Optimal Management of Barcelona Water Distribution Network using Non-linear Model Predictive Control ⋆

Y. Wang * **G. Cembrano** *,** **V. Puig** *,*** **M. Urrea** *,***
J. Romera *** **D. Saporta** **** **J. G. Valero** **** **J. Quevedo** ***

* *Institut de Robòtica i Informàtica Industrial (IRI), CSIC-UPC, Universitat Politècnica de Catalunya, C/. Llorens i Artigas 4-6, 08028 Barcelona, Spain (e-mail: {ywang, cembrano, vpuig}@iri.upc.edu)*
** *CETaqua, Water Technology Centre, Ctra. d'Esplugues 75, Cornellà de Llobregat, 08940 Barcelona, Spain*
*** *Automatic Control Department, Universitat Politècnica de Catalunya, Rambla de Sant Nebridi 11, 08222 Terrassa, Spain*
**** *Aigües de Barcelona, C/. Consell de Cent 396, 08009 Barcelona, Spain*

Abstract: This paper presents a non-linear optimal control strategy for the operational management of water distribution networks (WDNs) including both flow and hydraulic head/pressure constraints. The optimal operation of WDNs should guarantee water supply with suitable pressures at all the demand nodes in the network. The challenge for non-linear model predictive control in this context is to compute control strategies for the pumps and valves in a WDN to supply the required demand while optimizing performance goals related to cost and safety. A two-layer scheme is used in order to produce set-points that can be directly sent to the actuators: on-off schedules for pumps and pressure set-points for pressures reducing valves. Finally, the results of applying the proposed control strategy to a portion of the Barcelona real WDN are provided.

Keywords: Model predictive control, pump scheduling, non-linear hydraulic model, water distribution networks, Barcelona real network.

1. INTRODUCTION

Water distribution networks (WDNs) are complex multi-variable systems since they can contain a large number of pressurized pipes, storage tanks, pumping stations, valves, water sources (superficial and underground) and sectors of consumer demand (Brdys and Ulanicki, 1994). Electrical energy is the main source of operational costs, both for water production and water elevation to adequate pressure levels for consumptions. Recently, depending on the pumping station, its own importance, and the area of demand it covers, different bilateral contracts are established with energy supply companies, with a variety of prices and different cost periods (from two periods up to six different price periods per day, depending on working days/weekends, and on seasons). Accordingly, the current practice is to pre-allocate the pumping periods of each station when the energy prices arranged by the contract are the lowest possible for that station, guaranteeing that expected demand is satisfied, with the help of intermediate water storage capacity.

The optimal control strategies and optimisation techniques including predictive capabilities provide a suitable framework for the efficient management strategy of WDNs when also pressure constraints are considered. In particular, model predictive control (MPC) (Rawlings and Mayne, 2009) is an appropriate strategy to find optimal flow set-points for all the actuators while taking into account hydraulic heads in some specific points, as e.g., the inlet to demand sectors (Cembrano et al., 2000, 2011). The optimal strategies are computed by optimising a cost function describing all the operational objectives in a given prediction horizon subject to a representative model of the network dynamics (including flow and hydraulic head equations), as well as the required constraints on system variables. MPC is quite suitable to be used in the global control of networks related to the urban water cycle within a hierarchical control structure (Ocampo-Martinez et al., 2013). However, in Ocampo-Martinez et al. (2013); Grosso et al. (2014); Wang et al. (2015, 2016a), MPC strategy was successfully applied to the WDN using a control-oriented model that considers only flows, i.e., the pressure/head model of each element in the WDN (including water storage tanks/reservoirs, water demand sectors, pressurised

⋆ This work has been partially funded by the European Union through the project EFFINET (ref. FP7-ICT-2011-8-318556), by the Spanish Government and FEDER through the projects CICYT ECOCIS (ref. DPI2013-48243-C2-1-R), CICYT DEOCS (ref. DPI2016-76493-C3-3-R) and CICYT HARCRICS (ref. DPI2014-58104-R).

pipes, booster pumps and pressure/flow-controlled valves) is not considered explicitly. But, for general WDN, in order to satisfy water demands, it is also necessary to meet the required pressure/head at each water demand sector, which implies adding a set of non-linear constraints to the optimisation problem.

This paper presents a two-layer control scheme including a non-linear MPC (NMPC) strategy and a pump scheduling approach. The NMPC strategy is implemented by solving a non-linear optimisation problem to generate flow and head set-points for pumps and valves to transport water from the sources to the consumer areas meeting future water demands with appropriate pressures, while optimising a performance index related to operational goals such as economy and safety. The proposed optimal control strategy is applied to a high-fidelity simulator model of a portion of Barcelona WDN and compared with the current experience based operational strategies.

The remainder of this paper is structure as follows: In Section 2, the modelling methodology of components in WDNs is summarised. In Section 3, the problem statement is presented. In Section 4, the two-layer optimal control strategy including the NMPC strategy and the pump scheduling approach is proposed. In Section 5, a real application to Barcelona WDN is shown. In Section 6, some conclusions are presented.

2. CONTROL-ORIENTED MODELLING OF WDNS

2.1 Storage Tanks/Reservoirs

The mass balance expression related to the stored volume v in the m-th tank can be written in discrete-time as

$$v_m(k+1) = v_m(k) + \Delta t \left(\sum_i q_{i,m}^{\text{in}}(k) - \sum_j q_{m,j}^{\text{out}}(k) \right), \tag{1}$$

where $q_{i,m}^{\text{in}}(k)$ denotes the inflows from the i-th element to the m-th tank and $q_{m,j}^{\text{out}}(k)$ denotes the outflows from the m-th tank to the j-th element. Δt is the sampling time and k is the discrete-time instant. The physical limitation related to the storage volume in the m-th tank is described as

$$\underline{v}_m \leq v_m(k) \leq \overline{v}_m, \quad \text{for all } k, \tag{2}$$

where $\underline{v}_m$ and $\overline{v}_m$ denote the minimum and maximum admissible storage, respectively.

Besides, the hydraulic head related to the m-th tank can be formulated as

$$h_m(k) = \frac{v_m(k)}{\mathrm{S}_m} + E_m, \tag{3}$$

where S_m is the cross-sectional area of the m-th tank and E_m corresponds the m-th tank elevation.

2.2 Pumping Stations and valves

For a WDN, the parallel pumps are usually set in a pumping stations. In many WDNs, fixed-speed pumps are widely used. Since these pumps are operated in an ON/OFF manner, representing this behaviour directly would involve including discrete variables in the optimisation problem. The two layer scheme of this work solves this problem indirectly.

Firstly, pump and valve flows are chosen as the manipulated variables. Initially, pump flows are considered as continuous variables within a range of admissible values. Therefore, the physical limitations for pumps can be regarded as input constraints, which can be expressed as

$$\underline{q}_{u_n} \leq q_{u_n}(k) \leq \overline{q}_{u_n}, \quad \text{for all } k, \tag{4}$$

where q_{u_n} represents the manipulated flow of the n-th pump (or valve), $\underline{q}_{u_n}$ and $\overline{q}_{u_n}$ denote the minimum and maximum flow capacity of the n-th pump, respectively.

Moreover, the hydraulic characteristics are bounded by the following constraints:

$$\Delta h_p = h_d - h_s \geq 0, \tag{5a}$$
$$h_d \in \left[\underline{h}_d, \overline{h}_d\right], \tag{5b}$$
$$h_s \in \left[\underline{h}_s, \overline{h}_s\right], \tag{5c}$$

where h_d and h_s are called the *suction head* and the *delivery head*, respectively, with the physical limitation of $h_d \geq h_s$. Moreover, $\underline{h}_d$ and $\underline{h}_s$ denote the minimum values of the suction and delivery heads. $\overline{h}_d$ and $\overline{h}_s$ denote the maximum values of the suction and delivery heads.

Similarly, the static relationship between head and flow in a valve is constrained as follows:

$$\Delta h_v = h_{us} - h_{ds} \geq 0, \tag{6a}$$
$$h_{us} \in \left[\underline{h}_{us}, \overline{h}_{us}\right], \tag{6b}$$
$$h_{ds} \in \left[\underline{h}_{ds}, \overline{h}_{ds}\right], \tag{6c}$$

where h_{us} and h_{ds} denotes heads of valves in the upstream and downstream, respectively. $\underline{h}_{us}$ and $\underline{h}_{ds}$ denote the minimum values of the upstream and downstream heads. $\overline{h}_{us}$ and $\overline{h}_{ds}$ denote the maximum values of the upstream and downstream heads.

2.3 Nodes

The expression of the mass conservation in (non-storage) nodes can be written as

$$\sum_i q_{i,l}^{\text{in}}(k) = \sum_j q_{l,j}^{\text{out}}(k), \tag{7}$$

where $q_{i,l}^{\text{in}}$ represents the non-manipulated inflow through l-th node from the i-th element and $q_{l,j}^{\text{out}}$ represents the non-manipulated outflow through l-th node to the j-th element.

2.4 Water Demand Sectors

A water demand sector represents water demand of the network users of a certain physical area. At a certain time instant k, the consumed water in the r-th demand sector can be expressed as $d_r(k)$. Since the optimal control strategy must be predictive, short-term demand forecasts are required. These may be obtained using a suitable demand forecasting algorithm, such as (Quevedo et al., 2014; Wang et al., 2016b).

2.5 Interconnected Pipes

Water flows inside pressurized pipes occurs because of a difference in the hydraulic head at both ends of the pipe.

The head-flow relationship has been described experimentally through several approximations, as follows:

$$q_{i,j} = \Phi_{i,j}(h_i - h_j), \tag{8}$$

where $\Phi_{i,j}$ is a non-linear, monotonous function, such as a power function. One widely used approximation is the *Hazen-Williams* formula. According to this approximation, the headloss Δh_d through a pipe can be calculated as

$$\Delta h_d = h_i - h_j = R_{i,j} q_{i,j} \left| q_{i,j} \right|^{0.852}, \tag{9}$$

with

$$R_{i,j} \triangleq \frac{10.67 L_{i,j}}{C_{i,j}^{1.852} D_{i,j}^{4.87}},$$

where $L_{i,j}$, $D_{i,j}$ and $C_{i,j}$ denote the pipe length, diameter and roughness coefficient, respectively.

3. PROBLEM STATEMENT

Considering the modelling methodology of each component in a WDN, the control-oriented model of WDNs can be formulated by means of a set of differential algebraic equations (DAEs). The generalised discrete-time WDN model can be mathematically formulated as follows

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{\Xi}\boldsymbol{\mu}(k) + \mathbf{B}_d \mathbf{d}(k), \tag{10a}$$

$$0 = \mathbf{\Theta}\boldsymbol{\mu}(\boldsymbol{k}) + \mathbf{E}_d \mathbf{d}(k), \tag{10b}$$

$$0 = \mathbf{\Pi}\boldsymbol{\tau}(k) + \boldsymbol{\psi}\big(\mathbf{w}(k)\big), \tag{10c}$$

with

$$\mathbf{\Xi} \triangleq \begin{bmatrix}\mathbf{B}_u & \mathbf{B}_w\end{bmatrix}, \mathbf{\Theta} \triangleq \begin{bmatrix}\mathbf{E}_u & \mathbf{E}_w\end{bmatrix}, \mathbf{\Pi} \triangleq \begin{bmatrix}\mathbf{P}_z & \mathbf{P}_x\end{bmatrix},$$

$$\boldsymbol{\mu}(k) \triangleq \begin{bmatrix}\mathbf{u}(k)^T & \mathbf{w}(k)^T\end{bmatrix}^T, \boldsymbol{\tau}(k) \triangleq \begin{bmatrix}\mathbf{x}(k)^T & \mathbf{z}(k)^T\end{bmatrix}^T.$$

where $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^m$ represents the vector of hydraulic heads at storage tanks as system states, $\mathbf{z} \in \mathcal{Z} \subseteq \mathbb{R}^z$ represents the vector of hydraulic heads at non-storage nodes as algebraic states, $\mathbf{u} \in \mathcal{U} \subseteq \mathbb{R}^n$ denotes the vector of manipulated flow variables as control inputs, $\mathbf{w} \in \mathcal{W} \subseteq \mathbb{R}^w$ denotes the vector of non-manipulated flow variables and $\mathbf{d} \in \mathcal{D} \subseteq \mathbb{R}^d$ corresponds to the vector of water demands. $k \in \mathbb{N}$ denotes the time instant. All the considered variables of the WDN are assigned into control-oriented variables as summarised in Table 1. Moreover, $\mathbf{A}$, $\mathbf{B}_u$, $\mathbf{B}_w$, $\mathbf{B}_d$, $\mathbf{E}_u$, $\mathbf{E}_w$, $\mathbf{E}_d$, $\mathbf{P}_x$ and $\mathbf{P}_z$ are system matrices of appropriate dimensions. $\boldsymbol{\psi}(\cdot)$ denotes the vector of non-linear *Hazen-Williams* mapping functions.

Table 1. Variable assignments in the control-oriented model of the WDN

Type of variable	Related symbols
System states $\mathbf{x}$	h_m
Algebraic states $\mathbf{z}$	h_d, h_s, h_i, h_j
Control inputs $\mathbf{u}$	q_{u_n}
Non-control inputs $\mathbf{w}$	$q_{i,j}$
System disturbances $\mathbf{d}$	d_r

The control objectives for the management of WDNs are considered as (Ocampo-Martinez et al., 2013):

(1) *Economic*: To provide a reliable water supply with the required pressure minimising both water production and transport costs.
(2) *Safety*: To guarantee the availability of enough water in each storage tank to satisfy future stochastic water demands.
(3) *Smoothness*: To operate the WDN with smooth control actions.

These objectives are prioritized according to exploitation rules, using weights in the cost function. In general, safety is the first priority. The cost function related to the economic objective can be formulated as

$$\ell_1(k) \triangleq \big(\alpha_1 + \alpha_2(k)\big)^T \mathbf{u}(k), \tag{11}$$

where α_1 denotes the single-column vector of static economic costs of the water depending on the selected water sources and $\alpha_2(k)$ represents the vector of the time-varying electrical costs. Considering the variable daily electrical tariff, $\alpha_2(k)$ is time-varying.

For the purpose of maintaining the water supply in spite of the variation of water demands between two consecutive sampling steps, a suitable safety head for each storage tank must be maintained. Hence, the mathematical expression for this objective is formulated in a quadratic way as

$$\ell_2(k) \triangleq \begin{cases} \|\mathbf{x}(k) - \mathbf{x}_S\|_2^2, & \text{if } \mathbf{x}(k) \leq \mathbf{x}_S, \\ 0, & \text{otherwise,} \end{cases} \tag{12}$$

where $\mathbf{x}_S$ denotes the vector of safety heads for all the tanks and $\|\cdot\|_2^2$ is the squared 2-norm symbol. This cost function can also be implemented by means of a soft constraint with adding a slack variable $\boldsymbol{\xi}(k)$, which can be reformulated as

$$\ell_2(k) \triangleq \|\boldsymbol{\xi}(k)\|_2^2, \tag{13}$$

with the following soft constraint:

$$\mathbf{x}(k) \geq \mathbf{x}_S - \boldsymbol{\xi}(k), \tag{14a}$$

$$\boldsymbol{\xi}(k) \geq 0. \tag{14b}$$

Then, the cost function for the smoothness objective can be written as

$$\ell_3(k) \triangleq \|\Delta\mathbf{u}(k)\|_2^2, \tag{15}$$

where

$$\Delta\mathbf{u}(k) \triangleq \mathbf{u}(k) - \mathbf{u}(k-1). \tag{16}$$

In general, the general multi-objective cost function that gathers all the control objectives for the operational management of the WDN can be summarised as

$$\mathcal{J}(k) = \sum_{j=1}^{\mathit{\Gamma}} \lambda_j \ell_j(k), \tag{17}$$

where λ_j denotes the weighting term that indicates the prioritisation of control objectives and $\mathit{\Gamma}$ is the number of the selected control objectives.

As mentioned above, physical limitations for some system variables can be generally formulated as follows:

$$\underline{\mathbf{x}} \leq \mathbf{x}(k) \leq \overline{\mathbf{x}}, \tag{18}$$

$$\underline{\mathbf{u}} \leq \mathbf{u}(k) \leq \overline{\mathbf{u}}, \tag{19}$$

where $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$ are minimum and maximum values of system states. $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ are minimum and maximum values of control inputs.

In addition, the heads at non-storage nodes are required to be up to some minimum levels as in the case of the water demand sectors. Hence, the following inequality constraint must be considered:

$$\mathbf{z}(k) \geq \underline{\mathbf{z}}_i, \tag{20}$$

where $\underline{\mathbf{z}}$ denotes the vector of required heads at the water demand sectors.

In practice, most of the pumps in WDN are operated in ON-OFF discrete way. Thus, the flows in $\mathbf{u}$ become discrete values. Hence, the general control strategy can be formulated into the following mixed-integer optimisation problem:

Problem 1. (Mixed-integer NMPC for WDNs).

$$\min_{u^*(k|k),\ldots,u^*(k+H_p-1|k)} \sum_{i=0}^{H_p-1} \mathcal{J}(k+i \mid k), \tag{21a}$$

subject to

$$\mathbf{x}(k+i+1 \mid k) = \mathbf{A}\mathbf{x}(k+i \mid k) + \boldsymbol{\Xi}\boldsymbol{\mu}(k+i \mid k) + \mathbf{B}_d\mathbf{d}(k+i \mid k), \tag{21b}$$
$$0 = \boldsymbol{\Theta}\boldsymbol{\mu}(k+i \mid k) + \mathbf{E}_d\mathbf{d}(k+i \mid k), \tag{21c}$$
$$0 = \boldsymbol{\Pi}\boldsymbol{\tau}(k+i \mid k) + \boldsymbol{\psi}\Big(\mathbf{w}(k+i \mid k)\Big), \tag{21d}$$
$$\underline{\mathbf{x}} \leq \mathbf{x}(k+i+1 \mid k) \leq \overline{\mathbf{x}}, \tag{21e}$$
$$\underline{\mathbf{u}} \leq \boldsymbol{\zeta}_j(k)\mathbf{u}(k+i \mid k) \leq \overline{\mathbf{u}}, \tag{21f}$$
$$\mathbf{z}(k+i \mid k) \geq \underline{\mathbf{z}} \tag{21g}$$
$$\mathbf{x}(k+i+1 \mid k) \geq \mathbf{x}_s - \boldsymbol{\xi}(i), \tag{21h}$$
$$\boldsymbol{\xi}(i) \geq 0, \tag{21i}$$
$$\boldsymbol{\zeta}(i) \in [0,1], \tag{21j}$$
$$\Big(\mathbf{x}(k|k), \mathbf{d}(k|k)\Big) = \Big(\mathbf{x}(k), \mathbf{d}(k)\Big), \tag{21k}$$

where $\boldsymbol{\zeta}(k) \in \{0,1\}$ represents the vector of binary decision variables that correspond to the ON/OFF pump status at time instant k, where 1 means the ON-status of the pump and 0 means the OFF-status of the pumps. The ON-status pumping flow should be estimated based on the pressure conditions that the pumps should establish.

Solving a large-scale non-linear mixed-integer optimisation problem with a long horizon leads *Problem 1* to be computationally incompatible with real-time constraints. For this reason, this mixed-integer optimisation problem is replaced by a two-layer optimal control strategy in this work.

4. TWO-LAYER OPTIMAL CONTROL STRATEGY FOR MANAGEMENT OF WDNS

4.1 Two-layer NMPC strategy

In the upper layer, the NMPC problem is considered. In this layer, the WDN is operated on an hourly basis ($\Delta t_u = 3600$ s) over a prediction horizon of 24 to 48 hours, to find the optimal flow set-points for pumps and valves. In the lower layer, the hourly flow set-points for pumps are translated into ON-OFF schedules within each hour. Then, the sampling time of this layer is one minute ($\Delta t_l = 60$ s), and each hourly set-point provided by the upper layer NMPC strategy produces 60 values using a pump scheduling approach.

Taking into account the WDN model including the flow and pressure part, a NMPC strategy can be implemented by solving a finite-horizon optimisation problem over a prediction horizon H_p. The multi-objective cost function is minimised subject to the prediction model (10) and a set of system constraints presented above. Thus, the optimisation problem behind the NMPC strategy can be formulated as follows:

Problem 2. (NMPC for WDNs).

$$\min_{u^*(k|k),\ldots,u^*(k+H_p-1|k)} J = \sum_{i=0}^{H_p-1} \sum_{j=1}^{\Gamma} \lambda_j J_j\,(k+i \mid k)\,, \tag{22a}$$

subject to

$$\mathbf{x}(k+i+1 \mid k) = \mathbf{A}\mathbf{x}(k+i \mid k) + \boldsymbol{\Xi}\boldsymbol{\mu}(k+i \mid k) + \mathbf{B}_d\mathbf{d}(k+i \mid k), \tag{22b}$$
$$0 = \boldsymbol{\Theta}\boldsymbol{\mu}(k+i \mid k) + \mathbf{E}_d\mathbf{d}(k+i \mid k), \tag{22c}$$
$$0 = \boldsymbol{\Pi}\boldsymbol{\tau}(k+i \mid k) + \boldsymbol{\psi}\Big(\mathbf{w}(k+i \mid k)\Big), \tag{22d}$$
$$\underline{\mathbf{x}} \leq \mathbf{x}(k+i+1 \mid k) \leq \overline{\mathbf{x}}, \tag{22e}$$
$$\underline{\mathbf{u}} \leq \mathbf{u}(k+i \mid k) \leq \overline{\mathbf{u}}, \tag{22f}$$
$$\mathbf{z}(k+i \mid k) \geq \underline{\mathbf{z}} \tag{22g}$$
$$\mathbf{x}(k+i+1 \mid k) \geq \mathbf{x}_s - \boldsymbol{\xi}(i), \tag{22h}$$
$$\boldsymbol{\xi}(i) \geq 0, \tag{22i}$$
$$\Big(\mathbf{x}(k|k), \mathbf{d}(k|k)\Big) = \Big(\mathbf{x}(k), \mathbf{d}(k)\Big). \tag{22j}$$

Since the control-oriented model of the WDN includes the non-linear head-flow equations. *Problem 2* may be solved using non-linear programming techniques. One efficient class of non-linear programming techniques is gradient based optimization, such as the generalized reduced gradient search (Smeers, 1977).

Assuming that *Problem 2* is feasible, the sequence of control actions can be expressed as

$$\mathbf{u}^*(k) = \Big[\mathbf{u}^*(k \mid k),\ \mathbf{u}^*(k+1 \mid k),\ \cdots,\ \mathbf{u}^*(k+H_p-1 \mid k)\Big]. \tag{23}$$

Then, by applying the receding-horizon strategy, the optimal control action at time instant k is the first component of the sequence of control actions denoted by

$$\mathbf{u}_{opt}(k) \triangleq \mathbf{u}^*(k \mid k). \tag{24}$$

4.2 Pump scheduling approach

In fact, the pumping flow of the i-th pump in the j-th pumping station is affected by the suction and delivery heads. Hence, the real-time pumping flow is located within an interval, which can be formulated as

$$q_{i,j}^{real} \in \Big[q_{i,j}^{nom} - \sigma_{i,j},\ \ q_{i,j}^{nom} + \sigma_{i,j}\Big]\,, \tag{25}$$

where $q_{i,j}^{nom}$ denotes nominal pumping flows produced through the pumps, and $\sigma_{i,j}$ represents the variance of the pumping flows depending on the uncertainty of the heads in terms of the pumps.

If the actual pumping flow $q_{i,j}^{real}$ is approximated by the nominal pumping flow $q_{i,j}^{nom}$, then the total ON-status time can be calculated by

$$T_{i,j}^{ON} = \frac{Q_j^{opt}\Delta t_u}{q_{i,j}^{nom}\Delta t_l}, \tag{26}$$

where Q_j^{opt} denotes the optimal hourly flow as the set-point of the j-th pumping station and $T_{i,j}^{ON}$ denotes the total ON-status time of the pump. In some cases, only one pump cannot provide enough flow to maintain the optimal flow set-point. Hence, parallel pumps are set in each pumping station. Therefore, (26) can be rewritten as

$$Q_j^{opt}\Delta t_u = \sum_{i=1}^{\chi_j} T_{i,j}^{ON} q_{i,j}^{nom}\Delta t_l, \tag{27}$$

where $\chi_j \in [0, \gamma_j] \subset \mathbb{Z}^+$ denotes the number of the opened parallel pumps and γ_j is the total number of the parallel pumps located in the j-th pumping station. Note that parallel pumps are assumed to be opened following a prioritisation.

The pump scheduling approach is able to find a sequence value of the number of the ON-status parallel pumps in each pumping station at each sampling time. Hence, the variable χ_j is naturally considered as an admissible time-series vector, which can be written as

$$\boldsymbol{\chi}_j = [\chi_j(0), \chi_j(1), \cdots, \chi_j(t)]^T, \tag{28}$$

where $\chi_j(t)$ denotes the number of ON-status parallel pumps at time instant t. The vector of χ_j along the horizon of H_l can be obtained by solving an optimisation problem as follows:

Problem 3. (Pump scheduling approach).

$$\min_{\boldsymbol{\chi}_1^*, \cdots, \boldsymbol{\chi}_\Lambda^*} J = \sum_{j=1}^{\Lambda} \|\delta_j\|_2^2, \tag{29a}$$

subject to

$$\delta_j = V_j^p - V_j^{opt}, \tag{29b}$$

$$V_j^p = \sum_{i=1}^{\chi_j(0)} q_{i,j}^{real} \Delta t_l + \sum_{t=1}^{H_l-1} \sum_{i=1}^{\chi_j(t)} q_{i,j}^{real} \Delta t_l, \tag{29c}$$

$$V_j^{opt} = Q_j^{opt} \Delta t_u, \tag{29d}$$

$$0 \leq \chi_j(t) \leq \gamma_j, \tag{29e}$$

$$\sum_{i=1}^{\chi_j(0)-1} q_{i,j}^{nom} < Q_j^{opt}, \tag{29f}$$

$$\sum_{i=1}^{\chi_j(0)} q_{i,j}^{nom} \geq Q_j^{opt}, \tag{29g}$$

$$t \in [1, H_l - 1] \subset \mathbb{Z}^+, \tag{29h}$$

where Λ denotes the total number of the pumping stations. Assuming that the *Problem 3* is also feasible, the optimal sequence of decision variables can be written as

$$\boldsymbol{\chi}^* = \left[\boldsymbol{\chi}_1^{*T}, \ \boldsymbol{\chi}_2^{*T}, \ \cdots, \ \boldsymbol{\chi}_\Lambda^{*T}\right]^T, \tag{30}$$

where

$$\boldsymbol{\chi}_j^* = \left[\chi_j^*(0), \ \chi_j^*(1), \ \cdots, \ \chi_j^*(H_l-1)\right]^T, \ \text{ for } \forall j \in [1, \Lambda].$$

5. APPLICATION: BARCELONA WATER NETWORK

5.1 Description

The Barcelona WDN supplies water to approximately three million consumers, distributed in 23 municipalities in a 424 km^2 area. Water can be taken from both surface and underground sources. The most important ones in terms of capacity and use are the Ter, which is a surface source, and Llobregat rivers. Water from these sources comes at different elevations and also involve different production costs. From these sources, water is supplied to 218 demand sectors through around 4645 km of pipes. A portion of the Barcelona WDN has been considered in this paper. In particular, 12 pressure zones that belong to the municipality of Barcelona. Fig. 1 shows a conceptual diagram of the considered WDN part. In total, this network has ten tanks, 7 pumping stations, a flow valve, 10 pressure valves and 47 demands.

In order to emulate the on-line execution of this non-linear MPC, the following setup has been implemented:

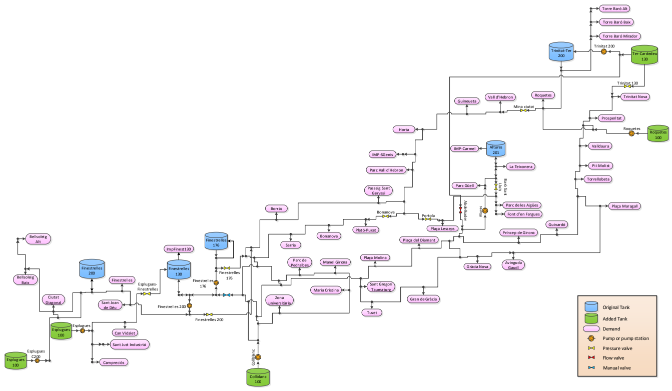

Fig. 1. Conceptual model of portion of Barcelona WDN

The NMPC problem is solved using a commercial solver (Brooke et al., 2004), which implements the reduced gradient method. Additionally, a detailed simulation of the distribution network is used as a virtual reality, which is implemented in PICCOLO [1]. MATLAB that is used for the lower layer translation of flow set-points into on-off pump schedules and also communication between the PICCOLO simulator.

5.2 Simulation Results

The obtained control strategies produced by the NMPC meet demands and operational constraints at all times, while optimizing the operational goals. Some illustrative results are presented in Fig 2 and Fig. 3, compared with the current operation by the water utility.

To test the two-layer NMPC controller, real data corresponding to the period between November 11 and December 2 of 2013 is used. To show the differences between the current heuristic control and the NMPC control, some tank volume and actuator flow evolutions are shown. In all the graphs, the curves present the current operation (real data) and the result of application of the NMPC strategy in the high-fidelity network simulator (simulation data).

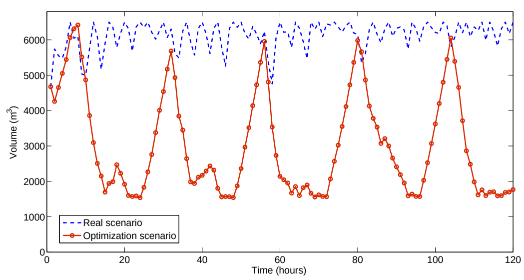

Fig. 2. Result of a selected tank in Barcelona WDN

Three tests scenarios were chosen based on real data. Each scenario lasts for 120 hours (5 days), and the three scenarios span the period from December 12 through December 27, 2013. The obtained control strategies produced by the NMPC and simulated with PICCOLO meet demands and operational constraints at all times. When compared with the actual strategies applied on the period, the NMPC

[1] http://www.safege.com/en/innovation/modelling-and-smart-solutions/

Table 2. Comparison costs with considered scenarios

		Scenario 1 12/12/2013-17/12/2013	**Scenario 2 17/12/2013-22/12/2013**	**Scenario 3 22/12/2013-27/12/2013**	**Total**	**Improvement (%)**
Total Energy Consumption	Real	486	495	440	1422	7.37
(K·Wh)	NMPC	473	437	408	1318	
Electrical Cost	Real	4659	4699	4280	13640	18.97
(e.u.)	NMPC	3985	3642	3425	11052	
Unitary Improvement ($\mathbf{e.u./m^3}$)		9.62%	15.27%	13.74%	12.83%	

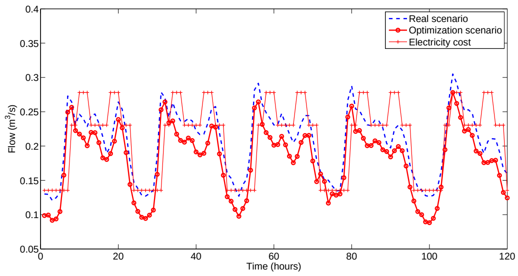

Fig. 3. Result of a selected pump in Barcelona WDN

strategies show significant reductions in energy use, as well as in energy cost, as shown in Table 2. Furthermore, in two of the three scenarios, it was also possible to reduce the total amount of water use to meet the same demands, by a different, but safe, use of tank storage.

In Table 2, costs achieved with the proposed NMPC when applied to the high fidelity simulator are compared with those obtained from the current control strategy in the period of time considered. The electrical energy and an associated cost is presented as well as the total accumulated energy and electrical cost in the considered days. The cost is given in economical units (e.u.) because of confidential reasons. The electrical cost obtained with the NMPC represents an improvement of almost 19% with respect to the current control in the time period considered. Taking into account the unitary cost of the water, the improvement because of the choice of the sources is between 9% to 15%.

6. CONCLUSION

In this paper, a two-layer NMPC control strategy for WDNs considering both flow and pressure models is proposed. The optimal set-points for actuators have been calculated by means of solving a non-linear optimisation problem and subsequently used for deploying the pump scheduling approach considering the ON/OFF management of individual pumps. Throughout a case study based on a portion of the Barcelona water distribution network, the on-line simulation results show the feasibility of the proposed control strategy and its economic cost improvement, while guaranteeing appropriate safety and stability objectives as well. Compared with current heuristic control strategies, the NMPC strategy of the WDN is able to meet all the demands in the considered sectors with their required pressure at all times, using less energy and cost. Thus, the proposed NMPC strategy is considered an adequate control strategy for the operational management of the WDN including both flow and pressure constraints.

REFERENCES

Brdys, M. and Ulanicki, B. (1994). *Operational Control of Water Systems: Structures, Algorithms and Applications*. Upper Saddle River. Prentice-Hall.

Brooke, A., Kendrick, D., Meeraus, A., and Raman, R. (2004). *GAMS*. A User's Guide. GAMS Development Corporation, Washington DC, USA.

Cembrano, G., Quevedo, J., Puig, V., Pérez, R., Figueras, J., Verdejo, J.M., Escaler, I., Ramón, G., Barnet, G., Rodríguez, P., and Casas, M. (2011). PLIO: a generic tool for real-time operational predictive optimal control of water networks. *Water Science and Technology*, 64(2), 448 – 459.

Cembrano, G., Wells, G., Quevedo, J., Perez, R., and Argelaguet, R. (2000). Optimal control of a water distribution network in a supervisory control system. *Control Engineering Practice*, 8(10), 1177 – 1188.

Grosso, J.M., Ocampo-Martinez, C., Puig, V., and Joseph, B. (2014). Chance-constrained model predictive control for drinking water networks. *Journal of Process Control*, 24(5), 504 – 516.

Ocampo-Martinez, C., Puig, V., Cembrano, G., and Quevedo, J. (2013). Application of MPC strategies to the management of complex networks of the urban water cycle. *IEEE Control Systems Magazine*, 33(1), 15 – 41.

Quevedo, J., Saludes, J., Puig, V., and Blanch, J. (2014). Short-term demand forecasting for real-time operational control of the barcelona water transport network. In *22nd Mediterranean Conference on Control and Automation (MED)*, 990 – 995. Palermo, Italy.

Rawlings, J. and Mayne, D. (2009). *Model predictive control: theory and design*. Madison, Wis. Nob Hill Pub. cop.

Smeers, Y. (1977). Generalized reduced gradient method as an extension of feasible direction methods. *Journal of Optimization Theory and Applications*, 22(2), 209 – 226.

Wang, Y., Ocampo-Martinez, C., and Puig, V. (2015). Robust model predictive control based on gaussian processes: Application to drinking water networks. In *2015 European Control Conference*, 3292 – 3297. Linz, Austria.

Wang, Y., Ocampo-Martinez, C., and Puig, V. (2016a). Stochastic model predictive control based on gaussian processes applied to drinking water networks. *IET Control Theory & Applications*, 10(8), 947 – 955.

Wang, Y., Ocampo-Martinez, C., Puig, V., and Quevedo, J. (2016b). Gaussian-process-based demand forecasting for predictive control of drinking water networks. In *Critical Information Infrastructures Security*, 69 – 80. Springer.