Centralized and Distributed Command Governor Approaches for Water Supply Systems Management

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***Abstract*—This paper evaluates the applicability of Command Governor (CG) strategies to the optimal management of Drinking Water Supply Systems (DWSS) in both centralized and distributed ways. It will be shown that CG approaches provide an adequate framework for addressing the management of these large-scale interconnected systems in the presence of periodically time-varying disturbances (water demands) that can be anticipated by using time-series forecasting approaches. The proposed centralized and distributed CG schemes are presented, discussed and compared when applied to the management of DWSS considering the same set of operational goals in all cases. The paper illustrates the effectiveness of all strategies using the Barcelona DWSS as a case study and highlighting the advantages of each approach.**

***Index Terms*—Optimisation-based control design, command governor, distributed control, drinking water networks, industrial applications**

I. Introduction

DRINKING water supply systems (DWSS) are of paramount importance for the life in modern societies. They are used for transferring water from production plants to consumers and their service is expected to be dependable and economically sustainable. From a dynamical systems point of view, DWSS are large-scale interconnected multivariable systems subject to several constraints related to the physical and operational limitations of reservoirs and actuators, and continuously varying customer demands (disturbances), which generally show a periodic behavior. The growing complexity of these networks, i.e., their size, the existing constraints on the information structure, the presence of model non-linearities and uncertainties and the requirement for higher performance make their management costly to be solved in real time. As a result, the corresponding control design problem has become an increasingly important environmental and socio-economic research topic worldwide.

Different approaches have been reported in the literature to cope with the operational control problems for DWSS. As discussed in [1], during the last years optimal operation of DWSS has been addressed by a wide variety of methods starting from heuristics and expert systems to more advanced control algorithms, as e.g. a variety of optimal control schemes or more recently, Model Predictive Control (MPC). In particular, the MPC framework as introduced in [2] is considered to be quite adequate for the optimal management of DWSS because of the possibility of considering constraints and multiple control objectives, see [3]. Examples of some of the aforementioned methods are reported in [3], [4], [5], [6], [7], [8], [9], among many others.

A further family of constrained control strategies based on the same receding horizon paradigm used in MPC is that of Command Governors (CG). A CG unit is a non-linear static module added to an asymptotically stable plant in charge of modifying, whenever necessary, the prescribed command signal when its unmodified application would lead to constraint violations and, in turn, possibly loss of stability. This modification is typically achieved by solving on-line at each time instant a constrained optimisation problem, whose constraints take into account future system predictions.

The use of CG is easily motivated in many practical industrial applications where only algorithms with reduced computational complexity are allowed. Moreover, in many cases it is possible to provide optimal setpoints only to existing regulatory controllers, which is the task performed by CG strategies [10]. In [11], a preliminary centralised CG solution (CCG) for the optimal management of DWSS has been proposed and compared with a centralised MPC scheme. Centralised solutions require to have a global dynamical model of the DWSS. Additionally, all measurements should be available in one place to estimate all network states and determine all control actions to be executed by actuators. However, when dealing with large DWSS, these conditions are difficult to be met because collecting all measurements in one location is not possible or because a centralised high-performance computing unit is not at disposal. Moreover, centralised implementations scale poorly when the size of a DWSS increases, requiring the complete readjustment of the controller every time the DWSS is updated. Thus, the cost of setting up and maintaining a monolithic centralised controller could be prohibitive. An effective choice to overcome these difficulties is the usage of either *decentralised* or *distributed* approaches, where the global control system is decomposed in a set of local controllers that are responsible of the supervision of each subsystem. Recent research following these ideas have been collected in [12].

In this paper, distributed versions of the CG approach are proposed for the optimal management of DWSS as an alternative to the CCG. In order to apply these distributed strategies, the DWSS should be split into several subsystems, each of which is locally controlled [13]. The main work on distributed CG strategies is based on a non-cooperative non-

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iterative game theoretical approach [14]. There a *sequential* scheme has been considered, where only one local controller is allowed to update its control actions at each sampling time and according to a fixed order, while all others continue applying their previous computed commands. The main advantage of this scheme is the low complexity and communication rates required for its implementation, significantly lower than other distributed approaches, e.g, those based on negotiation mechanisms. However, the above scheme could be quite conservative since the local controllers need to wait for several time instants before they can update their commands. In order to overcome this drawback, in the same work a more efficient implementation of the basic sequential CG is introduced, where at each step the commands of all local controllers are sequentially computed within the same sampling time according to a prefixed order. Moreover, in [15], a novel scheme has been presented where local controllers in the network are considered to belong to particular groups (turns). Then, this improved strategy exploits the fact that local controllers that are not in the same group can simultaneously update their control actions. Then, at each time instant on the basis of a round-robin policy, all local controllers belonging to a group are allowed to update simultaneously their commands while agents in other groups continue applying their current commands. Following the same lines of these works, a distributed non-iterative CG (DNI-CG) strategy is proposed in this paper, where agents are grouped into turns and where the entire decision process ends up within few sampling steps.

In [16], preliminary results of the design of the DNI-CG solution for the optimal management of DWSS has been proposed, presenting promising results considering the behaviour of such systems. In this paper, the further distributed iterative CG approach presented in [17], here referred as DI-CG, has been considered for supervision of the DWSS. In this case, the agents select their control action by performing an iterative decision procedure involving the resolution of several optimization problems. The main benefit related to the proposed class of distributed iterative CG strategies is the ability to achieve Pareto optimal coordination performance not only in steady-state conditions but also during transients.

Therefore, the main contribution of this paper relies on the evaluation of the applicability of both distributed and centralized CG strategies when considering the optimal management of a large-scale DWSS. From the obtained results, several relevant new issues have arisen such as the performance that can be achieved considering the management control objectives and the operational/physical constraints associated to the DWSS as well as the modification of standard CG formulations to deal with periodically time-varying disturbances.

The remainder of the paper is organized as follows. The optimal management problem for DWSS is stated in Section II outlining the most common operational goals considered. The proposed centralised CG approach is presented and discussed in Section III while, in Section IV, the distributed implementation is described and explained, both within the framework of DWSS optimal management. In Section V, results obtained for the Barcelona DWSS case study are presented including comparisons with the considered approaches. The simulations results have been performed on a water network model that is much larger than that considered in [16]. This fact implies considering a larger amount of data not only related to the newer and more complex topology of the proposed case study but also related to the exogenous signals (disturbances) represented here by the water demands. Moreover, given the different size of the case study in this journal paper, the results are totally new as expected. This clarification arises due the fact that the interactions between elements for the larger-scale system surely imply different behaviours for the system states and inputs, yielding to different conclusions about the performance of the proposed approaches in the closed-loop topologies. Finally, some conclusions and future research paths are given in Section VI.

II. DWSS Operational Control Problem Statement

A. Control-oriented Modelling

For completeness of exposition, DWSS modelling approach, already reported by [9] is recalled. In general terms, a DWSS system is represented by a directed graph $G(\mathcal{V},\mathcal{E})$, with a set of vertices $\mathcal{V}$ composed of n_s sources, n_x storage elements, n_q intersection nodes and n_d sinks, and a set of edges $\mathcal{E}$ that consists of directed links (pipes). For these systems, water flows along the links by using n_u flow actuators (i.e., pumps and valves), while is stored into reservoirs (or tanks). Moreover, water is taken from exogenous sources (that feed the overall network) with the aim of satisfying the customer demand (considered for the control system as disturbances). Complementary, the model considers operational constraints on its variables given by storage capacity and flow rate ranges.

Defining the system states $\mathbf{x}\in\mathbb{R}^{n_x}$ as the water volumes into the storage elements, the vector of commands $\mathbf{u}\in\mathbb{R}^{n_u}$ as the water flows through actuators and the *additive* disturbances $\mathbf{d}\in\mathbb{R}^{n_d}$ as the water demand flows, the DWSS control-oriented model is stated as the the following set of linear discrete-time difference-algebraic equations (DAE):

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k)+\mathbf{B}\mathbf{u}(k)+\mathbf{B}_d\mathbf{d}(k), \tag{1a}$$

$$\mathbf{0} = \mathbf{E}_u\mathbf{u}(k)+\mathbf{E}_d\mathbf{d}(k), \tag{1b}$$

where (1a) corresponds with the dynamics of storage tanks, the algebraic equations (1b) describe the network static relations (i.e., mass balance at junction nodes) and $k\in\mathbb{N}$ denotes the discrete time. Likewise, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_d$, $\mathbf{E}_u$, $\mathbf{E}_d$, are the time-invariant matrices of suitable dimensions describing the network topology.

Together with the model in (1), the following set of hard state and input constraints described in a convex polytopic way are defined:

$$\mathcal{X} \triangleq \{\mathbf{x}(k)\in\mathbb{R}^{n_x}|\mathbf{G}\mathbf{x}(k)\leq\mathbf{g}\}\subset\mathbb{R}^{n_x} \quad \forall k, \tag{2a}$$

$$\mathcal{U} \triangleq \{\mathbf{u}(k)\in\mathbb{R}^{n_u}|\mathbf{F}\mathbf{u}(k)\leq\mathbf{f}\}\subset\mathbb{R}^{n_u} \quad \forall k, \tag{2b}$$

where $\mathbf{G}\in\mathbb{R}^{c_x\times n_x}$, $\mathbf{g}\in\mathbb{R}^{c_x}$, $\mathbf{F}\in\mathbb{R}^{c_u\times n_u}$, $\mathbf{f}\in\mathbb{R}^{c_u}$, being c_x and c_u the amount of state and input constraints, respectively. Complementary and looking for water supply reliability, the following safety state constraints are defined:

$$\mathbf{x}(k)\geq\mathbf{d}_{\text{net}}(k) \quad \forall k, \tag{3}$$

where $\mathbf{d}_{\text{net}} \in \mathbb{R}^{n_x}$ is the vector of (possibly time-varying) lower-bounds on water storages (expressed in m^3) necessary to avoid water stock-outs.

B. Model decomposition

Given the model in (1) and constraints (2)-(3), a suitable system decomposition towards the application of the DCG approach proposed in this paper should be performed. The system partitioning seeks for achieving benefits such as the reduction of computational complexity or the system modularity while keeping the control performance of the centralised solution or, at least accepting a level of degradation.

This paper uses the partitioning algorithm proposed in [13], which automatically returns a set of compositional non-overlapped subsystems by splitting the corresponding graph $G(\mathcal{V},\mathcal{E})$ according to certain policies and criteria.

Therefore, the system (1) plus constraints (2)-(3) is decomposed in $M \triangleq |\mathcal{N}|$ subsystems collected in the set $\mathcal{N}$, which are output decentralised and input coupled. The model for the i-th subsystem S_i, for $i \in \{1,\ldots,M\}$, is expressed as

$$\mathbf{x}_i(k+1) = \mathbf{A}_i\mathbf{x}_i(k) + \mathbf{B}_{sh,i}\mathbf{u}_i(k) + \mathbf{B}_{d,i}\mathbf{d}_i(k), \tag{4a}$$

$$\mathbf{0} = \mathbf{E}_{sh,i}\mathbf{u}_i(k) + \mathbf{E}_{d,i}\mathbf{d}_i(k), \tag{4b}$$

where matrices $\mathbf{E}_{sh,i}$ and $\mathbf{E}_{sh,i}$ have suitable dimensions depending on the number of shared inputs of subsystem S_i. For simplicity, constraints

$$\mathbf{x}_{min,i} \leq \mathbf{x}_i(k) \leq \mathbf{x}_{max,i}, \quad \forall k \tag{5a}$$

$$\mathbf{u}_{min,i} \leq \mathbf{u}_i(k) \leq \mathbf{u}_{max,i}, \quad \forall k, \tag{5b}$$

correspond to a special case of the more general classes (2a) and (2b). Moreover, from (3) yields

$$\mathbf{x}_i(k) \geq \mathbf{d}_{\text{net},i}(k). \tag{6}$$

C. Control Objectives

The main management goal for a water supply system relies on fully satisfying the water demands of its customers while, at the same time, optimizing all operational objectives collected altogether into a multi-objective optimal control problem. According to [3], some of the most common operational goals for the management and control of a DWSS are:

- *Economic Objective*: consists in providing a reliable water supply while minimising both water production and transport costs.
- *Safety Objective*: consists in guaranteeing the availability of enough water (within the storage tanks) such as its underlying stochastic demand can be properly satisfied.
- *Smoothness*: consists in operating the supply systems under smooth control actions (avoiding abrupt changes in pumps and valves).

The economic and smoothness goals can be achieved by minimising the following cost functions, respectively:

$$J_E^i(k) \triangleq \|(\boldsymbol{\alpha}_1^i)^{\mathrm{T}}\mathbf{u}_i(k)\|_{1,\mathbf{W}_{e,1}^i} + \|(\boldsymbol{\alpha}_2^i(k))^{\mathrm{T}}\mathbf{u}_i(k)\|_{1,\mathbf{W}_{e,2}^i}, \tag{7a}$$

$$J_{\Delta U}^i(k) \triangleq \|\Delta\mathbf{u}_i(k)\|_{2,\mathbf{W}_u^i}^2, \tag{7b}$$

where $J_E^i \in \mathbb{R}_{\geq 0}$ takes into account the water production cost denoted by $\boldsymbol{\alpha}_1 \in \mathbb{R}^{n_u}$ and the water pumping cost denoted by $\boldsymbol{\alpha}_2 \in \mathbb{R}^{n_u^i}$. The latter cost may be time varying according to the variable electric tariff. On the other hand, $J_{\Delta U}^i \in \mathbb{R}_{\geq 0}$ represents the penalisation of control signal variations $\Delta\mathbf{u}_i(k) \triangleq \mathbf{u}_i(k) - \mathbf{u}_i(k-1)$, which is included to ensure a smooth actuators operation and, at the same time, to extend the lifetime of such devices. Here, $\|\cdot\|_{p,\mathbf{W}}$ denotes the weighted p-norm and $\mathbf{W}_{e,1}^i$, $\mathbf{W}_{e,2}^i$, $\mathbf{W}_u^i$ are diagonal matrices that weight each decision variable within the associated control objective.

Regarding the safety objective, it is reached by satisfying the safety constraints (3), which, in order to avoid numerical infeasibilities, can be conveniently reformulated as the following *soft* constraint

$$\mathbf{x}_i(k) \geq \mathbf{x}_s^i(k) - \boldsymbol{\xi}_i(k) \geq \mathbf{0}, \quad \forall k, \tag{8}$$

where $\mathbf{x}_{\text{s}}^i \in \mathbb{R}^{n_x^i}$ is a vector of safety-volume thresholds in m^3 (conveniently determined according to the management company policies related to the DWSS), and $\boldsymbol{\xi} \in \mathbb{R}^{n_x}$ represents the amount of volume going down from the desired thresholds. Hence, the cost functions $J_S^i \in \mathbb{R}_{\geq 0}$, defined as

$$J_S^i(k) \triangleq \|\boldsymbol{\xi}_i(k)\|_{2,\mathbf{W}_x^i}^2, \tag{9}$$

are consequently stated, with $\mathbf{W}_x^i$ being the corresponding weighting matrix related to the prioritisation of this control objective. A proper selection of the inputs of vector $\mathbf{x}_{\text{s}}^i$ should be carried out since they affect the conservativeness and the sub-optimality of the overall control problem solution.

D. Control Problem Statement

Finally, by merging all the elements described in previous subsections, the overall control problem related to a DWSS consists in the achievement of the system operational goals (i.e., their minimization), subject to the system model and physical/operational constraints. More formally, Problem 1 below can be stated.

Problem 1: The control problem related to a DWSS consists in determining at each time instant k command inputs $\mathbf{u}_i(k)$, $i = 1,\ldots,M$ along with volume relaxations $\boldsymbol{\xi}_i(k)$, $i = 1,\ldots,M$ that minimize (7) and (9) for the i-th subsystem according to the model in (4) and constraints in (5), (6) and (8).

III. Centralised CG Strategy for DWSS Operational Control

In this section, Problem 1 has been addressed by resorting to the well known CG method [10]. Because of its natural capability to handle in a systematic manner hard constraints on inputs and state-related variables, the CG approach seems to be quite suitable for solving Problem 1 in either a centralized manner or distributed manner (see Section IV).

In the centralized case, the CG scheme here considered is depicted in Figure 1. There, a unique CG device is in charge of accomplishing a supervision task on the DWSS consisting of satisfying time-by-time the water demand while minimizing the operative and other costs subject to operative constraints. In particular, at each time k, the command $\mathbf{u}(k)$ is computed

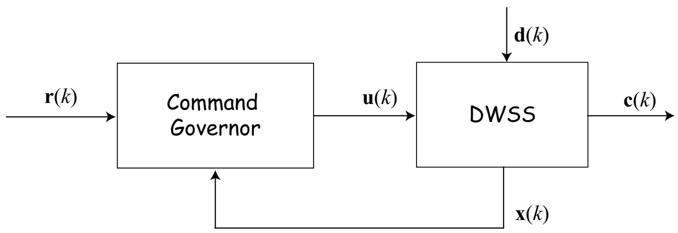

Figure 1. CG Scheme

as the best approximation of the desired command reference $\mathbf{r}(k)$ under the following pointwise-in-time constraints:

$$\mathbf{c}(k) \in \mathcal{C}, \forall k \tag{10}$$

where

$$\mathbf{c}(k) \triangleq \mathbf{C}\mathbf{x}(k) + \mathbf{L}\mathbf{u}(k), \tag{11}$$

and $\mathcal{C}$ is a convex set and $\mathbf{C} \triangleq [\mathbf{I}_{n_x}^T, \mathbf{0}_{n_u}^T]^T$ and $\mathbf{L} \triangleq [\mathbf{0}_{n_x}^T, \mathbf{I}_{n_u}^T]^T$.

For the problem of interest, the set $\mathcal{C}$ is defined as

$$\mathcal{C}(\mathbf{x}_s) \triangleq \left\{ (\mathbf{c}, \boldsymbol{\xi}) \in \mathbb{R}^{(n_x+n_u)\times(\mathbb{R}^{n_x})} \left| \begin{array}{l} \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \\ -\mathbf{I}_{n_x} & \mathbf{0} \end{bmatrix} \mathbf{c} \leq \begin{bmatrix} \mathbf{g} \\ \mathbf{f} \\ \boldsymbol{\xi} - \mathbf{x}_s \end{bmatrix} \\ \text{and} \\ \boldsymbol{\xi} - \mathbf{x}_s \leq 0 \end{array} \right. \right\}, \tag{12}$$

where matrices $\mathbf{G}, \mathbf{F}$ and vectors $\mathbf{g}, \mathbf{f}$ are the same as in (2).

From a mathematical point of view the CG action relies on the selection, at each time step of a *virtual* command $\mathbf{u}(l|k) \equiv \mathbf{w}$, $\forall l$, whose constant application over a semi-infinite horizon $l \in [0, \infty)$, from the initial state $\mathbf{x}(k)$, guarantees constraints fulfilment. More formally, the command $\mathbf{w}$ is chosen in order to prevent that the future predictions (*virtual evolutions*) of the $\mathbf{c}$-variable along the *virtual time l* under the action of $\mathbf{w}$ from the initial state $\mathbf{x}$ (at virtual time $l = 0$), i.e.,

$$\mathbf{c}(l, \mathbf{x}, \mathbf{w}, \mathbf{d}) \triangleq \mathbf{C}\left(\mathbf{A}^l \mathbf{x} + \sum_{j=0}^{l-1} \mathbf{A}^{l-j-1}(\mathbf{B}\mathbf{w} + \mathbf{B}_d \mathbf{d})\right) + \mathbf{L}\mathbf{w}, \tag{13}$$

violate constraints $\mathbf{c}(l, \mathbf{x}, \mathbf{w}, \mathbf{d}) \in \mathcal{C}, \forall l \in \mathbb{Z}_+$.

In order to take into account the algebraic-equations (1b), the following set is introduced:

$$\mathcal{W}(\mathbf{d}) \triangleq \{\mathbf{w} \in \mathbb{R}^{n_u} | \mathbf{E}_1 \mathbf{w} + \mathbf{E}_2 \mathbf{d} = 0\}. \tag{14}$$

In the case of a constant disturbance $\mathbf{d}(l|k) = \mathbf{d}$, $\mathbf{w}$ is selected as follows:

$$\begin{aligned} &\mathbf{w} \in \mathcal{V}(\mathbf{x}, \mathbf{d}) \triangleq \{\mathbf{w} \in \mathcal{W}(\mathbf{d}) | \\ &\quad \exists \boldsymbol{\xi}(l) \text{ such that } (\mathbf{c}(l, \mathbf{x}, \mathbf{w}, \mathbf{d}), \boldsymbol{\xi}(l)) \in \mathcal{C}(\mathbf{x}_s), \forall l \in \mathbb{Z}_+\}. \end{aligned}$$

It is worth mentioning that, if $\mathbf{A}$ is strictly Schur, the set $\mathcal{V}(\mathbf{x}, \mathbf{d})$, $\forall \mathbf{x} \in \mathbb{R}^{n_x}$, is convex and finitely determined, viz. there exists an *a-priori* known integer l_0 (see [18]) such that if $\mathbf{c}(i, \mathbf{x}, \mathbf{w}, \mathbf{d}) \in \mathcal{C}(\mathbf{x}_s)$, $i \in \{0, 1, \ldots l_0\}$, then $\mathbf{c}(i, \mathbf{x}, \mathbf{w}, \mathbf{d}) \in \mathcal{C}(\mathbf{x}_s)$, $\forall l \in \mathbb{Z}_+$.

Finally, the CG design problem is solved by selecting at each time instant k a command $\mathbf{u}^*(k) = \mathbf{w}^*$, with $\mathbf{w}^*$ solution of the following convex optimisation problem:

$$\mathbf{w}^* \triangleq \arg \min_{\mathbf{w} \in \mathcal{V}(\mathbf{x}(k), \mathbf{d})} \sum_{i=1}^{M} \sum_{l=k}^{k+H_p-1} \left[J_E^i(l) + J_S^i(l) + J_{\Delta U}^i(l)\right], r_i(l) = 0, \tag{15}$$

where $J_E^i(k) \triangleq \|(\boldsymbol{\alpha}_1^i)^{\mathrm{T}}(\mathbf{w}_i - \mathbf{r}_i(k))\|_{1,\mathbf{W}_{e,1}} + \|(\alpha_2^i(k))^{\mathrm{T}}(\mathbf{w}_i - \mathbf{r}_i(k))\|_{1,\mathbf{W}_{e,2}}$.

Notice that $\mathbf{r}(k) = 0, \forall k$ (and hence $\mathbf{u}(k) = 0, \forall k$) would provide a target vector of commands that would produce the lowest cost amongst all command policies. However, the constant command vector $\mathbf{u}(k) = 0, \forall k$ is usually not admissible and the CG is used to compute, instant by instant, the best admissible approximation of $\mathbf{r}(k) = 0, \forall k$ given the considered disturbances and constraints, usually achieving command sequences $\mathbf{u}(k) > 0$. When this sequence of commands is applied, optimized DWSS supervision operations are expected.

Whenever the disturbance predictions are time-varying (that is actually of interest in this paper), a more general CG formulation is needed. In this work we address this aspect and introduce a CG scheme where the commands $\mathbf{u}(i|k)$ are chosen in a set $\mathcal{V}$ depending on the entire disturbance sequence $\underrightarrow{\mathbf{d}}(k) \triangleq [\mathbf{d}(0|k), \ldots, \mathbf{d}(l_0|k)]$. Such a set $\mathcal{V}$ takes the form

$$\begin{aligned} \mathcal{V}(\mathbf{x}, \underrightarrow{\mathbf{d}}(k)) \quad \triangleq \quad &\Big\{\underrightarrow{\mathbf{w}} | \{\mathbf{w}(l) \in \mathcal{W}(\mathbf{d}_i)\} \text{ s.t. } \exists \boldsymbol{\xi}(l), \\ &(\mathbf{c}(l, \mathbf{x}, \mathbf{w}(l), \mathbf{d}_i(l)), \boldsymbol{\xi}(l)) \in \mathcal{C}(\mathbf{x}_s)\Big\}, \end{aligned} \tag{16}$$

$\forall i \in \{0, 1, \ldots, l_0\}$.

Then, a problem similar to (15) can be solved in this case and the obtained solution is the sequence

$$\underrightarrow{\mathbf{w}}^* \triangleq \arg \min_{\underrightarrow{\mathbf{w}} \in \mathcal{V}(\mathbf{x}, \underrightarrow{\mathbf{d}}(k))} \sum_{i=1}^{M} \sum_{l=k}^{k+H_p-1} \left[J_E^i(l) + J_S^i(l) + J_{\Delta U}^i(l)\right], r_i(l) = 0, \tag{17}$$

where the notation $\underrightarrow{\mathbf{u}}^*(k) = \underrightarrow{\mathbf{w}}^*$ denotes a command sequence of H_p not necessarily constant samples whose only the first one will be applied according to the RHC philosophy.

IV. Distributed CG Strategies for DWSS Operational Control

The centralised scheme described in previous section would require a central computational facility with access to all system information. On the contrary, in this section we are interested in the implementation of M computational nodes, each one with restricted information about the whole system.

In this case Problem 1 is solved by distributing the DWSS control/supervision task among the set $\mathcal{N}$ of M agents that have to locally compute a command sequences $\mathbf{u}_i(k)$. For this reason, it is of use to recast the discrete DAEs in (4) in these new expressions:

$$\mathbf{x}_i(k+1) = \mathbf{A}_i \mathbf{x}_i(k) + \mathbf{B}_i \mathbf{u}_i(k) + \sum_{j \in \mathcal{N}_i} \mathbf{B}_{ij} \mathbf{u}_j(k) + \mathbf{B}_{d,i} \mathbf{d}_i(k), \tag{18a}$$

$$\mathbf{0} = \mathbf{E}_{u,i} \mathbf{u}_i(k) + \sum_{j \in \mathcal{N}_i} \mathbf{E}_{u,i,j} \mathbf{u}_j(k) + \mathbf{E}_{d,i} \mathbf{d}_i(k), \tag{18b}$$

where the command $\mathbf{u}_i(k)$ is managed by the i-th agent while all the other flows $\mathbf{u}_j(k)$ by a related j-th agent each, all belonging to the set $\mathcal{N}_i$ of neighboring agents for the i-th agent. In order to consider a limited information scope for all agents, the notion of neighborhood of a given agent i is required, which is given in the definition below.

Definition 1: (**Neighborhood of the i-th agent**) The neighborhood of the i-th agent is defined as the set of all other agents j whose decision variables $\mathbf{u}_j$ are involved with $\mathbf{u}_i$ in some constraints and have a direct communication link with node i. □

As an immediate consequence of Definition 1, the sets of all commands $\mathbf{u}_j$ associated to the i-th agent can be characterised in the following way:

$$[\mathbf{u}]_i \triangleq \{\text{All subvectors } \mathbf{u}_j \text{ of } \mathbf{u} \text{ such that } j \in \mathcal{N}_i\}. \quad (19)$$

More formally, each agent at each time k is instructed at selecting the local command $\mathbf{u}_i(k)$, on the basis of the measured state $\mathbf{x}_i(k)$ and external input signals $\mathbf{u}_j(k)$ and $\mathbf{d}_i(k)$, as the best approximation of the local desired reference $\mathbf{r}_i(k)$. Moreover the following pointwise-in-time constraints have to be satisfied:

$$\mathbf{c}_i(k) \in \mathcal{C}_i. \quad (20)$$

In particular,

$$\mathbf{c}_i(k) \triangleq \mathbf{C}\mathbf{x}_i(k) + \mathbf{L}_i\mathbf{u}_i(k), \quad (21)$$

where $\mathcal{C}_i$ is a convex set, $\mathbf{C}^i \triangleq [\mathbf{I}_{n_x^i}^T, \mathbf{0}_{n_u^i}^T]^T$ and $\mathbf{L}_i \triangleq [\mathbf{0}_{n_x^i}^T, \mathbf{I}_{n_u^i}^T,]^T$. For the considered problem, the set $\mathcal{C}_i$ is defined as

$$\mathcal{C}_i(\mathbf{x}_s^i) \triangleq \left\{ \begin{array}{c|l} (c_i, \boldsymbol{\xi}_i) \in & \begin{bmatrix} \mathbf{I}_i & \mathbf{0} \\ -\mathbf{I}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_i \\ \mathbf{0} & -\mathbf{I}_i \\ -\mathbf{I}_{n_x^i} & \mathbf{0} \end{bmatrix} \mathbf{c}^i \leq \begin{bmatrix} \mathbf{x}_{max,i} \\ \mathbf{x}_{min,i} \\ \mathbf{u}_{max,i} \\ \mathbf{u}_{min,i} \\ \boldsymbol{\xi}_i - \mathbf{x}_{s,i} \end{bmatrix} \\ \mathbb{R}^{(n_x^i+n_u^i)\times(\mathbb{R}^{n_x^i})} & \text{and} \\ & \boldsymbol{\xi}_i - \mathbf{x}_{s,i} \leq 0 \end{array} \right\}. \quad (22)$$

Following the same lines as in the previous section, here the following local optimisation problem is defined for each agent:

$$\underset{\rightarrow}{\mathbf{w}}_i^* \triangleq \arg \min_{\underset{\rightarrow}{\mathbf{w}}_i \in \mathcal{V}_i(\mathbf{x}_i(k), [\underset{\rightarrow}{\mathbf{u}}(k)]_i, \underset{\rightarrow}{\mathbf{d}}_i)} \sum_{l=k}^{k+H_p-1} \left[J_E^i(l) + J_S^i(l) + J_{\Delta U}^i(l) \right],$$
$$r_i(l) = 0, \quad (23)$$

with

$$\begin{aligned} J_{E,i}(k) \quad &\triangleq \quad \|(\boldsymbol{\alpha}_{1,i})^{\mathrm{T}}(\mathbf{w}_i - \mathbf{r}_i(k))\|_{1,\mathbf{W}_{e,1}^i} \\ &\quad + \|(\boldsymbol{\alpha}_{2,i}(k))^{\mathrm{T}}(\mathbf{w}_i - \mathbf{r}_i(k))\|_{1,\mathbf{W}_{e,2}^i}. \end{aligned}$$

The above problem is aimed at selecting, at each time step, an open-loop *virtual* constant command sequence $\mathbf{u}_i(l|k) \equiv \mathbf{w}_i$, in such a way that the future predictions (*virtual evolutions*) of the $\mathbf{c}_i$-variable along the *virtual time l* under a constant *virtual command* $\mathbf{u}_i(l|k) \equiv \mathbf{w}_i$ from the initial state $\mathbf{x}_i$ (at virtual time $l = 0$), i.e.,

$$\mathbf{c}_i(l, \mathbf{x}_i, \mathbf{w}_i, [\mathbf{u}]_i, \mathbf{d}_i) \triangleq$$
$$\mathbf{C}\left(\mathbf{A}_i^l \mathbf{x}_i + \sum_{j=0}^{l-1} \mathbf{A}_i^{l-j-1} \left(\mathbf{B}_i \mathbf{w}_i + \sum_{j \in \mathcal{N}_i} \mathbf{B}_{j,i} \mathbf{u}_j + \mathbf{B}_{d,i} \mathbf{d}_i \right) \right) + \mathbf{L}_i \mathbf{w}_i, \quad (24)$$

do not violate constraints $\mathbf{c}_i(l, \mathbf{x}_i, \mathbf{w}_i, [\mathbf{u}]_i, \mathbf{d}_i) \in \mathcal{C}_i, \forall l \in \mathbb{Z}_+$. In order to take into account the algebraic equations (1b), the following set is introduced:

$$\begin{aligned} \mathcal{W}_i([\mathbf{u}]_i, \mathbf{d}_i) \triangleq \{ & \mathbf{w} \in \mathbb{R}^{n_u^i} | \mathbf{E}_{u,i}^i \mathbf{w}_i \\ & + \sum_{j \in \mathcal{N}_i} \mathbf{E}_{u,j,i} \mathbf{u}_j(k) + \mathbf{E}_{d,i} \mathbf{d}_i = 0 \}. \end{aligned} \quad (25)$$

In the case of time-varying disturbance predictions, commands $\mathbf{u}_i(l|k)$ need to be selected in a set that depends on the entire disturbance sequence $\underset{\rightarrow}{\mathbf{d}}_i(k) \triangleq [\mathbf{d}_i(0|k), ..., \mathbf{d}_i(l_0|k)]$ and has the following form:

$$\begin{aligned} & \mathcal{V}_i(\mathbf{x}_i, [\underset{\rightarrow}{\mathbf{u}}_j(k)]_i, \underset{\rightarrow}{\mathbf{d}}_i(k)) \triangleq \Big\{ \underset{\rightarrow}{\mathbf{w}}_i(l) | \{ \mathbf{w}_i(l) \in \mathcal{W}_i(\mathbf{d}_i(l)) \} \\ & \text{s.t. } \exists \boldsymbol{\xi}_i(l), (\mathbf{c}(l, \mathbf{x}_i, \mathbf{w}_i(l), [\mathbf{u}(l)]_i, \mathbf{d}_i(l)), \boldsymbol{\xi}_i(l)) \in \mathcal{C}(\mathbf{x}_{s,i}), \\ & \forall l \in \{0, 1, ..., l_0\} \}. \end{aligned} \quad (26)$$

Because the above set can be represented by a finite number of inequalities of the type

$$h_i(\underset{\rightarrow}{\mathbf{w}}_i | [\underset{\rightarrow}{\mathbf{u}}]_i, \mathbf{x}_i, \underset{\rightarrow}{\mathbf{d}}_i) \leq 0$$

for the purpose of this paper, it is convenient to redefine Problem (23) in the following quite general form:

$$\begin{array}{ll} \min_{\underset{\rightarrow}{\mathbf{w}}_i \in \mathbf{R}^{n_i}} & f_i(\underset{\rightarrow}{\mathbf{w}}_i) \\ s.t. & h_i(\underset{\rightarrow}{\mathbf{w}}_i | [\underset{\rightarrow}{\mathbf{u}}]_i, \mathbf{x}_i, \underset{\rightarrow}{\mathbf{d}}_i) \leq 0. \end{array} \quad (27)$$

A. Distributed Non-Iterative CG (DNI-CG)

The task introduced above is here addressed by resorting to the non-iterative optimisation procedure presented in [15], where agents that do not share any coupling constraint can update their control moves in parallel without affecting the feasibility retention. To this aim, agents are clustered into particular subsets here denoted as turns

Definition 2: (**Turn**) A turn $\mathcal{T} \subset \mathcal{A}$ is a *subset of non-neighboring nodes*, i.e., $\forall i, j \in \mathcal{T}$ such that $i \neq j$, $j \notin \mathcal{N}_i$ (none of them is a neighbor of the others). □

Following the ideas of [15], [14], the above definition is here exploited to design a distributed control strategy where the computation of all commands is performed, at each time instant, by means of a sequential procedure involving turns of agents. In particular, the simultaneous computation of the local commands $\mathbf{u}_i(k)$ is allowed only to agents belonging to the same turn. All the other agents await until their turn become active according to a precise order arranged beforehand. This approach requires that

- a communication network linking neighbouring agents exists and it is modelled as a graph (*communication graph*);
- a sequence of turns $\mathcal{T}_1, \mathcal{T}_2, ... \mathcal{T}_q$ that covers completely the communication graph has been determined.

Roughly speaking, at each time k, for each turn, agents carry out the following basic actions:

1) receive finite sequences $\mathbf{u}_j(l|k), l = 0, 1, ..., l_0$ of computed flows from previous updating neighboring agents and $\mathbf{u}_j(l|k-1), l = 0, 1, ..., l_0$ from next updating neighbouring agents;
2) compute $\mathbf{u}_i(k), l = 0, 1, ..., l_0$ according to the minimisation of a proper optimisation program;
3) transmit $\mathbf{u}_i(l|k), l = 0, 1, ..., l_0$ to neighbouring agents;
4) apply $\mathbf{u}_i(0|k)$.

This procedure can be recast in the following pseudo-code shown in Algorithm 1.

Algorithm 1 (DNI-CG)

At each time k

set $cnt = 1$
while $cnt \leq q$ **do**
  for $i \in \mathcal{T}_{cnt}$ **do** ▷ parallelisation
    receive $\mathbf{u}_j(l|k-1), l = 0,1,...,l_0$ from previous updating neighbours
    receive $\mathbf{u}_j(l|k), l = 0,1,...,l_0$ from previous updating neighbours
    compose $[\underrightarrow{\mathbf{u}}]_j$
    compute $\mathbf{u}_i^*(k)$ as solution of (27)
    transmit $\mathbf{u}_j(l|k), l = 0,1,...,l_0$ to the neighbours
    apply $\mathbf{u}_i(k) = \mathbf{u}_i^*(0|k)$
  end for
  $cnt \leftarrow cnt++$
end while

B. Distributed Iterative CG (DI-CG)

In this section, the above stated distributed CG supervision problem is solved through iterative approach presented in [19]. It has been shown in [17] that, unlike the above described non-iterative procedure, the proposed class of distributed iterative CG strategies is able to achieve Pareto optimal coordination performance not only in steady-state conditions but also during transients.

1) Distributed Optimization based on Penalty Methods: In this section, the distributed optimization method presented in [19] is recalled and it will be used to solve in a distributed way the above stated CG problem. To this end, consider for each i-th agent the following augmented cost function, related to problem (27), reinforced by the penalty function P_i:

$$\begin{aligned} F_i(\underrightarrow{\mathbf{w}}_i, \beta_i|[\underrightarrow{\mathbf{u}}]_j, \mathbf{x}_i, [\underrightarrow{\mathbf{d}}]_j) &\triangleq \beta_i f_i(\underrightarrow{\mathbf{w}}_i) + P_i(\underrightarrow{\mathbf{w}}_i|[\underrightarrow{\mathbf{u}}]_i, \mathbf{x}_i) \\ &= \beta_i[f_i(\underrightarrow{\mathbf{w}}_i) + \tfrac{1}{\beta_i}P_i(\mathbf{w}_i|[\underrightarrow{\mathbf{u}}]_i, \mathbf{x}_i)], \end{aligned} \tag{28}$$

where $\beta_i \geq 0$ is a local penalty parameter.

By using the above approach, local optimisation problems for each agent can then be defined as follows:

$$\min_{\mathbf{u}_i \in \mathbf{R}^{n_i}} F_i(\underrightarrow{\mathbf{w}}_i, \beta_i|[\underrightarrow{\mathbf{u}}]_i, \mathbf{x}_i, [\underrightarrow{\mathbf{d}}]_i), \tag{29}$$

with the optimal solution denoted by

$$\underrightarrow{\mathbf{w}}_i^* \triangleq \arg\min_{\underrightarrow{\mathbf{w}}_i} F_i(\underrightarrow{\mathbf{w}}_i, \beta_i|[\underrightarrow{\mathbf{u}}]_i, \mathbf{x}_i, \underrightarrow{\mathbf{d}}_i). \tag{30}$$

According to [19], it is possible to show that, for a local optimisation, the values of F_i and P_i decrease as β_i decreases. As a consequence, each agent can use the local β_i as a selection tool to achieve possibly less constraints violation (and indirectly, more cooperation) without resulting in an increase in $F_i(\underrightarrow{\mathbf{w}}_i, \beta_i|[\underrightarrow{\mathbf{u}}]_i, \mathbf{x}_i, \underrightarrow{\mathbf{d}}_i)$.

At each iteration, β_i are all decreased by the same factor $\lambda \in (0,1)$, i.e.,

$$\begin{aligned} \beta_i(1) &= \lambda\beta_i(0), \forall i \in \mathcal{A}, \\ &\vdots \\ \beta_i(p+1) &= \lambda\beta_i(p) = \lambda^p\beta_i(0), \forall i \in \mathcal{A}. \end{aligned} \tag{31}$$

Moving from these considerations, an algorithm that combines local subsystem optimisations with a bargaining scheme between subsystems can be implemented. Such a scheme uses iterative optimisations locally, where each agent i solves a sequence of local programs involving only its neighbourhood $\mathcal{N}_i$. In particular, at each iterate it optimises the cost (29) by a local selection of β_i, which is then made aware to all other agents $j \in \mathcal{N}_i$.

During the evolution of the distributed optimisation process, the subsystems are actually negotiating: during the p-th iteration they propose a solution $\underrightarrow{\mathbf{w}}_i^{(p)}$ on the basis of $[\underrightarrow{\mathbf{w}}]_i^{(p-1)}$ and receive a counter offer $[\underrightarrow{\mathbf{w}}^{(p+1)}]_i$ computed by taking into account $\underrightarrow{\mathbf{w}}_i^{(p)}$ when the other agents in the neighborhood change their individual moves. The selection of β_i gives each subsystem a way to "bargain": for large values of β_i the resulting solution provides minimal constraints satisfaction; as β_i decreases, the constraints satisfaction (and indirectly the cooperation) increases.

Below, a pseudo-code implementing the algorithm for the generic agent i with neighborhood $\mathcal{N}_i$ is reported in Algorithm 2.

Algorithm 2 (Distributed Optimisation Routine)

INPUTS: $\mathbf{x}_i$, $[\underrightarrow{\mathbf{u}}]_i$, $\underrightarrow{\mathbf{u}}_i$
OUTPUTS: $\underrightarrow{\mathbf{u}}_i^*$
initialisation:
$p \leftarrow 0$, $C_i \leftarrow 0$, $\underrightarrow{\mathbf{w}}_i^{(0)} \leftarrow \underrightarrow{\mathbf{u}}_i$, $[\underrightarrow{\mathbf{w}}]_i^{(0)} \leftarrow [\underrightarrow{\mathbf{u}}]_i$
main:
if $C_i == 0$ **then**
  if $p > 0$ **then**
    receive $\underrightarrow{\mathbf{w}}_j^{(p)}$ from each $j \in \mathcal{N}_i$
    $\underrightarrow{\mathbf{w}}_j^{(p)} \leftarrow \underrightarrow{\mathbf{w}}_j^{(p-1)}$ for agents that have notified convergence
    compose $[\underrightarrow{\mathbf{w}}]_i^{(p)}$
  end if
  select $\beta_i(p+1) = \lambda\beta_i(p)$
  $\underrightarrow{\mathbf{w}}_i^{(p+1)} \leftarrow \operatorname{argmin} F_i(\underrightarrow{\mathbf{w}}_i^{(p)}, \beta_i(p+1)|[\underrightarrow{\mathbf{w}_i}]_i^{(p)}, \underrightarrow{\mathbf{x}}_i, \underrightarrow{\mathbf{d}}_i)$
  $\Delta F_i(p+1) \leftarrow F_i(\underrightarrow{\mathbf{w}}_i^{(p)}, \beta_i(p)|[\underrightarrow{\mathbf{w}_i}]_i^{(p)}, \underrightarrow{\mathbf{x}}_i, \underrightarrow{\mathbf{d}}_i) - F_i(\underrightarrow{\mathbf{w}}_i^{(p+1)}, \beta_i(p+1)|[\underrightarrow{\mathbf{w}_i}]_i^{(p)}, \underrightarrow{\mathbf{x}}_i, \underrightarrow{\mathbf{d}}_i)$
  if $\Delta F_i(p) < \varepsilon$ **then**
    $C_i \leftarrow 1$
    notify local convergence to $\mathcal{N}_i$
  end if
  $p \leftarrow p+1$
  transmit $\underrightarrow{\mathbf{w}}_i^{(p)}$ to $\mathcal{N}_i$
  go to **main**
else
  $\underrightarrow{\mathbf{w}}_i^* \leftarrow \underrightarrow{\mathbf{w}}_i^{(p)}$
end if

The above described formulation allows one to present the main properties of the DI-CG approach in Algorithm 3 when executed at each time instant by all agents.

Algorithm 3 (Distributed Iterative CG Algorithm (DI-CG) - Agent i)

At each time k

1: receive $\underrightarrow{\mathbf{u}}_j(k-1)$, from each $j \in \mathcal{N}_i$
2: build up vectors $[\underrightarrow{\mathbf{u}}(k-1)]_i$
3: compute $\underrightarrow{\mathbf{u}}_i(k)$ by means of **Algorithm 2** with inputs $\mathbf{x}_i(k)$, $[\underrightarrow{\mathbf{u}}(k-1)]_i$, $\underrightarrow{\mathbf{u}}_i(k-1)$
4: apply $\mathbf{u}_i(0|k)$
5: transmit $\underrightarrow{\mathbf{u}}_i(k)$ to $\mathcal{N}_i$

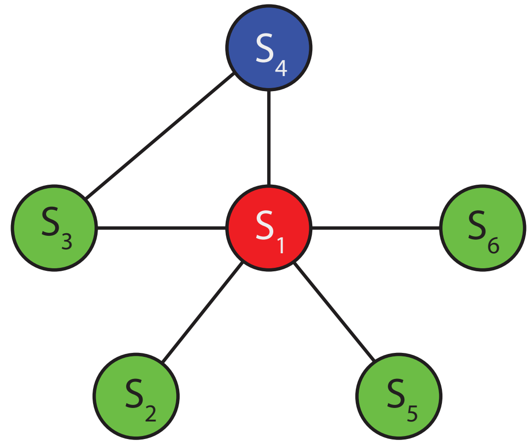

Figure 2. Turns partitioning

V. Simulation Results

A. Case Study Description

The case study proposed in this paper is the drinking water supply network of Barcelona (Spain), called in the sequel as the Barcelona DWSS [3]. In this network, water is taken from both superficial and underground sources (i.e., wells), providing together an inflow of about 7 m^3/s. The main supply comes from Llobregat, Ter, and Besòs rivers, which are regulated by dams with an overall capacity of 600 hm^3. Each water source is limited, implying different water prices depending on water treatments and legal extraction canons. Currently, there are four drinking water treatment plants (WTP) and several underground sources (wells) that can provide water through pumping stations. The reader is referred to [3] for further details of DWSS modelling and specific insights related to the case study of this paper. The case study model consists of 67 tanks and 121 actuators, these latter divided into 46 pumps and 75 valves. Moreover, the network has 88 water demand sectors and 16 nodes. Both the demand episodes and the network calibration/simulation setup are provided by AGBAR[1]. These water demands are characterised/modelled by patterns of water usage and can be forecasted by using time-series models, neural networks, among other methods [20], [21].

B. Closed-loop Setup

Considering [13], the control-oriented model of the Barcelona DWSS is decomposed into six subsystems, as depicted in Figure 3 using different colours. All reported results have been obtained by considering four-days water demand scenarios (with 1 hour of sampling time), and $H_p = l_0 = 24$ hours given the periodicity of the water demands. The prioritisation of control objectives in (7) and (9) has been determined by weights $W_{e,1} = 0.9$, $W_{e,2} = 0.5$, $W_x = 0.2$ and $W_u = 0.1$, following a exhaustive trial-and-error tuning procedure. The closed-loop system has been simulated by using the same model used to design the controller but fed with real water demands. The network model has been calibrated and validated by using real data provided by AGBAR. Regarding the DNI-CG approach, it is worth to mention that agents are grouped into 3 turns as depicted in Figure 2: $\mathcal{T}_1 = \{\text{Agent } 4\}$ (blue), $\mathcal{T}_2 = \{\text{Agent } 1\}$ (red), $\mathcal{T}_3 = \{\text{Agent } 2, \text{Agent } 3, \text{Agent } 5, \text{Agent } 6\}$ (green).

All simulations have been undertaken by using the Yalmip interpreter [22] and the CPLEX solver, all under Matlab© 8.2 environment, running on an Intel© Core i5-3330 machine with 3.3 GHz and 8 GB RAM.

C. Results, Comparison and Discussion

The results achieved by the CG-based approaches[2] are shown in Table I. From these results, it can be seen that the iterative distributed approaches achieve a quite similar performance with respect to the one obtained with the centralized CG scheme. Likewise, the DI-CG approach achieves better performance when compared with the DNI-CG approach since the DNI-CG does not envisage any information exchange among the agents during the optimization being a non-cooperative scheme.

Table I
Complete break-down of economic costs for the different approaches

		Day 1	Day 2	Day 3	Day 4
Water Cost	CG	28.73	32.05	32.46	32.71
	DNI-CG	32.25	33.45	35.40	35.56
	DI-CG	30.04	32.91	33.58	33.81
Electric Cost	CG	26.31	26.31	27.15	27.29
	DNI-CG	32.45	31.18	29.91	29.87
	DI-CG	26.92	27.13	27.11	27.22
Total Cost	CG	55.04	58.93	59.60	60.05
	DNI-CG	64.7	64.63	65.31	65.43
	DI-CG	56.96	60.04	60.74	61.08

[1]Aguas de Barcelona, S.A. (AGBAR) is the company that manages the drinking water supply and distribution in Barcelona (Spain).

[2]Costs in Table I are given in economic units rather than real values (Euro) due to confidentiality reasons.

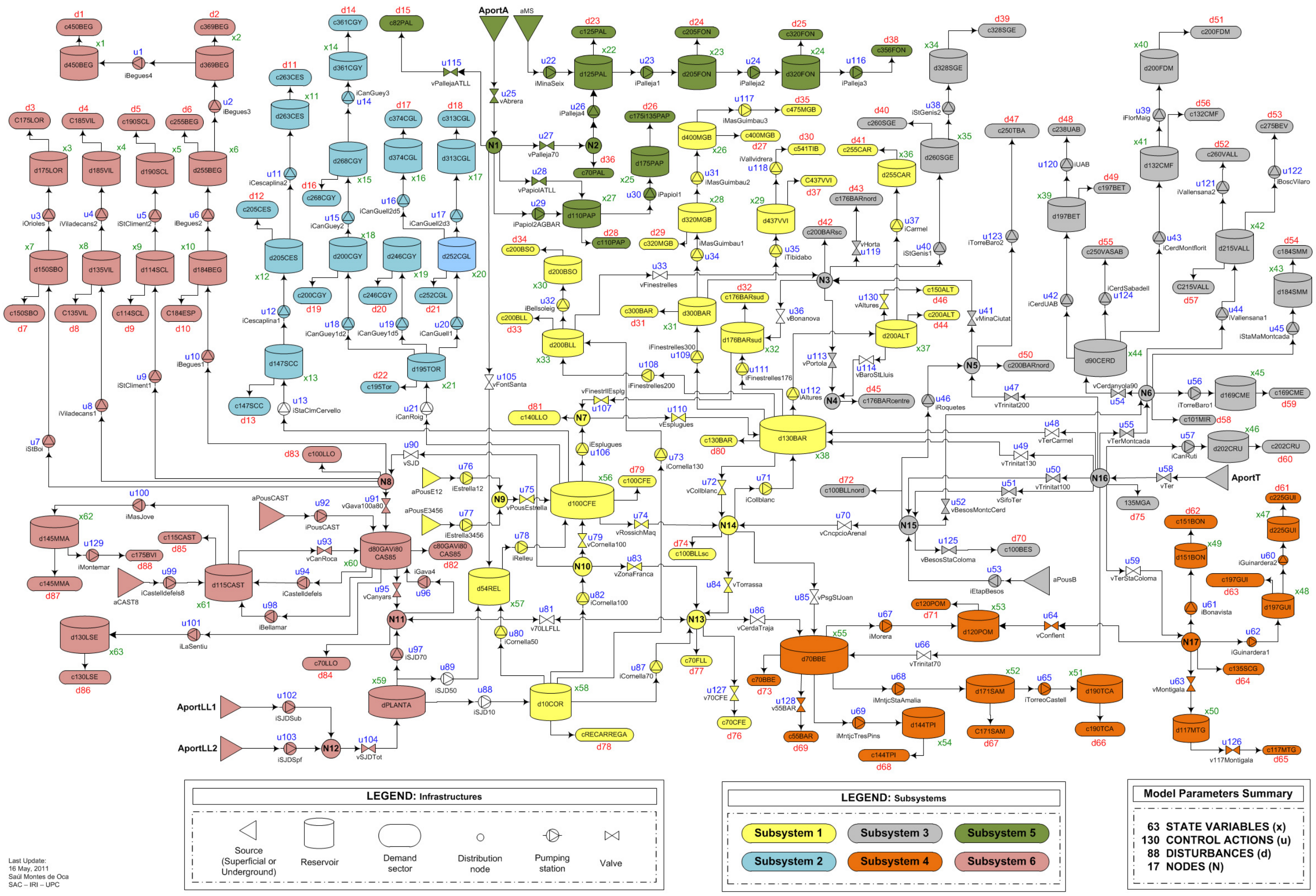

Figure 3. Partitioning of the Barcelona DWSS

Figures 4 and 5 show the comparison of tank volume evolutions and a water flow though a valve when considering the proposed CG approaches. The highlighted areas for all figures (in cyan colour) delimit the time intervals with the lowest electric costs (night hours).

Notice that all approaches try to fill the tanks up during the night, period when the energy is cheaper as seen in Figure 4. On the other hand, Figure 6 shows the different usage of the available water sources according to each CG approach proposed. In particular, the DNI-CG is not able to fully exploit the lower price sources as the other two approaches do. Such a behavior cannot be improved by acting on the available weights because in the DNI-CG scheme, each water source is governed by a different agent that works by minimizing its own local cost without any attempt of coordinating the solutions with the other agents, as, on the contrary, is accomplished in the centralized and the DI-CG schemes. For the sake of completeness, Figure 7 provides a closer look at the convergence of Algorithm 2 performed under the action of DI-CG. In particular, the iteration process at time $k = 11$ for the input u_{25} is depicted along with the evolution of the parameter β_5. Note that the input during the iteration process converges to a feasible value.

As expected, the different management behaviours produced by the proposed approaches are confirmed by the results collected in Table I. Specifically, the total cost achieved by both DCG and DI-CG are quite similar and better than the performance achieved by using DNI-CG.

Notice also that, although all simulations have considered a four-days scenario, the closed-loop system reaches its steady state behaviour at the end of the second day. After that, the behaviour becomes periodical and a single-day evaluation can be considered as performance indicator.

In order to motivate the choice of the tuning weights within the design of the multi-objective predictive controller, Table II presents a sensitivity analysis performed by changing such values and evaluating the total economic cost related to all four simulation days.

Table II
ECONOMIC COSTS COMPARISON WITH DIFFERENT SCENARIOS

Parameters				Economic Cost		
W_{e_1}	W_{e_2}	W_x	W_u	CG	DNI-CG	DI-CG
0.9	0.5	0.2	0.1	233.62	260.07	238.32
0.9	0.5	1	0.1	274.45	303.15	279.98
0.9	0.5	0.2	0.8	269.44	299.94	274.85
0.5	0.9	0.2	0.1	238.88	265.27	243.09

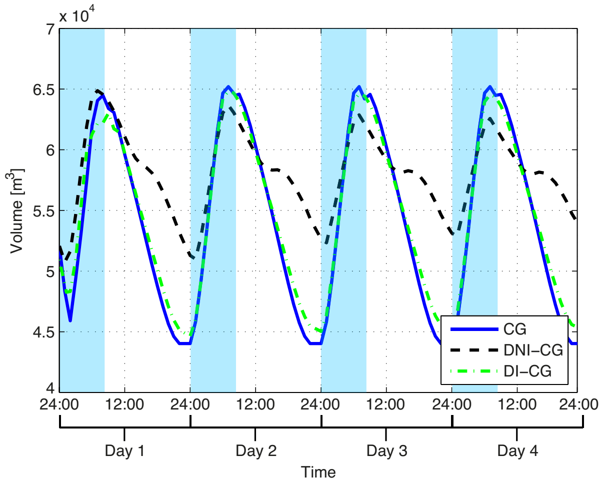

Figure 4. Resultant volume related to tank x_{60}

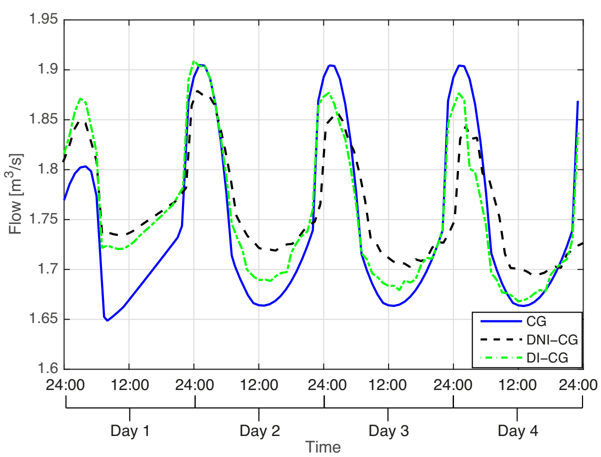

Figure 5. Computed flow related to valve u_{82}

VI. Conclusions

This paper evaluates the application of Command Governor (CG) strategies for the operational control of DWSS in both a centralized and distributed way, discussing the effectiveness and advantages of both strategies. The proposed CG approaches are shown to be suitable for optimal management of DWSSs because they allow one to consider the relevant effect of persistent and periodical disturbances (water demands) over the state evolutions of the network and their marginal stability feature. The Barcelona DWSS has been considered as the case study for the undertaken assessment analysis. From these results, it can be seen that iterative distributed approaches may achieve a performance that is quite close to that corresponding to the centralized CG scheme. In particular, the DI-CG approach achieves better management performance when compared with the DNI-CG approach. However, this improvement is achieved at the cost of larger computational times because of the iterative nature of the DI-CG scheme.

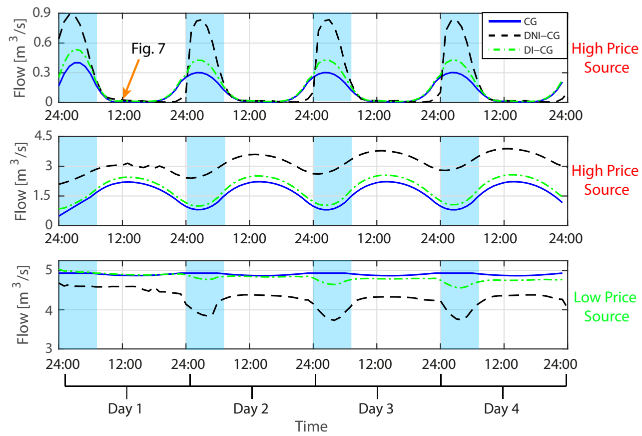

Figure 6. Comparison of inflows from the main water supply sources: (top) Abrera (u_{25}), (middle) Ter (u_{58}), (bottom) Llobregat (u_{104})

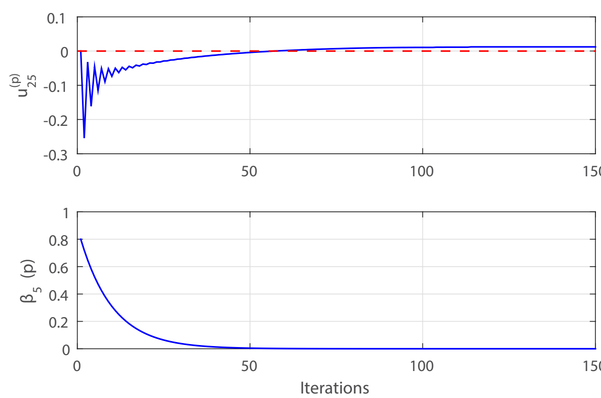

Figure 7. DI-CG: iteration process carried out by agent 5 for computing $u_{25}(11)$

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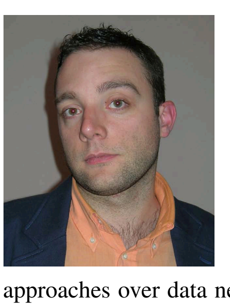

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