

# Different architectures to develop repetitive controllers

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Abstract: In this work several architectures to implement repetitive controllers are compared. A complete analytical analysis is performed for these different architectures and a simulation example with a power converter is included.

Keywords: Repetitive Control, Disturbance observer, Youla parametrization, inverter control

## 1. INTRODUCTION

Repetitive Control (RC) (Wang et al., 2009; Longman, 2010; Chen et al., 2008; Ahn et al., 2007) is an Internal Model Principle (IMP) (Francis and Wonham, 1976; Costa-Castelló et al., 2012) based control technique. RC it is specially suited for systems subject to periodical references and disturbances (Ramos et al., 2013). During last years this technique has been applied in different fields like power electronics (Ramos et al., 2013) or mechatronic systems (Park et al., 2005) among others.

The idea behind RC is to include inside the control-loop the generator of a periodical signal, which is a high order dynamic systems. Applying conventional stabilizing techniques to this type of systems would imply obtaining very high order controllers which would entail huge computational resources to implement these controllers. Due to this, RC uses specific architectures and anti-windup techniques (Ramos and Costa-Castelló, 2013; Wang, 2016) which allow to take profit from the steady-state nice properties of RC and obtain implementation structures which allow reducing computational requirements to a minimum.

Since the seminal work by Inoue et al. (1982), which introduced the plug-in architecture for RC, many architectures have been proposed, both in the input-output (Chen and Tomizuka, 2014, 2015) and the state-space (Wu et al., 2014) formalisms. In this work most relevant input-output based architectures are reviewed and compared. The comparison is illustrated in the case of a power inverter.

This paper is organized as follows: section 2 describes most relevant concepts about internal models used in RC, in section 3 the series architecture is described, in section 4 the plug-in structure is introduced, in section 5 a disturbance observer approach is used, in section 6 the controller parametrization is used, in section 7 a numerical

example is presented and finally section 8 provides some discussion about the different approaches.

## 2. INTERNAL MODEL

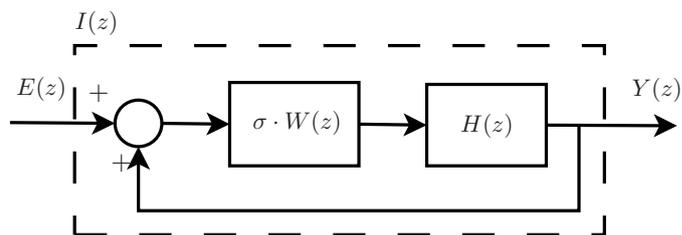


Figure 1. Internal model used in Repetitive Control.

The internal model is the core element of a RC system and it is composed by an element which can generate a periodic signal. Figure 2 shows the structure of a generic internal model used in RC:

$$I(z) = \frac{\sigma H(z)W(z)}{1 - \sigma H(z)W(z)},$$

where  $\sigma = \{-1, 1\}$ ,  $W(z)$  is the time delay function, and  $H(z)$  is a low-pass filter. As an example, for  $\sigma = 1$ ,  $W(z) = z^{-N}$  and  $H(z) = 1$  a N-periodic generator,  $I(z) = \frac{1}{z^N - 1}$ , is obtained and for  $\sigma = -1$ ,  $W(z) = z^{-\frac{N}{2}}$  and  $H(z) = 1$ , the odd-harmonic generator (Griño and Costa-Castelló, 2005) is obtained  $I(z) = \frac{-1}{z^{\frac{N}{2}} + 1}$ .

For those internal models obtained with  $H(z) = 1$ , the internal model,  $I(z)$ , introduces infinite gain at the signal frequency and all its harmonics. This high gain at high frequencies might be a problem in the presence of uncertainty. To reduce this gain a low-pass filter,  $H(z)$ , is usually used. A null-phase low pass filter is usually used (Griño and Costa-Castelló, 2005; Escobar et al., 2014).

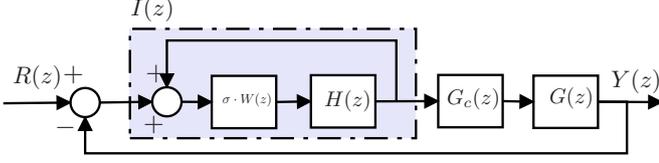


Figure 2. RC : Series approach.

### 3. SERIES APPROACH

The first approach to RC consists in placing the internal model in series connection with the plant. Additionally a stabilizing controller needs to be introduced to guarantee closed-loop stability (Figure 2). In this case the controller becomes:

$$C(z) = I(z)G_c(z)$$

the complementary sensitivity and sensitivity function become:

$$T(z) = \frac{I(z)G_c(z)G(z)}{1 + I(z)G_c(z)G(z)}$$

$$S(z) = \frac{1}{1 + I(z)G_c(z)G(z)}.$$

Usually, for minimum-phase plants

$$G_c(z) = \frac{k_r}{G(z)} \quad (1)$$

and :

$$T(z) = \frac{k_r \sigma W(z)H(z)}{1 + (k_r - 1)\sigma W(z)H(z)}$$

$$S(z) = \frac{1 - \sigma W(z)H(z)}{1 + (k_r - 1)\sigma W(z)H(z)}.$$

As expected,  $S(z)$  has in the numerator the denominator of  $I(z)$ . For non-minimum phase plants it is necessary to use phase cancellation techniques (Ye et al., 2009; Tomizuka, 1987) when designing  $G_c(z)$ .

In case of systems with multiplicative uncertainty:  $G(z) = G_n(1 + W_u^m(z)\Delta(z))$  (Sánchez-Peña and Szañer, 1998), the robust stability condition is:

$$\left\| W_u^m(z) \frac{k_r \sigma W(z)H(z)}{1 + (k_r - 1)\sigma W(z)H(z)} \right\|_{\infty} < 1.$$

### 4. PLUG-IN APPROACH

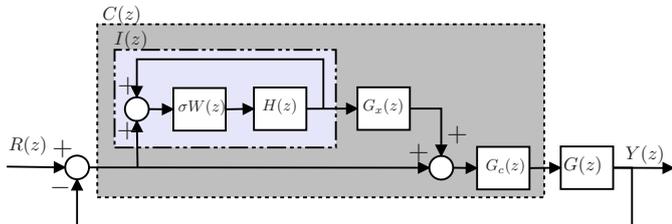


Figure 3. RC : Plug-in approach.

The most popular approach in RC is the plug-in architecture (Figure 3). This architecture introduces the internal model to a previously existing control system defined by an internal controller  $G_c(z)$  and the plant  $G(z)$ . The goal of

this internal controller is to guarantee closed-loop stability and robustness to the control system without the internal model. Later, the internal model  $I(z)$ , and the stabilizing controller,  $G_x(z)$ , are plugged in to the previous closed-loop system.

In this case the closed-loop transfer function can be constructed in terms of the closed-loop transfer function without the internal model:

$$S_o(z) = \frac{1}{1 + G_c(z)G(z)}, T_o(z) = \frac{G_c(z)G(z)}{1 + G_c(z)G(z)}$$

and a modifying term:

$$S_{Mod}(z) = \frac{1 - \sigma W(z)H(z)}{1 - \sigma W(z)H(z)(1 - G_x(z)T_o(z))}.$$

So the closed-loop transfer functions are:

$$S(z) = S_o(z)S_{Mod}(z)$$

$$T(z) = \frac{(1 - \sigma W(z)H(z)(1 - G_x(z)))T_o(z)}{1 - \sigma W(z)H(z)(1 - G_x(z)T_o(z))}.$$

For minimum-phase plants<sup>1</sup> the most popular form of for the stabilizing controller is:

$$G_x(z) = \frac{k_r}{T_o(z)}. \quad (2)$$

With this selection the closed-loop function becomes:

$$S(z) = S_o(z) \frac{1 - \sigma W(z)H(z)}{1 + (k_r - 1)\sigma W(z)H(z)}$$

$$T(z) = \frac{(T_o(z) - \sigma W(z)H(z)(T_o(z) - k_r))}{1 + (k_r - 1)\sigma W(z)H(z)}.$$

The following two conditions guarantee closed-loop stability (Ramos et al., 2013):

- (1)  $T_o(z)$  must be stable ( $G_c(z)$  can be designed to fulfill it)
- (2)  $\|W(z)H(z)(1 - k_r)\|_{\infty} < 1$  ( $k_r$  can be selected appropriately)

Even though these conditions are only sufficient it has been proved that they are close to the necessary ones in practice (Songschon and Longman, 2003).

It is important to emphasize that in (2) the inversion of  $T_o(z)$  is required while in (1) it is required the inversion of  $G(z)$ , as  $T_o(z)$  is a closed-loop system its uncertainty should be less than that of  $G(z)$ . Additionally, it is important to visualize that the sensitivity function in the series approach and the plug-in one are the same except the  $S_o(z)$  term.

In case of multiplicative uncertainty the robust stability condition becomes:

$$\left\| W_u^m(z) \frac{(T_o(z) - \sigma W(z)H(z)(T_o(z) - k_r))}{1 + (k_r - 1)\sigma W(z)H(z)} \right\|_{\infty} < 1.$$

This condition is quite similar to the one obtained in the series approach but it contains  $T_o(z)$ . This term can be shaped using  $G_c(z)$ , so it is simpler to fulfill this constraint than the one obtained in the series approach.

<sup>1</sup> For nonminimum phase plants a phase cancellation approach is usually used

## 5. DISTURBANCE REJECTION APPROACH

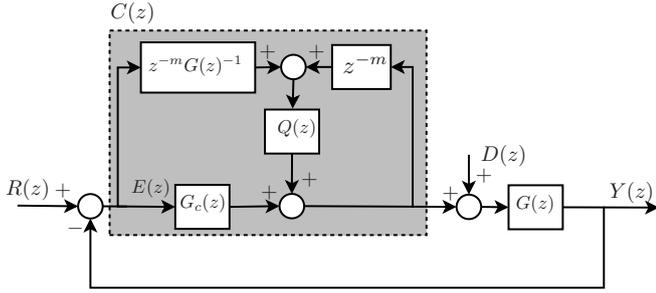


Figure 4. RC : Disturbance rejection approach.

During last years a great effort in the disturbance rejection mechanisms has been made (Chen et al., 2016). In this context a RC has been proposed based on this methodology (Chen and Tomizuka, 2014) and its characteristics are shown in this section. The controller scheme for an  $m$ -relative degree minimum phase plant,  $G(z)$ , is shown in Figure 4. This system uses a baseline controller,  $G_c(z)$ , plus a disturbance observer composed by the plant model and a filter,  $Q(z)$ . The sensitivity function for this closed-loop system is:

$$S(z) = \frac{1 - z^{-m}Q(z)}{1 + G(z)G_c(z)}.$$

In order to transform this in a RC it is necessary to choose  $Q(z)$  appropriately, so that the denominator of the internal model,  $I(z)$ , appears in the numerator of  $S(z)$ . Consequently,  $Q(z)$  must be isolated from the equation:

$$\frac{1 - z^{-m}Q(z)}{1 + G(z)G_c(z)} = S_o(z) (1 - \sigma W(z)H(z)) \frac{N_s(z)}{D_s(z)}$$

where  $N_s(z)$  and  $D_s(z)$  are polynomials that must be fixed (this selection requires  $S_o(z)$  to be stable, which can be achieved by selecting an appropriate  $G_c(z)$ ). So:

$$Q(z) = z^m \frac{(1 - \sigma W(z)H(z)) N_s(z) - D_s(z)}{D_s(z)}.$$

A simple option is choosing  $N_s(z) = 1$  and  $D_s(z) = 1 - \alpha \cdot \sigma \cdot W(z)H(z)$  with  $|\alpha| < 1$  which generates:

$$Q(z) = z^m \frac{-(1 + \alpha) \cdot \sigma W(z)H(z) N_s(z)}{1 - \alpha \cdot \sigma \cdot W(z)H(z)}.$$

Although this is not the conventional shape of  $Q(z)$  in disturbance observer based control it allows to reject periodical signals and behave as RC. Finally the controller becomes:

$$C(z) = \frac{G_c(z) + Q(z)z^m G(z)^{-1}}{1 - z^{-m}Q(z)}$$

This controller has a degree of freedom which corresponds to the value of  $\alpha$ . This value plays a similar role to  $k_r$  in the plug-in approach.

In case of multiplicative uncertainty the robust stability condition becomes:

$$\|W_u^m(z)T(z)\|_\infty < 1,$$

with

$$T(z) = 1 - S_o(z) \frac{1 - \sigma W(z)H(z)}{1 - \alpha \cdot \sigma W(z)H(z)}.$$

Clearly, other selections for  $N_s(z)$  and  $D_s(z)$  are possible. Although these other options might provide additional

degrees of freedom they might increase the controller complexity.

## 6. YOULA PARAMETRIZATION

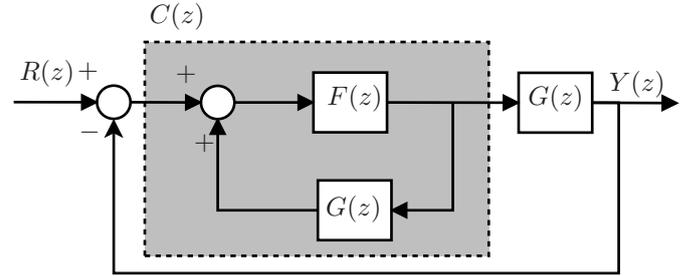


Figure 5. RC : Youla parametrization.

It is well-known that all stabilizing controllers for a given stable plant,  $G(z)$ , can be written in terms of the Youla parametrization (Sánchez-Peña and Sznajer, 1998) shown in Figure 5. In this case, the sensitivity and complementary sensitivity functions are:

$$S(z) = 1 - F(z)G(z), T(z) = F(z)G(z)$$

where  $F(z)$  is a stable system. And the controller is  $C(z) = \frac{F(z)}{1 - F(z)G(z)}$ .

In case of minimum-phase plants it is possible to select  $F(z) = F'(z)G^{-1}(z)$ , so the closed-loop functions become:

$$S(z) = 1 - F'(z), T(z) = F'(z).$$

In order to impose that the controller behaves as a RC it is necessary to impose the appropriate shape for  $F'(z)$ . It is necessary that:

$$S(z) = 1 - F'(z) = (1 - \sigma W(z)H(z)) \frac{N_s(z)}{D_s(z)}$$

with  $N_s(z)$  and  $D_s(z)$  arbitrary elements. So

$$F'(z) = \frac{D_s(z) - (1 - \sigma W(z)H(z)) N_s(z)}{D_s(z)}.$$

A simple solution is  $N_s(z) = 1$  and  $D_s(z) = 1 - \alpha \cdot \sigma \cdot W(z)H(z)$  with  $|\alpha| < 1$ , which generates:

$$F'(z) = \frac{(1 - \alpha) \cdot \sigma \cdot W(z)H(z)}{1 - \alpha \cdot \sigma \cdot W(z)H(z)}.$$

This option generates the following closed-loop transfer functions:

$$T(z) = \frac{(1 - \alpha) \cdot \sigma \cdot W(z)H(z)}{1 - \alpha \cdot \sigma \cdot W(z)H(z)}$$

$$S(z) = \frac{1 - \sigma \cdot W(z)H(z)}{1 - \alpha \cdot \sigma \cdot W(z)H(z)}.$$

Which can be considered a generalized version of the series approach (section 3).

Finally, robust stability condition for multiplicative uncertainty is :

$$\|W_u^m(z)T(z)\|_\infty < 1.$$

Clearly, this approach can generate more complex closed-loop transfer function, in particular the closed-loop poles

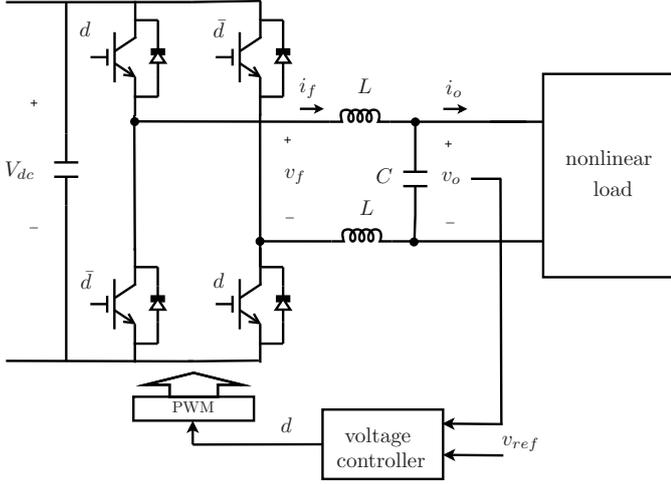


Figure 6. Circuit diagram.

could be arbitrarily placed by choosing an appropriate value for  $D_s(z)$ . Alternatively, these degrees of freedom could be used to optimize any criteria such as robustness. This increase in the controller complexity would increase the computational resources required to implement the controller.

## 7. NUMERICAL EXAMPLE

The numerical example is based on the system depicted in Figure 6. It consists of a Voltage Source Inverter (VSI) which transforms a DC supply,  $V_{dc}$ , in an AC power source,  $v_o$ . The duty cycle,  $d$ , of a Pulse Width Modulation (PWM) signal is the control action used to switch the power transistors and the produced signal is filtered by an LC network to obtain the output AC voltage  $v_o(t)$ . The control objective is providing a sinusoidal waveform voltage signal with specific amplitude and frequency ( $v_{ref}(t)$ ). At the same time, it is needed to reject disturbances caused by the load. Nonlinear loads, as the one produced by full-bridge diode rectifiers, are of special interest since they produce disturbances with high harmonic content. A way of measuring the system performance is through the Total Harmonic Distortion (THD), thus a low THD in voltage waveform ( $v_o(t)$ ) is desirable.

The system transfer function is based on the LC filter circuit equations to obtain:

$$V_o(s) = G_p(s)V_f(s) + G_d(s)I_o(s), \quad (3)$$

with

$$G_p(s) = \frac{R_C}{2LCR_Cs^2 + (2CR_LR_C + 2L)s + (2R_L + R_C)}, \quad (4)$$

and

$$G_d(s) = -\frac{2LR_C + 2R_L + R_C}{2LCR_Cs^2 + (2CR_LR_C + 2L)s + (2R_L + R_C)}, \quad (5)$$

where  $G_p(s)$  is the plant transfer function,  $G_d(s)I_o(s)$  is the disturbance signal caused by the load,  $L = 300 \mu\text{H}$  and  $C = 80 \mu\text{F}$  are the inductive and capacitive part of the filter,  $R_L = 0.1 \Omega$  and  $R_C = 8200 \Omega$  are the parasitic resistance of inductance and capacitance respectively.

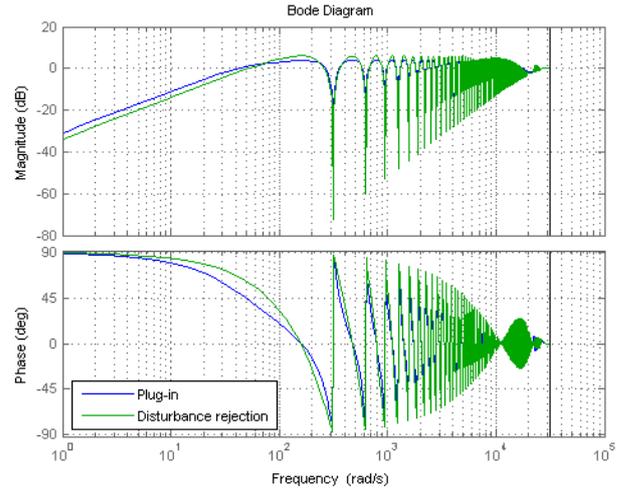


Figure 7. Sensitivity function for RC based on plug-in and disturbances observer approaches.

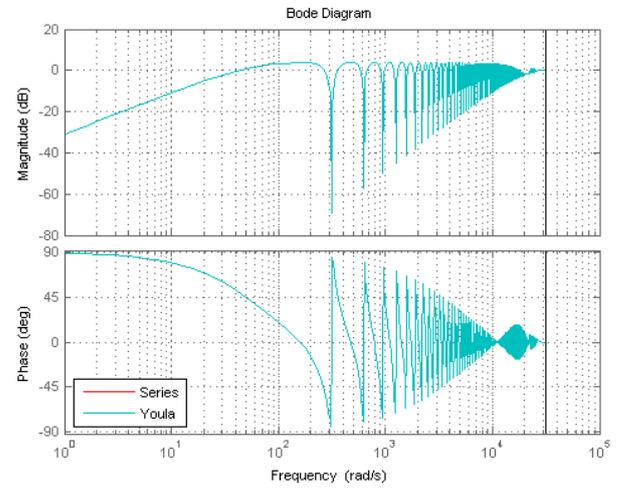


Figure 8. Sensitivity function for RC based on series and Youla parametrization approaches.

Additionally, the control action is  $V_f(s) = (2d - 1)V_{dc}$ , where  $d \in [0, 1]$  is the duty cycle of the PWM signal and  $V_{dc}$  is the DC input source. The PWM signals has a frequency of 10KHz, and the duty cycle is updated each period, so a sampling time of  $T_s = 0.0001\text{s}$  is used.

Although the inverter is designed to feed generic loads, it is difficult to design a control system without some assumptions about the concrete load being used. The load has been modeled by means of an inductance and a resistor in series placed in parallel with the capacitor. For this reason, an impedance  $Z(s)$  to relate the current introduced to the load  $I_o(s)$  and the voltage supplied to it  $V_o(s)$ . This impedance is as can be seen in equations (6) and (7):

$$I_o(s) = Z(s)V_o(s) \quad (6)$$

$$Z(s) = \frac{1}{L \cdot s + R_L} \quad (7)$$

Taking this into account the nominal model is constructed as:

$$G_n(s) = \frac{G_p(s)}{1 + G_d(s)Z(s)}.$$

This model will be the one used to design the controller.

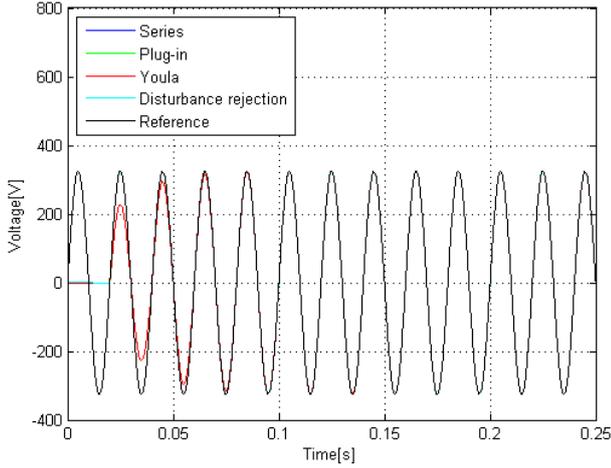


Figure 9. Bode diagram of  $1/T(z)$  for all four approaches.

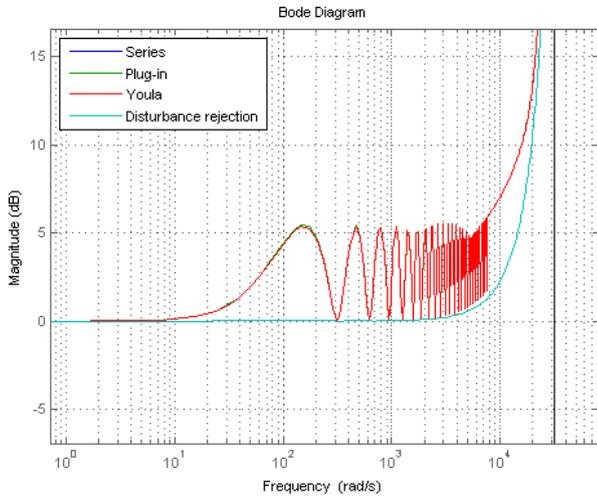


Figure 10. Bode diagram of  $1/T(z)$  for all four approaches.

The goal in this type of system is feeding the load with a sinusoidal voltage of frequency  $f = 50$  Hz and amplitude  $220\sqrt{2}$  V. Consequently, the period signal of to be tracked is  $T_p = \frac{1}{50} = 0.02$ s. Consequently, the discrete time period,  $N$ , can be computed as  $N = T_p/T_s = 200$ . This allows to obtain a good reconstruction of the continuous time signal.

In the following, 4 different RC systems, corresponding to the previously introduced architectures will be designed. In each approach different parameters have been tuned.

In the series (section 3) and the plug-in (section 4) approaches,  $k_r$  is the most relevant degree of freedom. It is related with the most relevant closed-loop poles. Approximately, these are the solutions of  $z^N = 1 - k_r$ . These poles are homogeneously distributed over a circle of radius  $|1 - k_r|$ , which directly defines the closed-loop settling time (Yeol et al., 2008; Garimella and Srinivasan, 1994). Due to this, the selection of  $k_r$  is a trade-off between settling time and robustness. A reasonable value is  $k_r = 0.7$ .

In the disturbance rejection (section 5) and the Youla parametrization (section 6) a parameter called  $\alpha$  is introduced. Similarly to  $k_r$ ,  $\alpha$  is directly related with the

closed-loop poles. In fact, a relation between  $k_r$  and  $\alpha$  can be established,  $\alpha = 1 - k_r$ , so that the closed-loop poles from different architectures are placed in the same location. For consistency with previous cases,  $\alpha = 0.3$  is chosen.

An element present in all these architectures is the low-pass filter. This element's most relevant goal is introducing robustness in the high frequency range. In this work the following filter has been used:

$$H(z) = \frac{0.25z^2 + 0.5z + 0.25}{z}, \quad (8)$$

it has null-phase, a gain close to 1 in low and medium frequency range and a important attenuation in the high frequency range.

Finally, the plug-in approach (section 4) and disturbance rejection one (section 5) contain an internal controller in charge of providing robustness. To prevent increasing the complexity and allowing a better comparison a proportional controller like the following one has been selected:

$$G_c(z) = 0.1. \quad (9)$$

Taking this into consideration and noting that the discrete-time plant obtained when applying the z-transform to  $G_n(s)$  is minimum-phase plant, all the controllers introduced in this work are completely defined. Figure 7 shows the frequency response of the sensitivity function for the plug-in approach and the disturbance observer one. As it can be seen, both responses are similar, have small values for the working frequency and all the harmonics. The effect of the filter can also be seen, the small values at harmonics increase as frequency increases. Additionally, as the sensitivity function does not take values over 6dB it can be stated that both of them are quite robust.

Figure 8 shows the sensitivity function for the series and Youla architectures. As it can be seen, both schemes provide the same result and it is quite similar to the one observed in Figure 7.

Figure 9 shows the closed-loop time response for all described architectures when a sinusoidal reference of the working frequency is introduced in the system. As it can be seen, after a small transient all closed-loop schemes converge to the reference with null-steady state error. No relevant differences can be observed.

Finally, the attention is turned to robustness issues. In this type of application, the real load is not know. A way to analyze the robustness of the control system against changes in the load characteristics is to model the variation in the load as a multiplicative uncertainty  $W_u^m(z)$  in the plant and perform a robust stability analysis. The robust stability condition is  $\|W_u^m(z)T(z)\|_\infty < 1$ , which can be analyzed frequency by frequency as :

$$|W_u^m(e^{j\omega T_s})| < \frac{1}{|T(e^{j\omega T_s})|}. \quad (10)$$

Figure 10, shows the frequency response of  $T(z)^{-1}$ . As it can be seen it is always over 0dB in all the frequency range, and specially in the high frequency range. Consequently, a 100% change in the impedance at each frequency can be handled. It can be stated that the closed-loop system is very robust. Additionally, it is possible to see that all architectures provide a similar robustness.

## 8. CONCLUSIONS

In this paper most relevant architectures used to implement RC in input-output form have been reviewed and analyzed for the case of minimum-phase plants and its applications to the case of an inverter has been illustrated.

As it can be shown, most architectures achieve similar results. Obtaining good steady-state results and reasonably robustness conditions. Although it has not been shown in this work, the plug-in approach and the disturbance observer contains an additional degree of freedom which can be used to slightly improve robustness and transient behavior. Youla parametrization is the most generic by far because it can be used to reduce all the other approaches. Finding new approaches based on Youla parametrization which improve transient and robustness with a limited complexity increase is a current research area.

## ACKNOWLEDGEMENTS

This work was partially supported by the spanish Ministerio de Educación project DPI2015-69286-C3-2-R (MINECO/FEDER) and the catalan AGAUR project 2014 SGR 267.

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