A Novel Design of Unknown Input Observers using Set-theoretic Methods for Robust Fault Detection

Feng Xu[1], Junbo Tan[1], Xueqian Wang[1,*], Vicenç Puig[2], Bin Liang[1,3,*] and Bo Yuan[4]

***Abstract*— This paper proposes a novel unknown input observer (UIO) design method, which incorporates the set-theoretic notions into the design of UIOs. In this way, we can take advantage of both UIOs and set-theoretic methods in fault detection (FD). The main advantage of UIOs is that they can be insensitive to unknown inputs affecting a system. However, a critical limitation is the satisfaction of the UIO design conditions for a monitored system. The core idea of this paper is that, even though we cannot design a UIO insensitive to all unknown inputs, we can at least design a UIO insensitive to as many unknown inputs as possible. In this case, although the effect of all unknown inputs on FD cannot be completely removed, we can at least partially remove the effect of unknown inputs. Furthermore, for the remaining unknown inputs whose effect cannot be removed, the set-theoretic methods can be employed to specify them and obtain FD robustness against their effect. At the end of this paper, the effectiveness of the proposed method is illustrated by a numerical example.**

I. INTRODUCTION

It is inevitable for a real system to operate in situations under the effect of unknown inputs originated from modeling errors, linearization, disturbances, faults and among other things. Generally, it is difficult to obtain sufficient information about these unknown inputs that can result in practical issues when using models including them for estimating, diagnosing and controlling dynamic systems. The UIOs, as a variant of Luenberger observers, are specifically developed to handle the unknown inputs. In the past decades, various UIOs have been designed under different conditions [5]–[7]. Moreover, the applications of UIOs have been extended from state estimation to fault diagnosis. The principle of the UIO-based fault diagnosis is to use UIOs to decouple the effect of disturbances on a residual vector from faults. Then, the fault diagnosis task can be carried out by comparing the residual signals with their given thresholds [3].

Unfortunately, for the system with unknown inputs, the residual signals are simultaneously affected by unknown inputs and faults. In order to obtain reliable fault diagnosis results, it is essential that the residual vector can be decoupled from unknown inputs so that they are only sensitive to faults, which is the most important advantage of the UIO-based fault diagnosis. However, the UIO has an important weakness, that is, the satisfaction of the UIO design conditions for a system perturbed by unknown inputs. The UIOs are designed to asymptotically converge to the system states by choosing a proper group of observer parametric matrices. However, if the number of independent measurable outputs is less than that of unknown inputs, it is impossible to design a conventional UIO insensitive to all unknown inputs except for the case that extra conditions regarding unknown inputs or the system can be satisfied (see [4], [5], [8]). The proposed solutions in [4], [8] assume that some prior knowledge of unknown inputs is obtained (e.g., mathematical relations between unknown inputs and states are known). However, these conditions based on extra knowledge are still quite restrictive when applied to for real systems.

The objective of this paper is to propose a novel design of UIOs to handle the case when not all unknown inputs can be decoupled because the UIO design conditions are not satisfied. In this case, by assuming part of unknown inputs are bounded, this idea is that, instead of designing a UIO insensitive to all unknown inputs, we can design a UIO only insensitive to a part of all unknown inputs. This means that the set of all unknown inputs can be divided into two groups: one group to which the UIO is insensitive and the other group to which the UIO is sensitive. Thus, the effect of the first group on fault diagnosis can be eliminated by designing a proper UIO insensitive to this group, while the effect of the second group on fault diagnosis cannot be removed since the UIO in use is sensitive to it.

In order to obtain robustness of fault diagnosis, we should properly consider the effect of the second group on the residual vector. In this paper, we choose the set-theoretic methods to tackle this problem [2]. The set-theoretic methods have been successfully used for robust fault diagnosis, which obtain robustness by propagating the sets of unknown inputs through a system model or considering invariant sets to describe their effect on the system [9]–[12]. However, the set-theoretic methods have drawbacks. The first drawback is related to the complexity originated from on-line set computation. The second drawback is its conservatism because it considers the worst-case uncertainty bounds such as unknown inputs. That is, if there exist too many unknown inputs, the size of sets will increase and consequently the sensitivity to faults will decrease.

This paper proposes to combine set-theoretic approaches with the UIOs. Eventually, the residuals generated by the designed observer are insensitive to the first group of unknown inputs as well as robust to the second group of unknown

[1]Center of Intelligent Control and Telescience, Graduate School at Shenzhen, Tsinghua University, 518055 Shenzhen, P.R.China.

[2]Institut de Robòtica i Informàtica Industrial (CSIC-UPC), Technical University of Catalonia (UPC), 08028 Barcelona, Spain.

[3]Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, 100084 Beijing, P.R.China.

[4]Intelligent Computing Lab, Division of Informatics, Graduate School at Shenzhen, Tsinghua University, 518055 Shenzhen, P.R.China.

*Corresponding authors: Xueqian Wang and Bin Liang

E-mail:{xu.feng, wang.xq}@sz.tsinghua.edu.cn, bliang@tsinghua.edu.cn

inputs. With this distinct feature, we can make full use of the advantages of the UIO-based and set-based methods and simultaneously mitigate their disadvantages, which is the core contribution of this paper.

The remainder of this paper is organized as follows. Section II introduces the system model and the UIO. Section III proposes a novel FD method by combining the UIO and the set theory. Section IV illustrates the effectiveness of the proposed FD method with a numerical example. This paper is concluded in Section V including future research directions.

II. Unknown Input Observers

This section introduces the notion of UIOs and presents the existence conditions of UIOs.

A. Plant Models and UIOs

The linear discrete time-invariant plant is modeled as

$$x_{k+1} = Ax_k + Bu_k + E\omega_k, \tag{1a}$$
$$y_k = Cx_k + F\eta_k, \tag{1b}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $E \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{q\times n}$ and $F \in \mathbb{R}^{q\times s}$ are time-invariant matrices, k denotes the k-th discrete-time instant, $x_k \in \mathbb{R}^n$ and $y_k \in \mathbb{R}^q$ are the state and output vectors, respectively, $u_k \in \mathbb{R}^p$ and $\omega_k \in \mathbb{R}^r$ are known and unknown inputs, respectively, and $\eta_k \in \mathbb{R}^s$ represents the measurement noise vector.

Remark 1: In (1), ω_k is generally used to model unknown inputs originated from modeling errors, linearization, disturbances and among other unknown effects.

Assumption 2.1: The unknown input vector η_k is bounded by a known set (i.e., $\eta_k \in V$):

$$V = \{\eta \in \mathbb{R}^s : \; |\eta - \eta^c| \leq \bar{\eta}\},$$

where η^c and $\bar{\eta}$ are known and constant vectors.

Definition 2.1: In the presence of unknown inputs, an observer is defined as a UIO for the system (1) if the state-estimation-error vector asymptotically converges to zero.

Due to the existence of ω_k, when estimating the states of (1), the ability of a conventional observer to compensate the effect of ω_k is limited. Thus, we design a UIO to estimate the states of (1) as

$$z_{k+1} = Nz_k + Tu_k + Ky_k, \tag{2a}$$
$$\hat{x}_k = Mz_k + Hy_k, \tag{2b}$$
$$\hat{y}_k = C\hat{x}_k, \tag{2c}$$

where $z_k \in \mathbb{R}^n$, $\hat{x}_k \in \mathbb{R}^n$ and $\hat{y}_k \in \mathbb{R}^q$ are the state of (2), the state and output estimations, respectively, $N \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{n\times p}$, $K \in \mathbb{R}^{n\times q}$, $M \in \mathbb{R}^{n\times n}$ and $H \in \mathbb{R}^{n\times q}$.

Based on (1) and (2), the state-estimation-error vector can be defined as

$$e_k = x_k - \hat{x}_k. \tag{3}$$

Furthermore, with (1) and (2), the dynamics of the state-estimation error e_k can be derived as

$$\begin{aligned} e_{k+1} =&(A - HCA - MK_1C)e_k + [(A - HCA \\ &- MK_1C)M - MN]z_k + [(A - HCA \\ &- MK_1C)H - MK_2]y_k + (B - MT - HCB)u_k \\ &+ (E - HCE)w_k - HF\eta_{k+1} - MK_1F\eta_k, \end{aligned} \tag{4}$$

with

$$K = K_1 + K_2. \tag{5}$$

Based on (4), if we can design a group of observer matrices such that

$$E - HCE = \mathbf{0}, \tag{6a}$$
$$B - MT - HCB = \mathbf{0}, \tag{6b}$$
$$(A - HCA - MK_1C)M - MN = \mathbf{0}, \tag{6c}$$
$$(A - HCA - MK_1C)H - MK_2 = \mathbf{0}, \tag{6d}$$

then the dynamics of e_k can be rewritten as

$$e_{k+1} =(A - HCA - MK_1C)e_k - HF\eta_{k+1} - MK_1F\eta_k. \tag{7}$$

Remark 2: Due to the effect of η_k, the observer (2) cannot rigidly satisfy Definition 2.1 (i.e., the state-estimation-error vector cannot converge to zero), which can be observed in (7). However, we should emphasize that the observer (2) can be designed to be insensitive to the unknown input vector ω_k under some existence conditions of UIOs. Thus, without loss of generality, we still treat (2) as a UIO of (1) in this paper.

B. Existence Conditions of UIOs

By analyzing (4) and (6), in order to ensure the existence of the observer (2), we should simultaneously guarantee:

- $E - HCE = \mathbf{0}$ is solvable,
- $A - HCA - MK_1C$ is a stable matrix.

Remark 3: The first condition means the possibility of obtaining N, T, K, M and H, while the second condition guarantees the convergence of the observer (2).

Theorem 2.1 is given to show the necessary and sufficient conditions of the existence of the observer (2).

Theorem 2.1: The necessary and sufficient conditions for the observer (2) to exist for the system (1) are

- $rank(CE) = rank(E)$,
- $(C,\, A_1)$ is a detectable pair, where

$$A_1 = A - E[(CE)^T CE]^{-1}(CE)^T CA.$$

Proof: The proof of Theorem 2.1 can be found in [5]. □

Remark 4: In Theorem 2.1, the first and second conditions are equivalent to the solvability of the equation $E - HCE = \mathbf{0}$ and the stability of the matrix $A - HCA - MK_1C$, respectively. Moreover, the solution of the equation $E - HCE = \mathbf{0}$ is $H = E[(CE)^T CE]^{-1}(CE)^T + H_0\{I - CE[(CE)^T CE]^{-1}(CE)^T\}$, where H_0 is an arbitrary matrix with proper dimensions.

Under Theorem 2.1, it can be guaranteed that the observer (2) exists. The procedure to calculate N, T, K, M and H is given as follows:

- use (6a) to compute a value of H,
- design a pair (M, K_1) such that $A - HCA - MK_1C$ is a stable matrix,
- calculate T by using (6b),
- calculate N by using (6c),
- calculate K_2 by using (6d).

Remark 5: In order to guarantee that $A-HCA-MK_1C$ is a stable matrix, we can consider MK_1 as a whole. Then, by choosing a value of M, we can determine both M and K_1. Based on (6b), (6c) and (6d), the remaining parametric matrices of (2) can be further computed.

The state-estimation-error vector e_k is originated from η_k, which can be seen in (7). Thus, although we cannot eliminate the effect of η_k, we can reduce it as much as possible by selecting H and MK_1 such that the sets HFV and MK_1FV of $HF\eta_{k+1}$ and $MK_1F\eta_k$ are as small as possible.

III. Robust Fault Detection

A set-based design of UIOs is proposed to handle the existence problem of UIOs and reduce conservatism of the conventional set-theoretic FD methods by making the observer insensitive to a subset of the unknown inputs.

A. Design of Set-theoretic UIOs

In Section II, the UIOs have been systematically introduced. However, they have an important weakness, which consists in their existence for a monitored system. According to the existence conditions in Theorem 2.1, the number of independent rows of C should not be less than that of independent columns of E, which means that the maximal number of unknown inputs that can be decoupled cannot be larger than that of independent measurable outputs.

In this paper, we focus on the case that the UIOs satisfying Theorem 2.1 do not exist, which means that the system has too many unknown inputs and cannot completely decouple their effect from the residual vector in an active way. In this case, it is impossible to achieve robust FD by only the active decoupling way. However, we can turn to the passive method to further consider the effect of unknown inputs from the residual vector to obtain robust FD. Particularly, we consider using the set-theoretic methods to implement the passive managing of unknown inputs by bounding and propagating their effect.

Since we cannot actively decouple all unknown inputs described by ω_k, we divide them into two groups. Thus, we rewrite the unknown input vector into

$$\omega_k = \begin{bmatrix} \omega_{1,k} \\ \omega_{2,k} \end{bmatrix}, \tag{8}$$

where $\omega_{1,k} \in \mathbb{R}^{n_a}$ and $\omega_{2,k} \in \mathbb{R}^{n_p}$. Note that n_a is the number of unknown inputs that the observer (2) is designed to be insensitive to, while n_p denotes the remaining number of unknown inputs, where

$$n_p = r - n_a.$$

Remark 6: The n_a unknown inputs can be actively decoupled from the residual, while the n_p unknown inputs cannot be actively decoupled by the same UIO. Instead, we can use the passive method to decouple them from the residual.

Furthermore, we can rewrite the matrix E into

$$E = \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \tag{9}$$

where $E_1 \in \mathbb{R}^{n\times n_a}$ and $E_2 \in \mathbb{R}^{n\times n_p}$.

By substituting (9) into (4), we can obtain

$$\begin{aligned} e_{k+1} =&(A - HCA - MK_1C)e_k + [(A - HCA- \\ &MK_1C)M - MN]z_k + [(A - HCA- \\ &MK_1C)H - MK_2]y_k + (B - MT - HCB)u_k+ \\ &(E_1 - HCE_1)\omega_{1,k} + (E_2 - HCE_2)\omega_{2,k} - HF\eta_{k+1} \\ &- MK_1F\eta_k. \end{aligned} \tag{10}$$

Similarly, according to Theorem 2.1, we can design a matrix E_1 to satisfy

$$E_1 - HCE_1 = \mathbf{0}. \tag{11}$$

Remark 7: Since n_a denotes unknown inputs able to be actively decoupled, (11) can be satisfied and the design conditions of the traditional UIOs can be overcome.

Thus, a UIO insensitive to $\omega_{1,k}$ can be designed and the dynamics of the corresponding state-estimation-error vector can be further derived as

$$\begin{aligned} e_{k+1} =&(A - HCA - MK_1C)e_k + (E_2 - HCE_2)\omega_{2,k} \\ &- HF\eta_{k+1} - MK_1F\eta_k. \end{aligned} \tag{12}$$

Without loss of generality, a group of parametric matrices N, T, K, K_1, K_2, M and H satisfying (6b), (6c), (6d) and (11) are further obtained for (2). Thus, the designed UIO insensitive to $\omega_{1,k}$ can be obtained by substituting these parametric matrices into (2).

B. FD Strategy

In the proposed FD method, considering that the set-theoretic methods will be used to cope with the unknown input vector $\omega_{2,k}$, we should make Assumption 3.1.

Assumption 3.1: The unknown input vector $\omega_{2,k}$ is bounded by a known set (i.e., $\omega_{2,k} \in W_2$), where

$$W_2 = \{\omega_2 \in \mathbb{R}^{n_p} : |\omega_2 - \omega_2^c| \le \bar{\omega}_2\}$$

with $\omega_2^c \in \mathbb{R}^{n_p}$ and $\bar{\omega}_2 \in \mathbb{R}^{n_p}$ being constant vectors.

The initial set of the state-estimation-error vector is denoted as E_0^r. Thus, with (10), we can obtain that the sets of the state-estimation-error vector for all $k \ge 0$ as

$$\begin{aligned} E_{k+1}^r =&(A - HCA - MK_1C)E_k^r \oplus (E_2 - HCE_2)W_2 \\ &\oplus HF(-V) \oplus MK_1F(-V), \end{aligned} \tag{13}$$

where $\oplus$ denotes the Minkowski sum of two sets.

Remark 8: If $e_0 \in E_0^r$ holds, we can have $e_k \in E_k^r$ for all $k \ge 0$, as long as no faults occur in the system.

In this paper, we use the set-theoretic notion to passively decouple the unknown inputs that cannot be actively decoupled from the residual vector. Thus, the residual vector is defined as

$$\begin{aligned} r_k =& y_k - \hat{y}_k \\ =& Ce_k + F\eta_k. \end{aligned} \tag{14}$$

By using (13), we can construct the set of the residual vector at time instant k as

$$R_k = CE_k^r \oplus FV. \tag{15}$$

Remark 9: Based on the assumption $e_0 \in E_0^r$, we can construct the residual set R_k to contain the current residual vector for time instants $k > 0$.

By means of the above analysis, the robust FD criterion used for the proposed FD method is designed as

$$r_k \in R_k, \tag{16}$$

where both the residual vector r_k and residual set R_k are obtained in real time. This criterion implies that if at any time instant, a violation of (16) is detected, it means that a fault has occurred in the system. Otherwise, we assume that the system is still in healthy operation.

C. Computational Implementation

In this paper, the implementation of set computation is based on zonotopes, because they have relatively low computational complexity in comparison with some other geometric objects [2]. The definition and properties of zonotopes used in this paper are given below [1], [2].

Definition 3.1: An r-order zonotope Z is defined as $Z = g \oplus H\mathbb{B}^r$, where g and H are its center and segment matrix and $\mathbb{B}^r$ is a box composed of r unitary intervals.

Property 3.1: Given $X_1 = g_1 \oplus H_1\mathbb{B}^{r_1}$ and $X_2 = g_2 \oplus H_2\mathbb{B}^{r_2}$, $X_1 \oplus X_2 = \{g_1 + g_2\} \oplus [H_1 \quad H_2]\mathbb{B}^{r_1+r_2}$.

Property 3.2: Given $X = g \oplus H\mathbb{B}^r$ and a suitable matrix K, $KX = Kg \oplus KH\mathbb{B}^r$.

Property 3.3: Given a zonotope $Z = g \oplus H\mathbb{B}^m \subset \mathbb{R}^n$ and an integer s (with $n < s < m$), denote by $\hat{H}$ the matrix resulting from the reordering of the columns of the matrix H in decreasing Euclidean norm. Then $Z \subseteq g \oplus [\hat{H}_T \quad Q]\mathbb{B}^s$ where $\hat{H}_T$ is obtained from the first $s - n$ columns of the matrix $\hat{H}$ and $Q \in \mathbb{R}^{n\times n}$ is a diagonal matrix whose elements satisfy $Q_{ii} = \sum_{j=s-n+1}^{m} \mid \hat{H}_{ij} \mid$, $i = 1, \ldots, n$.

Remark 10: For the proposed FD method, we use Properties 3.1 and 3.2 to implement set computation. Besides, since the order of zonotopes dramatically increases during the online propagation, we further need Property 3.3 to reduce and control the order of zonotopes.

When using Properties 3.1 and 3.2 to unfold (13) and (15) into the center-segment form of zonotopes, (13) and (15) can be equivalently transformed into

$$\begin{aligned} e_{k+1}^c =&(A - HCA - MK_1C)e_k^c + (E_2 - HCE_2)\omega_2^c \\ &- HF\eta^c - MK_1F\eta^c, \end{aligned} \tag{17a}$$

$$\begin{aligned} H_{k+1}^e =&[(A - HCA - MK_1C)H_k^e \;\; (E_2 - HCE_2)H_{\omega_2} \\ &- HFH_\eta \;\; - MK_1FH_\eta], \end{aligned} \tag{17b}$$

$$r_k^c = Ce_k^c + F\eta^c, \tag{17c}$$

$$H_k^r = [CH_k^e \;\; FH_\eta], \tag{17d}$$

where e_k^c, r_k^c, H_k^e and H_k^r are the centers and segment matrices of E_k^r and R_k, respectively, and H_{ω^2} and H_η are the segment matrices of W_2 and V, respectively.

Remark 11: According to Assumptions 2.1 and 3.1, W_2 and V can be rewritten into zonotopes. Moreover, if E_0^r is a zonotope, all sets generated should be zonotopes.

IV. Illustrative Example

In this paper, we use a numerical example to illustrate the proposed FD method. The discrete-time dynamics of the example is presented as

$$\begin{aligned} x_{k+1} &= Ax_k + BF^a u_k + Ew_k, \\ y_k &= Cx_k + F\eta_k, \end{aligned}$$

with

$$A = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \; C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$E = \begin{bmatrix} 0.5 & 0 & 0.3 & 0 \\ 0.2 & 0.4 & 0 & 0.5 \end{bmatrix}, \; F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where F^a, a diagonal matrix, models actuator faults.

Remark 12: When the system is healthy, F^a is the identity matrix. If a fault occurs in an actuator, the corresponding diagonal entry of F^a will be a value inside $[0, 1)$.

In order to show the effectiveness of the proposed method, we consider a fault magnitude in the second actuator:

$$F^a = \begin{bmatrix} 1 & 0 \\ 0 & 0.95 \end{bmatrix}.$$

The considered system has four unknown inputs in total. In this example, we will design a UIO to be insensitive to the first unknown input. Thus, we have

$$E_1 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \; E_2 = \begin{bmatrix} 0 & 0.3 & 0 \\ 0.4 & 0 & 0.5 \end{bmatrix}.$$

According to the procedure described before, the corresponding parametric matrices of the UIO for the monitored system are designed as

$$N^* = \begin{bmatrix} 0.4097 & 0.0179 \\ -0.7594 & -0.1122 \end{bmatrix}, \; T^* = \begin{bmatrix} -0.8681 & 1.0851 \\ 1.3194 & -1.6492 \end{bmatrix},$$
$$K_1^* = \begin{bmatrix} 0.3167 & 0.3135 \\ 0.6999 & 0.2940 \end{bmatrix}, \; K_2^* = \begin{bmatrix} 0.3993 & 0.0668 \\ -1.8536 & -0.7081 \end{bmatrix},$$
$$K^* = \begin{bmatrix} 0.7160 & 0.3803 \\ -1.1537 & -0.4141 \end{bmatrix}, \; M^* = \begin{bmatrix} 0.6825 & 0.5746 \\ 0.7279 & 0.2220 \end{bmatrix},$$
$$H^* = \begin{bmatrix} 8.3436 & 4.1409 \\ 3.3899 & 1.5253 \end{bmatrix}.$$

The initial conditions in this simulation are given as

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{X}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \mathbb{B}^2,$$
$$z_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; E_0^r = \begin{bmatrix} -0.0739 \\ -0.0296 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbb{B}^2.$$

We use two sinusoidal inputs to excite the system:

$$u_{1,k} = 10\sin(0.2k), \; u_{2,k} = 10\sin(0.2k).$$

Additionally, we assume that the bounding sets of unknown inputs and measurement noises are as follows:

$$W = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \oplus \begin{bmatrix}1 & 0 & 0 & 0\\0 & 0.1 & 0 & 0\\0 & 0 & 0.1 & 0\\0 & 0 & 0 & 0.1\end{bmatrix} \mathbb{B}^4,$$
$$W_1 = [0] \oplus [1]\, \mathbb{B}^1,$$
$$W_2 = \begin{bmatrix}0\\0\\0\end{bmatrix} \oplus \begin{bmatrix}0.1 & 0 & 0\\0 & 0.1 & 0\\0 & 0 & 0.1\end{bmatrix} \mathbb{B}^3,$$
$$V = \begin{bmatrix}0\\0\end{bmatrix} \oplus \begin{bmatrix}0.01 & 0\\0 & 0.01\end{bmatrix} \mathbb{B}^2.$$

In this example, a fault scenario is defined to show the effectiveness of the proposed method as follows: from $k = 0$ to $k = 20$, the system is healthy, while from $k = 21$ to $k = 50$, the actuator fault is present.

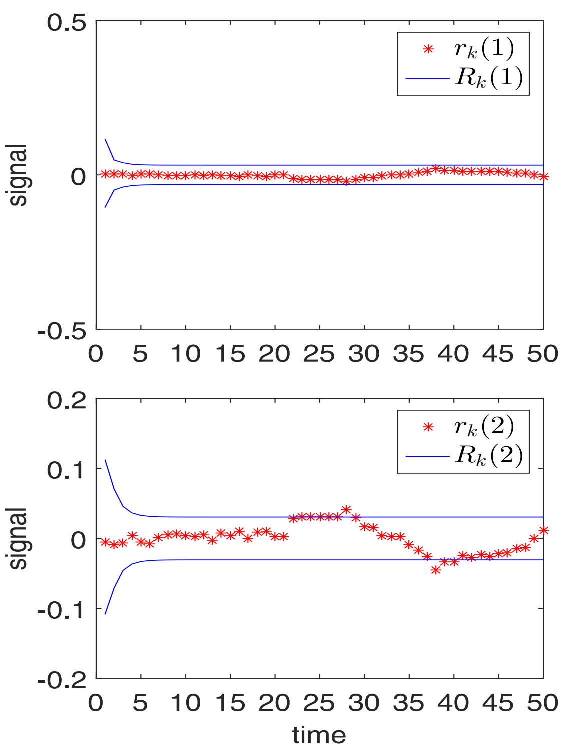

Fig. 1. UIO-based FD

For the fault scenario, the results corresponding to the set-theoretic UIO-based FD method are shown in Figure 1. It can be observed that the set-theoretic UIO-based method can detect the actuator fault at time instant $k = 22$, which has shown its effectiveness. Without loss of generality, in Figure 1, the presentation of results is based on interval hulls of zonotopes instead of using zonotopes for simplicity of plotting. Note that, $r_k(1)$, $R_k(1)$, $r_k(2)$ and $R_k(2)$ in Figure 1 denote the intervals of the first and second components of the signals r_k and R_k, respectively.

V. Conclusions

In this paper, a novel design method of UIOs for FD is proposed, which combines both the UIO and the set theory. Both the UIO-based and the conventional set-based FD methods can be robust to unknown inputs and are able to decouple the effect of unknown inputs from the residual vector. The difference is that, the former uses the active decoupling while the latter is based on the passive decoupling. We have to mention that both methods have their advantages and limitations. The former requires the existence of UIOs for the decoupled system, which is not always the case, while the latter considers that all unknown inputs are bounded. In practice, the existence conditions of the UIOs are generally harsher than the boundedness conditions of unknown inputs and measurement noises. The contribution of this paper is that it proposes a novel method that combines both the active and the passive decoupling methods to obtain FD robustness. Eventually, the existence conditions of UIOs can be removed and the advantages of the active and passive decoupling methods can be kept. In the future, the proposed method will be further extended to fault isolation (FI) and fault-tolerant control (FTC).

Acknowledgements

This work was supported by the Natural Science Foundation of Guangdong (No.2015A030313881) and the Shenzhen Key Lab of Space Robotic Technology and Telescience.

References

[1] T. Alamo, J.M. Bravo, and E.F. Camacho. Guaranteed state estimation by zonotopes. *Automatica*, 41(6):1035–1043, 2005.

[2] F. Blanchini and S. Miani. *Set-theoretic Methods in Control*. Birkhäuser Boston, 2008.

[3] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. *Diagnosis and Fault-Tolerant Control*. Springer-Verlag, Berlin, Germany, 2006.

[4] B.A. Charandabi and H.J.Marquez. A novel approach to unknown input filter design for discrete-time linear systems. *Automatica*, 50(11):2835–2839, 2014.

[5] J. Chen, R.J. Patton, and Hong-Yue Zhang. Design of unknown input observers and robust fault detection filters. *International Journal of Control*, 63(1):85–105, 1996.

[6] P. Guerra, V. Puig, and M. Witczak. Robust fault detection with unknown-input interval observers using zonotopes. In *Proceedings of the 17th World Congress, The International Federation of Automatic Control*, COEX, Seoul, South Korea, July 2008.

[7] Stefen Hui and Stanisaw H. Żak. Observer design for systems with unknown inputs. *International Journal of Applied Mathematics and Computer Science*, 15(4):431–446, 2005.

[8] T.G. Park. Designing fault detection observers for linear systems with mismatched unknown inputs. *Journal of Process Control*, 23(8):1185–1196, 2013.

[9] V. Puig, J. Quevedo, T. Escobet, and A. Stancu. Passive robust fault detection using linear interval observers. In *IFAC Safe Process*, Washington, USA, 2003.

[10] M.M. Serona, J.A. De Doná, and J.J. Martinez. Invariant set approach to actuator fault tolerant control. In *the 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, Barcelona, Spain, 2009.

[11] F. Xu. *Diagnosis and Fault-tolerant Control using Set-based Methods*. PhD thesis, Technical University of Catalonia, Barcelona, Catalonia, Spain, November 2014.

[12] F. Xu, V. Puig, C. Ocampo-Martinez, S. Olaru, and F. Stoican. Set-theoretic methods in robust detection and isolation of sensor faults. *International Journal of Systems Science*, pages 1–18, 2014.