC as constraints of state variables to meet hydraulic requirement, as shown in Figure 10 *(b)*.

5.4. Comparison with Nonlinear MPC

In order to validate the proposed CSP-MPC approach, results obtained from CSP-MPC will be compared to nonlinear MPC which is implemented with PLIO tool. As described in Cembrano et al. (2011), PLIO is a graphical real-time decision support tool based on non-linear MPC which allows the flow and pressure control management for the real-time operative control of DWNs.



(a) Domains of *demand-5* (b) Calibration of water penalty of *tank D*

Figure 10: Results of Modelling Uncertainty

PLIO was developed using standard GUI (graphical user interface) techniques and objective oriented programming using Visual Basic.NET. In PLIO, models are built using the GAMS optimization modelling language. The resulting non-linear optimization problem is solved using CONOPT, which is a solver for large-scale nonlinear optimization problem (NLP) and is developed and maintained by ARKI Consulting and Development in Denmark. CONOPT is a feasible path solver based on the proven GRG method as in Flores-Villarreal & Rios-Mercado (2003) with many newer extensions. All components of CONOPT have been designed for large and sparse models with over $10,000$ constraints. Figure 11 is the PLIO model of Richmond water distribution network.

In the CSP-MPC control scheme, both linear and non-linear constraints of DWNs should be satisfied. Besides that, optimal solution produced by CSP-MPC should be in consistency with that from non-linear MPC in tanks dynamic evolution, pump flows and also demand node pressure.

Results shown in Figure 12, 13 and 14 provide the evolution comparisons of

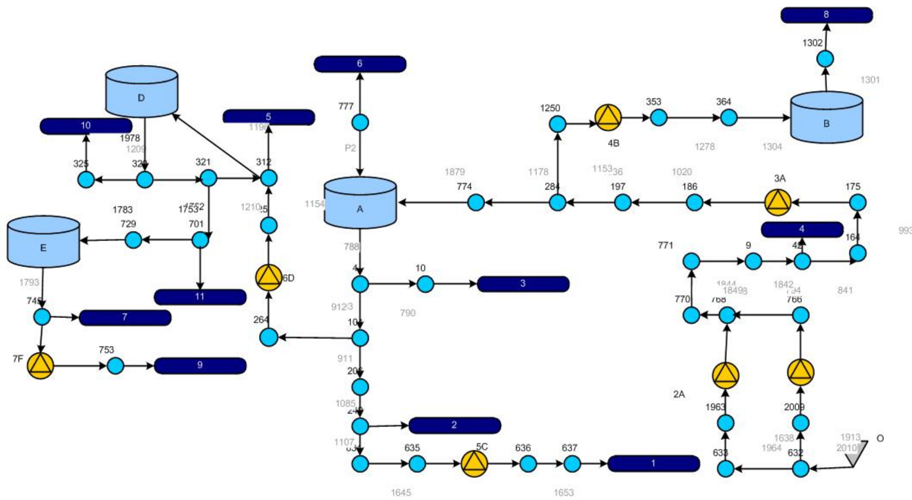
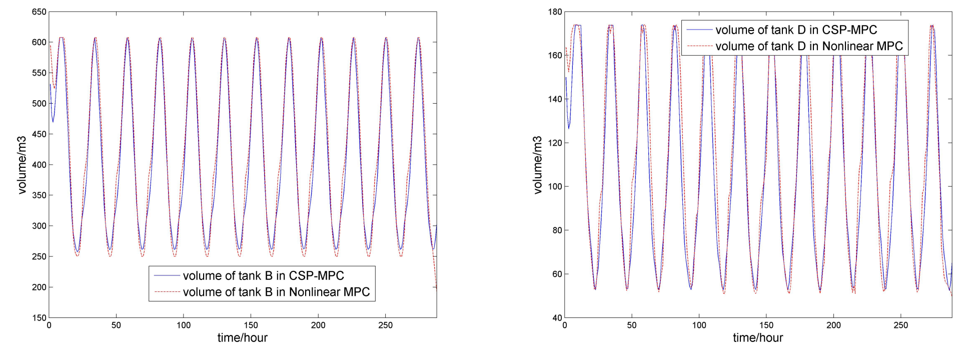


Figure 11: The PLIO model of Richmond water distribution network

tank volumes, pump flows and demand node pressure between CSP-MPC and PLIO. The similarity of these comparisons validates the functionality of the proposed CSP-MPC control approach, which optimize the non-linear MPC problem using a linear MPC with tightened constraints.

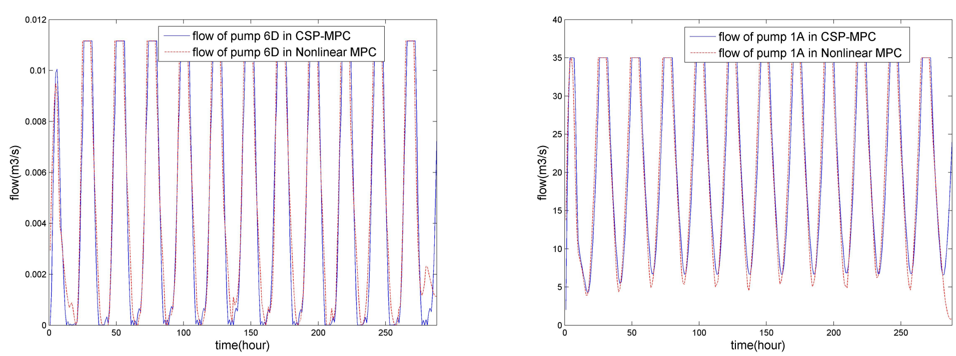
Besides consistency validation between non-linear MPC and CSP-MPC, Table 1 shows in further detail the operational cost and computational load comparisons between non-linear MPC and CSP-MPC using a 288 hours iteration and a 70% pump efficiency. The indices representing costs are given in British pounds. The row of *Comput. time* compares the computational time for every iteration between non-linear MPC and CSP-MPC with the time unit of second (s). Since the sampling time used by the controller is 1 hour, consequently real-time operation can be clearly guaranteed. The column of *Improvement* provides the variation in percentage of the results from non-linear MPC to the CSP-MPC control.

The results presented in this table confirm that the operational costs obtained



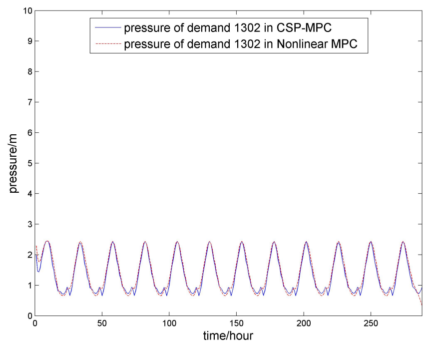
(a) *Tank B* (b) *Tank D*

Figure 12: Comparison of water evolution in tank between CSP-MPC and non-linear MPC

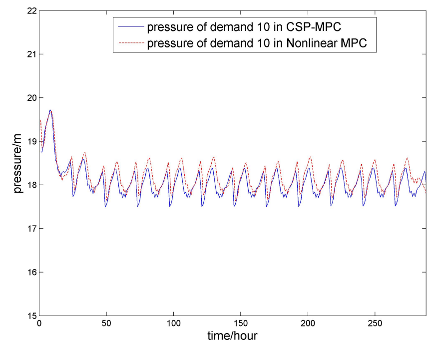


(a) *Pump 6D* (b) *Pump 1A*

Figure 13: Comparison of pump flow between CSP-MPC and Non-linear MPC



(a) *Demand 1302*



(b) *Demand 10*

Figure 14: Comparison of demand node pressure between CSP-MPC and Non-linear MPC

Table 1: Compar. betw. Non-linear MPC and CSP-MPC

Define Name	**Non-linear MPC**	**CSP-MPC**	**Improvement**
J_{cost}(£)	1079.1	1110.3	2.89%
$Comput.time(S)$	83	29.2	-185.71%

from nonlinear MPC and CSP-MPC are similar, which confirm the consistency between nonlinear MPC and CSP-MPC. Besides, the computational time of non-linear MPC is shown nearly more than twice longer than the one needed by CSP-MPC, which confirms the advantage of CSP-MPC in the computing efficiency. The relative improvement of this execution time is expected to increase in a larger network and therefore, this reduced execution time would imply a potential advantage in large scale systems is foreseen using CSP-MPC.

5.5. *Comparison with other approaches.*

Operation of the Richmond water distribution system was optimized previously using Hybrid GA in van Zyl et al. (2004) and ACO (Ant Colony Optimization) in López-lbañez et al. (2008), whose optimal annual operational costs are £35, 296 and £33, 683, respectively. A comparison of results of CSP-MPC with these methods is included below, to the extent possible with the information provided in the referenced papers.

The calculation for estimating annual operational cost of Richmond system is

$$C_{ann.} = \frac{g \sum_{j=1}^{7} \sum_{i=1}^{365} \rho \tilde{u}(i,j) \Delta H(i,j) a_2(i,j)}{\eta} \tag{25}$$

where g is the gravity, η is efficiency for the pumps, ρ is density of water, ΔH is the head gain provided by the pump, $\tilde{u}$ is the pump flow and a_2 contains the cost of pumping.

In practice, considering that efficiency e ranges from 65% to 75%, the operational annual cost by CSP-MPC is ranging from £31, 520 to £36, 369, which is in order of the results obtained using Hybrid GA (£35, 296) in van Zyl et al. (2004) and ACO (£33, 683) in López-lbañez et al. (2008).

5.6. *Application Limitations of CSP-MPC*

Considering the definition and interval implementation characteristics of CSP explained in above sections, the building of the constraints C in DWNs can only be directly implemented in networks that do not present bidirectional flows, as initially proposed in Sun et al. (2014a). In DWNs, some pipes are bidirectional, which adds difficulty to build the pressure constraints set for CSP. In order to apply successfully CSP-MPC to the bidirectional DWNs, a Network Aggregation

Method (NAM) may be used to simplify a complex water network into an equivalent simplified conceptual one. Then, the non-linear pressure constraints may be transformed into safety volumes for the tanks. This is illustrated in the next section.

6. Application Example: D-Town Water Network

In order to test the applicability of CSP-MPC to a complex water network with many bidirectional elements, the D-Town network is used as a supplementary case study. D-Town network shown in Figure 15 is a complex benchmark DWN with 388 nodes, 405 actuators and 7 tanks and multiple bidirectional links, which has already been used in Price & Ostfeld (2014) and P.L.Iglesias-Rey et al. (2014).

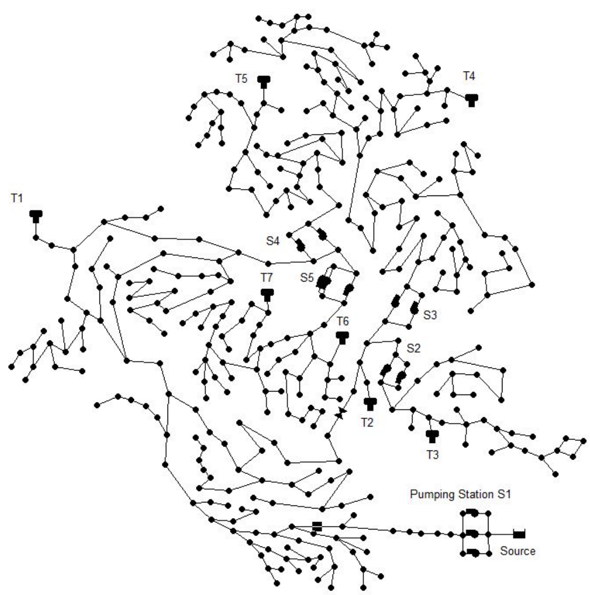


Figure 15: Original D-Town network

6.1. Results of NAM for D-Town

The conceptual one-directional network model of D-Town was obtained using a Network Aggregation Method (NAM) described in Sun et al. (2015). This model is shown in Figure 16, where all the demand nodes have been aggregated inside one demand node and related directly with the tanks. This model can be optimized using CSP-MPC approach proposed in this paper.

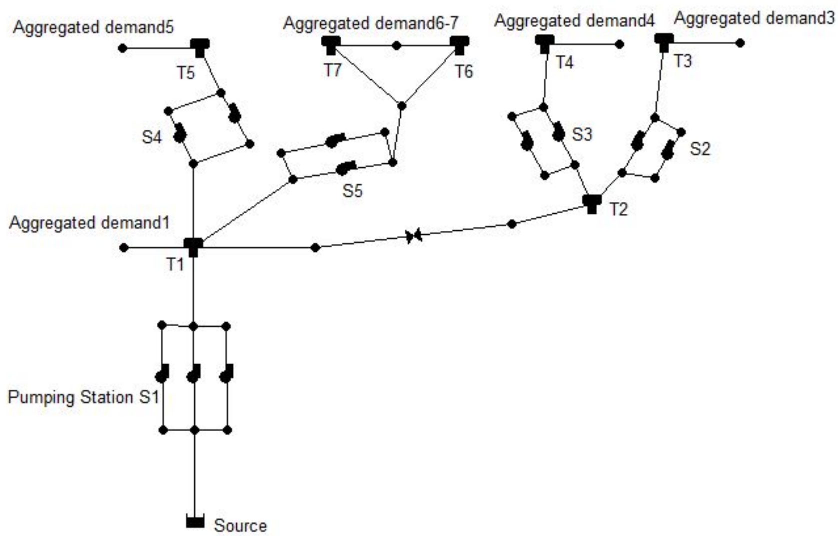
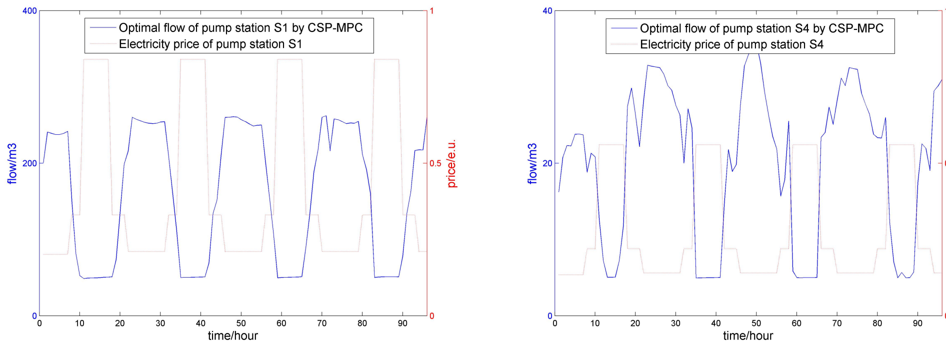


Figure 16: Conceptual D-Town network

6.2. Results of CSP-MPC for D-Town

As discussed in Section 3, the objective function of MPC includes the economical water transportation cost associated to the pumps that should be minimized. Figure 17 shows in the same plot the pump flows after applying MPC and electricity tariffs for pump stations S_1 and S_4. From this figure, it is clear that, in order to



(a) *Pump S1*

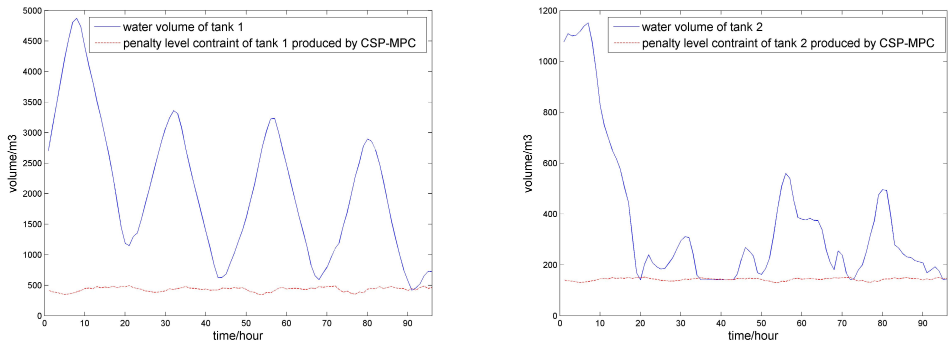
(b) *Pump S4*

Figure 17: Comparison between pump flow and its electricity price

reduce the operational cost of the whole network, the MPC strategy pumps more water when the electricity prices are low while less or no water when the prices are high.

By means of CSP, non-linear pressure equations of D-Town could be transferred into linear constraints which imposes new limitations for both tanks and actuators in order to fit the non-linear pressure dynamics. Figure 18 shows the evolutions of real tank volumes compared with their updated minimal safety volume, which has been produced by CSP in order to satisfy the required pressure of demand nodes. From this figure, it can be noticed that the added constraints for tanks determine their water volume evolutions, which guarantee the required pressure for the demand node.

The above results using the D-town case study confirm that the applicability of CSP-MPC is not restricted to simple case studies, but it may be applied to complex networks with multiple bi-directional connections.



(a) *Tank 1* (b) *Tank 2*

Figure 18: Comparison of tank volume and the safety volume by CSP-MPC

6.3. Comparison with other Approaches

Operations of the D-Town network were optimized previously by a Pseudo-Genetic Algorithm (PGA) proposed by P.L.Iglesias-Rey et al. (2014) and a successive linear programming proposed by Price & Ostfeld (2014), whose optimal annual pump costs are $168,118$ and $117,740$ Euros respectively, according to the information in the referenced papers. Using equation (25), considering pump efficiency e as 70% here, the operational annual cost for D-Town with CSP-MPC is $137,880$ Euros, which is in order of the results obtained by Price & Ostfeld (2014) and P.L.Iglesias-Rey et al. (2014).

6.4. Advantages of CSP-MPC Application in DWNs

The main advantages of the CSP-MPC compared with the existing approaches reported in the literature are the following: Regarding methods based on evolutionary/genetic algorithms, the computation time is considerable reduced allowing the online optimization required in the real-time operational control. This is especially important in large scale networks where the computation time could be prohibitive when the real-time implementation is based on evolutionary/genetic algorithms.

Concerning non-linear MPC, since the associated optimization problem is non-convex, current existing non-linear programming algorithms although they scale better than evolutionary/genetic algorithms, they can only guarantee local optima. Moreover, most of the theoretical properties (as stability, robustness, feasibility) of linear MPC (that are preserved in the CSP-MPC approach) can be more easily guaranteed than in the case of the non-linear MPC. For all these reasons, in spite of a small amount of decrease of performance, the proposed CSP-MPC approach over-performs the current existing approaches.

7. Conclusions

This paper presents a control scheme integrating CSP and linear MPC for the optimal management of DWNs, considering both flow and pressure dynamics, as an alternative to solve a non-linear MPC problem. The CSP-MPC control scheme successfully solves the non-linear optimization problem of DWNs by creating an efficient linear approximation of the nonlinear problem and solving it. The proposed CSP-MPC approach provides a significant reduction of the computational load compared to that of nonlinear MPC, which is an important feature for efficiency and scalability for large-scale networks.

The Richmond water distribution network has been used as an illustrative example and the D-Town benchmark network as a more realistic and challenging application. Non-linear MPC implemented with PLIO tool has been used to verify the proposed control scheme, while Epanet has been used as the water network simulator to reproduce the water network behavior in a highly realistic manner.

The result comparison between non-linear MPC and CSP-MPC confirms that the CSP-MPC control scheme produces optimization results that are comparable to those obtained from nonlinear MPC.

The CSP-MPC method achieves a significant improvement in computation time. The results of the proposed approach has also been compared to those of ACO and Hybrid GA producing similar results regarding the operational cost. Operational cost comparisons among CSP-MPC, ACO and Hybrid GA also confirm that, the CSP-MPC scheme is also economically feasible and reasonable.

Finally, the supplementary application of D-Town network has proved that, the CSP-MPC control scheme provides good results even for the complex and realistic case study presenting bidirectional flows, if combined with a network aggregation approach. As future work, the effect of the uncertainty in the demands or network parameters on the performance will be studied. Moreover, distributed implementations of the proposed CSP-MPC approach will be investigated.

Acknowledgements

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