

Adaptive Optimal Parameter Estimation and Control of Servo Mechanisms: Theory and Experiments

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Abstract—Most of classical adaptive laws used in adaptive control have been developed based on the gradient descent algorithm to minimize the control error. Hence, the sluggish convergence of tracking error may affect the online learning, making accurate parameter estimation difficult. The aim of this paper is to present a new adaptive law to achieve optimal parameter estimation, and then to showcase its application to adaptive control for a benchmark servo system to retain simultaneous convergence of both the estimation error and tracking error. For this purpose, an auxiliary filter is introduced to extract the estimation error, which is used to drive the adaptive law with a time varying gain to minimize a cost function of the estimation error to achieve fast, accurate parameter estimation. Finally, this new adaptation is incorporated into an adaptive nonsingular terminal sliding mode control (ANTSMC) for the considered servo system to obtain tracking control and parameter estimation simultaneously. The effectiveness of the developed method is validated by means of comparative simulations and experiments.

Index Terms—Servo mechanisms, optimal parameter estimation, adaptive control, sliding mode control.

NOMENCLATURE

θ_m	Motor angular position.
θ_l	Load angular position.
ω	Load angular speed.
B	$= B_l + n^2 B_m$ Total viscous damping constant.
B_l	Damping constants of the load.
B_m	Damping constants of the motor.
J	$= J_l + n^2 J_m$ Total inertia.
J_l	Load inertia.
J_m	Motor inertia.
K_e	Electromotive force constant.
K_t	Motor torque constant.
n	Transmission ratio.
T_m	Motor output torque.
T_a	Motor torque.
T_f	Friction torque.
T_L	Load torque.

T_l	Output torque of the gearbox.
F_c	Friction coefficient.
$d(\cdot)$	Nonlinear function.
τ_d	$= d(\cdot) - T_L$, lump disturbance.

I. INTRODUCTION

High performance modeling and motion control of servo mechanisms have been of great importance in practical engineering applications, and thus drawn significant attentions in the past decades [1]–[6]. However, there are usually unknown dynamics such as friction, system uncertainties and external disturbances in such systems, which can degrade the control performance. To address these problems, many advanced control algorithms have been developed [7]–[9]. Moreover, artificial intelligent techniques, such as neural network (NN) [10]–[13], fuzzy logic control (FLC) [14]–[17], have also been incorporated into adaptive control designs for uncertain systems.

However, in most of existing adaptive control designs, the adaptive laws used to update parameters are usually derived based on the gradient algorithm to minimize the tracking control error [18]. These methods may trigger bursting phenomena when the system is subject to disturbances, i.e., the estimated parameters may go to infinity, leading to the instability of the constructed control system. To address the robustness issue, several modified adaptive laws have been proposed, such as e -modification and σ -modification [18]. However, the parameter estimation error may not converge to zero due to the included damper terms in these robust adaptive laws. Following this observation, a composite estimation method was incorporated into adaptive control to enhance the estimation response for unknown parameters [19]. Again, the estimated parameters stay around the pre-set values only, such that the control error convergence may be affected by the nonconvergent parameters [18]. In this respect, some efforts have been made toward developing novel parameter estimation methods, which can guarantee that the estimated parameters converge to the true values. In [20], an adaptive finite-time parameter estimation was proposed for nonlinear systems with the persistent excitation (PE) condition. In [21], [22], a novel filter operation was further introduced to design adaptive laws based on the extracted estimation error, where the widely used observer is avoided. This idea has established a new parameter estimation framework, which is independent of the control or observer designs, while guaranteeing exponential or even finite-time estimation error convergence. However, the adaptive laws proposed in [21], [22] cannot obtain optimal convergence (i.e., to minimize a predefined estimation error cost function).

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On the other hand, from the perspective of control design for servo systems, sliding model control (SMC) has been proved as an effective method to accommodate the unknown, bounded dynamics [23]–[26]. However, the induced chattering phenomenon of SMC cannot be completely eliminated. To reduce the chattering and enhance the transient control response, a nonsingular terminal sliding mode control (NTSMC) approach has been reported in [27]. In the subsequent work, by using a new sliding mode surface design, the NTSMC has been further modified to achieve faster convergence rate, i.e., nonsingular fast TSMC (NFTSMC) [28], [29]. The key merit is that the NFTSMC can retain the advantages of NTSMC and provide a faster convergence rate. Nevertheless, when the NTSMC is combined with adaptive control with classical adaptive laws to address the tracking control for servo systems, the convergence of the estimated parameters cannot be guaranteed [30], and the tuning of adaptive learning gains in these adaptive estimation and control schemes is not a trivial task.

Motivated by the above discussions, the aim of this paper is to develop a new adaptive law to achieve optimal parameter estimation (i.e., to minimize the estimation error in an optimal manner), and then to showcase its application to adaptive control synthesis to obtain simultaneous convergence of both the estimation error and tracking error. We take a servo system with unknown parameters as the benchmark example in this paper. To obtain optimal parameter estimation, we first introduce filter operations and auxiliary variables to extract the parameter estimation error (the error between the unknown parameters and their estimates), which is used to construct an estimation error cost function. Then by minimizing this cost function, a new optimal adaptive law with a time-varying gain is obtained to eliminate the effect of the regressor and thus improve the transient estimation response. Hence, the advancement over the previous work [21], [22] is not trivial. Moreover, to achieve finite-time tracking control as well as the parameter estimation simultaneously, the proposed adaptive estimation scheme is incorporated into a modified NFTSMC method, where the potential singularity problem in the conventional TSMC methods [31], [32] is avoided. Comparative simulations and experiments are given to validate the effectiveness of the developed methods.

The contributions of this paper are summarized as follows:

1) A novel adaptive optimal parameter estimation method is developed, where a cost function of the derived estimation error is defined to derive a time-varying gain to improve the transient estimation response.

2) This new optimal parameter estimation scheme is further incorporated into an adaptive NFTSMC control synthesis to achieve simultaneous convergence of estimation error and tracking error in finite-time (FT).

The paper is organized as follows. The system model is shown in Section II. The optimal parameter estimation method is given in Section III. Section IV introduces the control design. Section V provides the stability and convergence analysis. Simulations are given in Section VI, and experiments are provided in Section VII. Section VIII draws some conclusions.

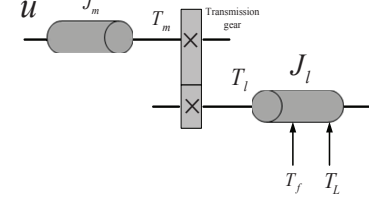


Fig. 1. Block diagram of the servo mechanism.

II. DYNAMIC MODEL AND PROBLEM FORMULATION

This paper considers a servo mechanism driven by a servo motor (See Fig.1), which is modeled as

$$J_m \ddot{\theta}_m = T_a - B_m \dot{\theta}_m - T_m \quad (1)$$

$$T_a = K_t \frac{u - K_e \dot{\theta}_m}{R} \quad (2)$$

$$J_l \ddot{\theta}_l = T_l - B_l \dot{\theta}_l - T_L - T_f \quad (3)$$

The definitions of variables as shown in Fig.1 and the above equation have been defined in nomenclature. The friction force is described as:

$$T_f = F_c \text{sgn}(\omega) \quad (4)$$

The backlash dynamics are defined as

$$T_l = nT_m + d(T_m) \quad (5)$$

The system dynamics given in (1)-(5) can be written as

$$\begin{cases} \dot{\theta}_l = \omega \\ J_t \dot{\omega} = -(B + \frac{n^2 K_t K_e}{R}) \omega + \frac{n K_t}{R} u - F_c \text{sgn}(\omega) + \tau_d \end{cases} \quad (6)$$

Define the parameter vector $\Theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T = [(B + \frac{n^2 K_t K_e}{R})/J_t, \frac{n K_t}{J_t R}, F_c/J_t, \tau_d/J_t]^T$, and state variable as $x = [x_1, x_2] = [\theta_l, \omega]$, then the model (6) is rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\theta_1 x_2 + \theta_2 u - \theta_3 \text{sgn}(x_2) + \theta_4 \end{cases} \quad (7)$$

The aim of this paper is to propose an optimal parameter estimation method to estimate the unknown parameters $\theta_i, i = 1, \dots, 4$ and design an adaptive controller to achieve position tracking for the servo mechanism, i.e., x_1 tracks a given reference x_d .

III. ADAPTIVE OPTIMAL PARAMETER ESTIMATION

In this section, we will propose an optimal parameter estimation (OPE) method to estimate unknown system parameters. Differing from the classical adaptive estimation schemes, we will introduce a cost function of the derived estimation error, which provides a new analytical method to obtain optimal parameter estimation with a time-varying gain rather than a manually tuned constant gain as [21], such that faster, optimal error convergence can be achieved by minimizing this cost function.

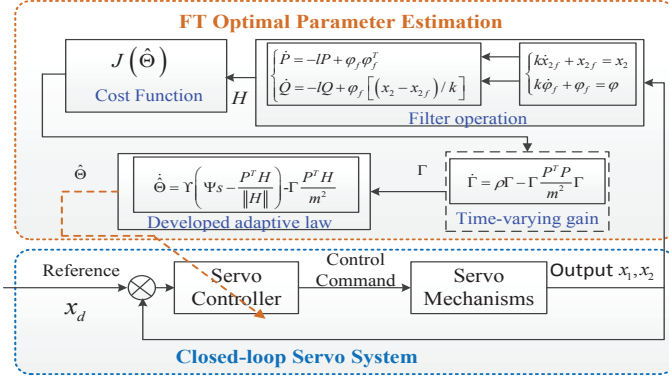


Fig. 2. Structure of the proposed control system

A. Filter Design

To facilitate the design of adaptive law, the following dynamics of system (7) with the unknown parameter Θ is considered

$$\dot{x}_2 = \Theta^T \Psi \quad (8)$$

where $\Psi = [-x_2, u, -\text{sgn}(x_2), 1]^T$ is the regressor.

The filtered variables x_{2f} and Ψ_f of x_2 and Ψ are defined as

$$\begin{cases} \kappa \dot{x}_{2f} + x_{2f} = x_2, & x_{2f}(0) = 0 \\ \kappa \dot{\Psi}_f + \Psi_f = \Psi, & \Psi_f(0) = 0 \end{cases} \quad (9)$$

where $\kappa > 0$ is the filter constant. From (8), one can obtain

$$\dot{x}_{2f} = \frac{x_2 - x_{2f}}{\kappa} = \Theta \Psi_f \quad (10)$$

Then, the auxiliary matrix P and vector Q are defined as

$$\begin{cases} \dot{P} = -lP + \Psi_f \Psi_f^T, & P(0) = 0 \\ \dot{Q} = -lQ + \Psi_f [(x_2 - x_{2f})/\kappa], & Q(0) = 0 \end{cases} \quad (11)$$

where $l > 0$ is a constant.

The solution of (11) is given as

$$\begin{cases} P(t) = \int_0^t e^{-l(t-r)} \Psi_f(r) \Psi_f^T(r) dr \\ Q(t) = \int_0^t e^{-l(t-r)} \Psi_f(r) [(x_2 - x_{2f})/\kappa] dr \end{cases} \quad (12)$$

Finally, another auxiliary vector H is defined as

$$H = P\hat{\Theta} - Q \quad (13)$$

where $\hat{\Theta}$ is the estimate of the unknown parameter Θ , which will be updated online by the developed adaptive law.

Lemma 1: For the variable H defined in (13), which can be obtained based on P, Q defined in (13), we have

$$H = P\hat{\Theta} - Q = -P\tilde{\Theta} \quad (14)$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$ denotes the estimation error.

Proof: From (12), we can verify that

$$Q = P\Theta. \quad (15)$$

Then, substituting (15) into (13), we can have $H = P\hat{\Theta} - P\Theta = -P\tilde{\Theta}$.

Remark 1: It is shown in Lemma 1 that the derived variable H is a function of the unknown estimation error $\tilde{\Theta}$, which can be online calculated based on measurable dynamics x_2, Ψ by using the filter operations in (9) - (10) and algebraic calculation in (13). As shown in our previous work [21], the variable H can be used to design adaptive laws to obtain $\hat{\Theta}$ with guaranteed convergence. However, the adopted constant learning gains in [21] cannot address the effect of the regressor P , and thus the transient convergence response may be sluggish. This paper aims to develop a new adaptive law design based on the extracted estimation error H to obtain optimal parameter estimation. Specifically, we will introduce a cost function of the estimation error with H , which can be minimized to derive a time-varying gain in the adaptive law to achieve improved estimation response. Thus, the following developments are essentially different to the previous work [21].

B. Adaptive Optimal Parameter Estimation

In this section, a novel adaptive optimal parameter estimation (AOPE) method is designed by minimizing a cost function of the extracted error information $\tilde{\Theta}$, which leads to a time-varying gain in the adaptive law to compensate the effect of regressor P and improve the estimation performance. According to [18], a cost function is defined as:

$$\begin{aligned} J(\hat{\Theta}, t) = & \frac{1}{2} \int_0^t e^{-\rho(t-\tau)} \frac{[Q(\tau) - P(\tau)\hat{\Theta}(t)]^T [Q(\tau) - P(\tau)\hat{\Theta}(t)]}{m^2(\tau)} d\tau \\ & + \frac{1}{2} e^{-\rho t} (\hat{\Theta}(t) - \hat{\Theta}(0))^T R_0 (\hat{\Theta}(t) - \hat{\Theta}(0)) \end{aligned} \quad (16)$$

where $m^2 = I_4 + \|P^T P\|$ is utilized for the normalization of P , $R_0 = R_0^T > 0$ and $\rho > 0$ are constants. The cost function $J(\hat{\Theta}, t)$ includes the discounting of the past estimation error based on the current parameter estimate $\hat{\Theta}$ and penalizes the parameter change in $[0, t]$ weighted by $e^{-\rho t} R_0$. The constant ρ serves as a forgetting factor, which implies that the effect of old data and the initial error $\hat{\Theta}(0)$ are discarded exponentially as time t increases.

Note that the above cost function is different to the one used in the derivation of the least-squares algorithm [18], which is a function of the observer error rather than the estimation error $\tilde{\Theta}$. The cost function $J(\hat{\Theta}, t)$ is a convex function of $\hat{\Theta}$ at each time t , thus we can update the parameter estimation $\hat{\Theta}$ to minimize the cost function $J(\hat{\Theta}, t)$ to obtain the optimal parameter estimation $\hat{\Theta}(t)$, which satisfies the minimum condition [33]

$$\frac{\partial J(\hat{\Theta}, t)}{\partial \hat{\Theta}} = 0, \quad \forall t \geq 0 \quad (17)$$

where $\partial J(\hat{\Theta}, t)/\partial \hat{\Theta}$ denotes the partial derivative of the cost function with respect to $\hat{\Theta}$.

Then from (17), we can obtain

$$\begin{aligned} \frac{\partial J(\hat{\Theta}, t)}{\partial \hat{\Theta}} = & \int_0^t e^{-\rho(t-\tau)} \frac{-P^T(\tau)Q(\tau) + P^T(\tau)P(\tau)\hat{\Theta}(t)}{m^2} d\tau \\ & + e^{-\rho t} R_0 (\hat{\Theta}(t) - \hat{\Theta}(0)) = 0 \end{aligned} \quad (18)$$

By solving the above equation, we can obtain the following solution

$$\hat{\Theta}(t) = \left(\int_0^t e^{-\rho(t-\tau)} \frac{P^T(\tau)P(\tau)}{m^2} d\tau + e^{-\rho t} R_0 \right)^{-1} \left(\int_0^t e^{-\rho(t-\tau)} \frac{P^T(\tau)Q(\tau)}{m^2} d\tau + e^{-\rho t} R_0 \hat{\Theta}(0) \right) \quad (19)$$

The equation (19) gives a non-recursive algorithm. To facilitate online updating the estimated parameter, we further take the derivative of $\hat{\Theta}$ given in (19) with respect to time t . For the simplicity of notation, we define

$$\Gamma(t) = \left(\int_0^t e^{-\rho(t-\tau)} \frac{P^T(\tau)P(\tau)}{m^2} d\tau + e^{-\rho t} R_0 \right)^{-1}, \quad W(t) = \left(\int_0^t e^{-\rho(t-\tau)} \frac{P^T(\tau)Q(\tau)}{m^2} d\tau + e^{-\rho t} R_0 \hat{\Theta}(0) \right). \quad (20)$$

Then the equation (19) can be rewritten as $\hat{\Theta}(t) = \Gamma(t)W(t)$. Considering the following matrix equality [34]:

$$\frac{d}{dt} \Gamma \Gamma^{-1} = \dot{\Gamma} \Gamma^{-1} + \Gamma \frac{d}{dt} \Gamma^{-1} = 0 \quad (21)$$

then we can obtain

$$\dot{\Gamma} = -\Gamma \left(\frac{d}{dt} \Gamma^{-1} \right) \Gamma \quad (22)$$

Then according to definition of $\Gamma(t)$ as given above, we have

$$\begin{aligned} \frac{d}{dt} \Gamma^{-1} &= \frac{d}{dt} \left(\int_0^t e^{-\rho(t-\tau)} \frac{P^T(\tau)P(\tau)}{m^2} d\tau + e^{-\rho t} R_0 \right) \\ &= -\rho \int_0^t e^{-\rho(t-\tau)} \frac{P^T(\tau)P(\tau)}{m^2} d\tau - \rho e^{-\rho t} R_0 \\ &\quad + \frac{P^T P}{m^2} = -\rho \Gamma^{-1} + \frac{P^T P}{m^2} \end{aligned} \quad (23)$$

Substituting (23) into (22) will yield

$$\dot{\Gamma} = \rho \Gamma - \Gamma \frac{P^T P}{m^2} \Gamma, \quad \Gamma^{-1}(0) = R_0 > 0 \quad (24)$$

On the other hand, similar to (23), we can also obtain from (20) that

$$\frac{dW}{dt} = -\rho W + \frac{P^T Q}{m^2} \quad (25)$$

Now by differentiating (19), we can obtain the following adaptive law for online parameter estimation

$$\dot{\hat{\Theta}} = -\Gamma \frac{P^T H}{m^2} \quad (26)$$

Note the above adaptive law can be obtained based on the following mathematical manipulators

$$\begin{aligned} \dot{\hat{\Theta}} &= \dot{\Gamma} W + \Gamma \dot{W} \\ &= \left(\rho \Gamma - \Gamma \frac{P^T P}{m^2} \Gamma \right) W + \Gamma \left(-\rho W + \frac{P^T Q}{m^2} \right) \\ &= \Gamma \frac{P^T Q - P^T P \hat{\Theta}}{m^2} \\ &= -\Gamma \frac{P^T H}{m^2} \end{aligned} \quad (27)$$

In the proposed adaptive law (26), the extracted estimation error $\tilde{\Theta}$ is used to drive the parameter estimation via the term H . Hence, the estimation error dynamics of (26) can be obtained as $\dot{\tilde{\Theta}} = -\Gamma \frac{P^T P \tilde{\Theta}}{m^2}$. The exponential or even finite-time convergence properties of adaptive laws using the estimation error has been studied in the previous work [21]. However, a notable advancement of this adaptive law (26) is that the time-varying gain Γ updated by (24) is introduced to eliminate the effect of the induced filtered regressor P in the term H in (26) on the transient convergence response of $\tilde{\Theta}$. Specifically, the gain Γ given in (24) converges exponentially to the weighted average of $P^T P$ as shown in (20). Nevertheless, since $R_0 = R_0^T > 0$ and the matrix P is semi-positive definite [35] according to its definition given in (11), then Γ exists for any $t > 0$. Moreover, different to the classical adaptive laws derived based on the gradient algorithm, the proposed adaptive law is driven by the estimation error $\tilde{\Theta}$ by minimizing the constructed cost function to achieve optimal estimation performance.

It is also well-recognized that the persistent expiation (PE) condition imposed on the regressor is essential for proving the parameter estimation convergence. Hence, we first establish the relationship between the the standard PE condition and the positive definiteness of the introduced matrix P .

Lemma 2 [21], [36]: The matrix P is positive definite satisfying $\lambda_{\min}(P) > \sigma_1 > 0$ for a positive constant $\sigma_1 > 0$, if the regressor Ψ is PE.

Before we present the main results of this section, we first evaluate the boundedness of the time-varying gain Γ .

Lemma 3: For the time-varying gain Γ defined in (24) with the PE condition of Ψ being true, then we know

$$\gamma_1 I \leq \Gamma(t) \leq \gamma_2 I \quad (28)$$

where $\gamma_1 = 1/(\lambda_{\min}(R_0) + 1/\rho)$ and $\gamma_2 = e^{\rho T} m^2 / \sigma_1^2$.

Proof: By calculating the solution of equation (23), we have

$$\Gamma^{-1}(t) = e^{-\rho t} \Gamma^{-1}(0) + \int_0^t e^{-\rho(t-\tau)} \frac{P^T P}{m^2} d\tau \quad (29)$$

Consider the facts $\frac{P^T P}{m^2} \leq I$ and $\int_0^t e^{-\rho(t-\tau)} d\tau \leq 1/\rho$, we can further obtain

$$\Gamma^{-1}(t) \leq \Gamma^{-1}(0) + I \int_0^t e^{-\rho(t-\tau)} d\tau \leq R_0 + I/\rho \quad (30)$$

On the other hand, when the PE condition of Ψ holds, the fact $\lambda_{\min}(P) > \sigma_1 > 0$ is true. Hence, one can further verify from (29) that

$$\begin{aligned} \Gamma^{-1}(t) &\geq \int_0^t e^{-\rho(t-\tau)} \frac{P^T P}{m^2} d\tau \geq \int_{t-T}^t e^{-\rho(t-\tau)} \frac{P^T P}{m^2} d\tau \\ &\geq \frac{\sigma_1^2}{m^2} e^{-\rho T} I \end{aligned} \quad (31)$$

for any $t > T > 0$. Then the boundedness of the learning gain Γ given in (28) can be verified.

The convergence of the proposed adaptive law can be summarized as the following theorem:

Theorem 1: For the parameter estimation of system (8) under the PE condition, the adaptive law (26) is used with the

variables P, Q, H defined in (11)-(13) and the time-varying gain given in (24), then the estimation error $\tilde{\Theta}$ converges to zero exponentially.

Proof: Consider a Lyapunov function as follows:

$$V_1 = \frac{1}{2} \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta} \quad (32)$$

The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= \tilde{\Theta}^T \Gamma^{-1} \dot{\tilde{\Theta}} + \frac{1}{2} \tilde{\Theta}^T \dot{\Gamma}^{-1} \tilde{\Theta} \\ &= -\tilde{\Theta}^T \frac{P^T P}{m^2} \tilde{\Theta} + \frac{1}{2} \tilde{\Theta}^T (-\rho \Gamma^{-1} + \frac{P^T P}{m^2}) \tilde{\Theta} \\ &\leq -\frac{1}{2} (\sigma_1^2 / m^2 + \rho / \gamma_2) \|\tilde{\Theta}\|^2 \\ &\leq -\mu V_1 \end{aligned} \quad (33)$$

where $\mu = \gamma_1 (\sigma_1^2 / m^2 + \rho / \gamma_2)$ is a positive constant. Then, from the Lyapunov theorem, we can conclude that the estimation error $\tilde{\Theta}$ can converge to zero exponentially.

Remark 2: Lemma 2 shows that the minimum eigenvalue condition $\lambda_{\min}(P) > \sigma_1 > 0$ can be fulfilled under the conventional PE condition of Ψ . This positive definiteness property will also be used to prove the convergence of the proposed adaptive law (26). Hence, instead of validating the PE condition directly, which is difficult to conduct online, Lemma 2 provides an alternative method to verify the required excitation condition by computing the minimum eigenvalue of matrix P to test for $\lambda_{\min}(P) > \sigma_1 > 0$, which can be carried out online.

Remark 3: From (24), if we set $\rho = 0$, then the gain Γ given in $\frac{d}{dt} \Gamma^{-1} = \frac{P^T P}{m^2} \geq 0$ will converge to zero, which makes the proposed adaptive law (26) switched off. This has been recognized as the gain wind-up issue in the adaptive estimation. Hence, as inspired by the least square algorithm we include the forgetting factor ρ to remedy this issue and to guarantee the boundedness of the gain Γ as shown in Lemma 3.

IV. ADAPTIVE SLIDING MODE CONTROLLER DESIGN

In this section, we will incorporate the proposed adaptive algorithm into the adaptive control to achieve tracking control and parameter estimation simultaneously. To retain faster control error convergence, we first present a modified FNTSMC without the singularity issue.

To construct the sliding mode surface, the tracking error is defined as

$$e = x_d - x_1 \quad (34)$$

Then a modified sliding mode surface s is constructed as

$$s = \dot{e} + \lambda_1 e + \lambda_2 \beta(e) \quad (35)$$

where

$$\beta(e) = \begin{cases} |e|^\nu \text{sgn}(e) & s = 0 \text{ or } s \neq 0, |e| > \mu \\ \beta_1 e + \beta_2 |e|^2 \text{sgn}(e) & s \neq 0, |e| \leq \mu \end{cases} \quad (36)$$

where $\beta_1 = (2 - \nu)\mu^{\nu-1}$, $\beta_2 = (\nu - 1)\mu^{\nu-2}$, and $\nu > 0$ is a positive constant. $\lambda_1 > 0$ and $\lambda_2 > 0$ are positive constants.

Then the derivative of s is calculated based on (7) as

$$\dot{s} = \ddot{x}_1 - \dot{x}_r = -\theta_1 x_2 + \theta_2 u - \theta_3 \text{sgn}(x_2) + \theta_4 - \dot{x}_r \quad (37)$$

where

$$\dot{x}_r = \begin{cases} \ddot{x}_d + \lambda_1 \dot{e} + \lambda_2 \nu |e|^{\nu-1} \dot{e} & s = 0 \text{ or } s \neq 0, |e| > \mu \\ \ddot{x}_d + \lambda_1 \dot{e} + \lambda_2 \beta_1 \dot{e} + 2\lambda_2 \beta_2 |e| \dot{e} & s \neq 0, |e| \leq \mu \end{cases} \quad (38)$$

Then, an adaptive nonsingular terminal sliding mode control (ANTSMC) is designed as

$$u = \frac{1}{\hat{\theta}_2} [-k_1 s - k_2 |s|^\gamma \text{sgn}(s) + \hat{\theta}_1 x_2 + \hat{\theta}_3 \text{sgn}(x_2) - \hat{\theta}_4 + \sigma_2 \text{sgn}(s) + \dot{x}_r] \quad (39)$$

where $\hat{\theta}_i, i = 1, \dots, 4$ is the estimation of θ_i , $-k_1 s - k_2 |s|^\gamma \text{sgn}(s)$ represents the feedback control which is used to ensure the finite-time convergence of the sliding mode surface s . $\gamma = \gamma_1 / \gamma_2$ with $\gamma_1 > 0, \gamma_2 > 0$ and $\gamma_1 < \gamma_2$ are all positive constants. $\sigma_2 \text{sgn}(s)$ denotes a robust term used to reject the potential bounded estimation error and other disturbances. Thus, the control gains k_1 and k_2 are positive constants.

By substituting the control (39) into (37), we can obtain the following tracking error dynamics

$$\dot{s} = -k_1 s - k_2 |s|^\gamma \text{sgn}(s) - \tilde{\Theta}^T \Psi - \sigma_2 \text{sgn}(s) \quad (40)$$

Remark 4: Compared with conventional sliding mode control designs, e.g., [37], this paper suggests a modified NTSMC given in (35)-(36), which can avoid the singularity problem since the introduced term $\beta(e)$ can also overcome the singularity problem in the case $s \neq 0$ and $e = 0$.

In the above control (39), we need to obtain the estimated parameters $\hat{\theta}_i$. Since the tracking error has also been derived in this section by using the sliding mode surface s in (35), it can also be used in the design of adaptive law, so as to guarantee the convergence of both the estimation error $\tilde{\Theta}$ and tracking error s simultaneously.

Then, when the adaptive parameter estimation is incorporated into the control design, we can design the following adaptive law:

$$\dot{\tilde{\Theta}} = \Upsilon \left(\Psi s - \frac{P^T H}{\|H\|} \right) - \Gamma \frac{P^T H}{m^2} \quad (41)$$

where $\Upsilon > 0$ is a positive constant.

Remark 5: In this paper, we utilized the NTSMC to achieve finite-time convergence of both the estimation error and control error due to the finite-time property of NTSMC. It is true that the proposed adaptive law can be incorporated into other adaptive control designs.

Remark 6: In the proposed adaptive law (41), the first term is the gradient term of the tracking error s , the second term is used to achieve finite-time convergence of $\tilde{\Theta}$ together with s , and the third term stemming from the optimal estimation algorithm shown in the above section is used to enhance the transient estimation response. It is noted that the obtained parameters are smooth since an integration is adopted to obtain the parameter estimates though there are high-frequency switchings in the right side of (41). This is different to the

SMC schemes. Finally, to avoid the potential singularity issue in the control (39) (when $\hat{\theta}_2 = 0$), we should set the initial condition as $\hat{\theta}_2(0) > 0$ and/or impose the projection method.

V. STABILITY ANALYSIS

To analyze the stability of the closed-loop system, we have the following lemma.

Lemma 4 [38]: Assume that there exists a continuous, positive-definite function $V(t)$ satisfying

$$\dot{V}(t) + \eta V(t) + \varpi V^\ell(t) \leq 0, \quad \forall t \geq t_0, \quad V(t_0) \geq 0 \quad (42)$$

where η , ϖ , and $0 < \ell < 1$ are positive constants. Then, $V(t)$ satisfies

$$V^{1-\nu}(t) \leq (\eta V^{1-\ell}(t_0) + \varpi) e^{-\eta(1-\ell)(t-t_0)} - \varpi, \quad t_0 \leq t \leq t_s \quad (43)$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_s \quad (44)$$

with t_s is

$$t_s = t_0 + \frac{1}{\eta(1-\ell)} \ln \frac{\eta V^{(1-\ell)}(t_0) + \varpi}{\varpi} \quad (45)$$

Now the stability of the proposed control system and the convergence of the errors can be summarized as:

Theorem 2: Consider the close-loop system constituting of the plant (6), adaptive control (39), and the parameter updating law (41), then

- 1) The closed-loop system is stable.
- 2) The sliding mode surface s and the estimation error $\tilde{\Theta}$ can converge to zero in finite-time.
- 3) The tracking error e will converge to zero in finite-time.

Proof: 1) Consider a Lyapunov function as follows:

$$V_2 = \frac{1}{2} \tilde{\Theta}^T \Upsilon^{-1} \tilde{\Theta} + \frac{1}{2} s^2 \quad (46)$$

The time derivative of V_2 is then derived as

$$\begin{aligned} \dot{V}_2 &= -k_1 s^2 + \tilde{\Theta}^T \Psi s - k_2 |s|^{\gamma+1} - \sigma_2 |s| - \tilde{\Theta}^T \Psi s \\ &\quad - \frac{\tilde{\Theta}^T P^T P \tilde{\Theta}}{\|P \tilde{\Theta}\|} - \tilde{\Theta}^T \Upsilon^{-1} \Gamma \frac{P^T H}{m^2} \\ &\leq -k_1 s^2 - k_2 |s|^{\gamma+1} - \sigma_2 |s| - \sigma_1 \|\tilde{\Theta}\| \\ &\leq -\sigma_2 |s| - \sigma_1 \|\tilde{\Theta}\| \\ &\leq -a \sqrt{V_2} \end{aligned} \quad (47)$$

where $a = \min\{\sqrt{2}\sigma_1, \sqrt{2}\sigma_2\}$ is a positive constant. According to the Lyapunov theorem, we know that $\dot{V}_2 \leq 0$ and thus s and $\tilde{\Theta}$ are all bounded and converge to zero in finite-time. Moreover, we can easily verify that the control u and the system state x are all bounded. Hence, the closed-loop system is stable.

2) According to the above proof, we know that s converges to zero in finite-time. When the sliding surface $s = 0$, one has

$$\dot{e} = -\lambda_1 e - \lambda_2 \beta(e) \quad (48)$$

Select a Lyapunov function as follows

$$V_3 = \frac{1}{2} e^2 \quad (49)$$

and differentiating V_3 along (48) yields

$$\begin{aligned} \dot{V}_3 &= -\lambda_1 e^2 - \lambda_2 |e|^{\nu+1} \\ &= -2\lambda_1 V_3 - \lambda_2 2^{\frac{\nu+1}{2}} V_3^{\frac{\nu+1}{2}} \end{aligned} \quad (50)$$

If $s \neq 0$, then (50) can be written as

$$\begin{aligned} \dot{V}_3 &= e(-\lambda_1 e - \lambda_2 \beta_1 e - \lambda_2 \beta_2 |e|^2 \text{sgn}(e)) \\ &= -(\lambda_1 + \lambda_2 \beta_1) e^2 - \lambda_2 \beta_2 |e|^3 \\ &= -2(\lambda_1 + \lambda_2 \beta_1) V_3 - \lambda_2 \beta_2 2^{\frac{3}{2}} |e|^{\frac{3}{2}} \end{aligned} \quad (51)$$

Define $\alpha_1 = 2\lambda_1$ or $= 2(\lambda_1 + \lambda_2 \beta_1)$, $\alpha_2 = \lambda_2 2^{\frac{\nu+1}{2}}$ or $= \lambda_2 \beta_2 2^{\frac{3}{2}}$, and $\alpha_3 = \frac{\nu+1}{2}$ or $= 3/2$, then, we can obtain

$$\dot{V}_3 + \alpha_1 V_3 + \alpha_2 V_3^{\alpha_3} \leq 0 \quad (52)$$

Then based on Lemma 4, we can obtain that the tracking error e will converge to zero in finite-time given by

$$t_1 = \frac{1}{\alpha_1(1-\alpha_1)} \ln \frac{\alpha_1 V_3^{1-\alpha_3}(t_0) + \alpha_2}{\alpha_2} \quad (53)$$

In the engineering applications, the parameter tuning of the proposed control algorithm can be conducted in a straightforward way. The parameters to be tuned include two sets: adaptive estimation parameters κ , l and $\hat{\Theta}(0)$, and controller parameters k_1, k_2 and λ_1, λ_2 . The tuning guidelines can be briefly summarized as follows:

- 1) Choose proper initial parameter condition $\hat{\Theta}(0)$, which should satisfy $\hat{\theta}_2(0) > 0$.
- 2) Large feedback gains k_1, k_2 and λ_1, λ_2 can lead to fast convergence of tracking error while triggering oscillations in the control actions. The learning gain Γ can improve the estimation performance of $\hat{\Theta}$, but may excite the parameter oscillations.
- 3) The filter constants κ , l cannot be set too large to seek for a trade-off between the robustness and convergence rate.

The flowchart for the practical implementation of the proposed control algorithm can be summarized as follows:

The implementation of the proposed algorithm	
1:	Initialization: Set initial condition $\hat{\Theta}(0)$, $\gamma(0)$, filter constant κ and l , sliding mode surface parameters λ_1, λ_2 , and control parameters k_1 and k_2 ;
2:	Start procedure;
3:	Compute the sliding mode surface s from (35);
4:	Compute the control law u via Eq.(39) for integration interval $t \in [t_i, t_{i+1}]$, $i \in N$;
5:	Online adaptation: online calculate the P and Q based on (11);
6:	Compute the parameter error information H given in (13);
7:	Update adaptive estimation $\hat{\Theta}$ according to (26) for integration interval $t \in [t_i, t_{i+1}]$, $i \in N$;
8:	Continuation: let $i := i + 1$;
9:	End procedure

VI. SIMULATION RESULTS

To verify the effectiveness of the developed estimation and control algorithms, simulations are implemented on a servo mechanism. A PC with CPU i7 and 8 G memory is used to run the simulation. The system parameters in (6) are chosen as $J_t = 0.2$, $K_t = 0.185$, $K_e = 0.3$, $R = 1.5\Omega$, $T_L = 0.1$ and $n = 10$. The friction force is $T_f = T_c \text{sgn}(x_2)$, where

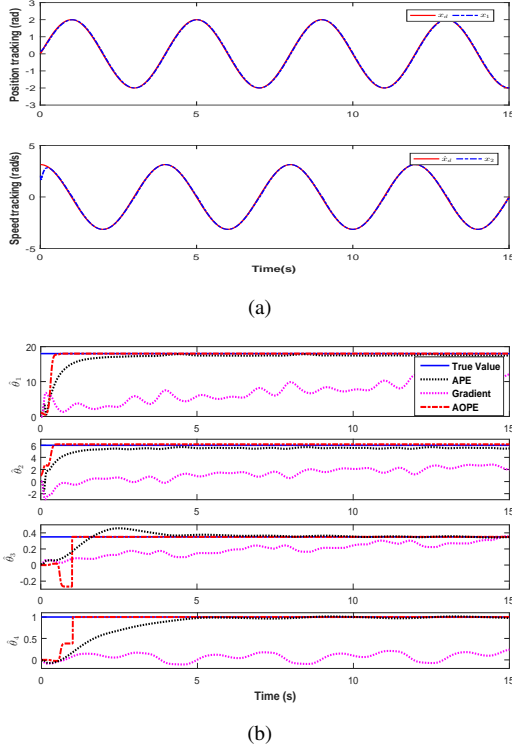


Fig. 3. Simulation results (a) Control performance; (b) Parameters estimates for $\theta_1, \theta_2, \theta_3, \theta_4$.

the Coulomb friction coefficient is $T_c = 0.07$. Then, the unknown parameters can be set as $\Theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T = [18, 6.16, 0.35, 1]^T$. The controller parameters are $k_1 = 20$, $k_2 = 1.5$, $\lambda_1 = 11$, $\lambda_2 = 5$, $\nu = 17/12$, $\sigma_2 = 0.1$. A desired trajectory $x_d = 2 \sin(0.5\pi t)$ is first utilized in simulation.

In order to validate the estimation performance, three parameter estimation methods are adopted in this simulation.

1) *Adaptive Optimal Parameter Estimation (AOPE)*: The parameter update law (41) is designed in Section IV. The initial conditions are set as $\hat{\Theta}(0) = [0 \ 1 \ 0 \ 0]^T$. The filter time constant is selected as $k = 0.01$ $l = 1$. The adaptive parameters are chosen as $\rho = 20$, $\Upsilon = 0.5$ and $\Gamma(0) = 100 \text{diag}[1 \ 1 \ 1 \ 1]$.

2) *Adaptive Parameter Estimation (APE)* [21]: The adaptive parameter estimation method with finite-time convergence is tested. The parameter update law is $\dot{\hat{\Theta}} = \Gamma_1(\Psi s - \frac{P^T H}{\|H\|})$ with a constant gain $\Gamma_1 = \text{diag}[2.5 \ 4 \ 3 \ 0.5]$. The other parameters are the same as AOPE.

3) *Gradient algorithm*: The adaptive law is $\dot{\hat{\Theta}} = \Gamma_1 \Psi s$ with a constant $\Gamma_1 = \text{diag}[2.5 \ 4 \ 3 \ 0.5]$ is also tested for comparison.

Simulation results are depicted in Fig.3, where the position tracking performance and speed tracking performance are depicted in Fig.3 (a), and Fig.3 (b) shows the results of the above three parameter estimation methods. One can find in Fig.3 (a) that the position tracking and speed tracking can achieve satisfactory performance with the proposed algorithms. Fig.3(b) illustrates that the parameters from the proposed updating law converge to the actual values very fast. By comparing the estimation results of AOPE and APE, one can find that the transient performance of the proposed parameter updating law

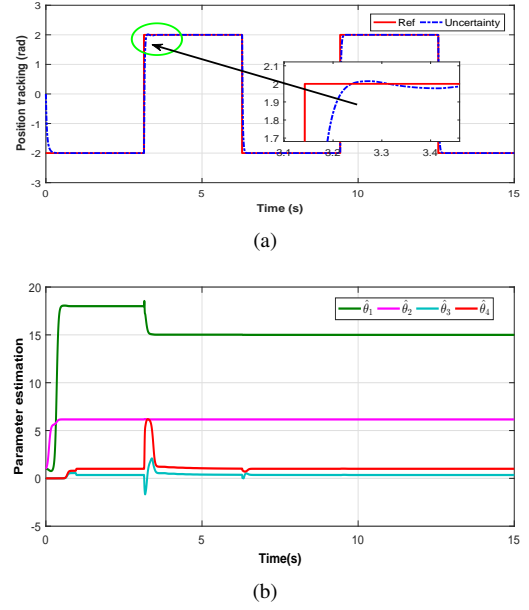


Fig. 4. Simulation results (a) Position tracking and tracking error (b) Parameters estimates for $\theta_1, \theta_2, \theta_3$ and θ_4

with a time-varying gain is faster than APE with a constant gain. This shows that the time-varying gain given in (24) can address the effect of regressor and thus improve the transient performance. Among three parameter estimation methods, the gradient methods provides inaccurate estimation since it does not contains the information of the estimation error $\tilde{\Theta}$. These results show that the use of parameter estimation error in the adaptive law can contribute to achieving fast convergent and accurate estimation performance.

To study the effect of parameter uncertainties on the dynamic response of the servomechanism system, we change the parameter θ_1 at $t = 2.5$ s as

$$\Theta = \begin{cases} [18, 6.16, 0.35, 1], & 0 \leq t < 3 \\ [15, 6.16, 0.35, 1], & 3 \leq t \leq 15 \end{cases} \quad (54)$$

It should be noted that since the unknown parameters and the variations can be accurately estimated via the proposed adaptive law as guaranteed in Theorem 1, the effect of parameter uncertainties can be estimated and then compensated in the proposed control. Fig.4 shows the position tracking (Fig.4(a)) and parameter estimation (Fig.4(b)) under the parameter changes. From Fig.4(a), we can see that the transient response has a small overshoot at $t = 3$ s. Moreover, the sudden change in the parameters can be tracked after very short transient at 3 sec.

Moreover, to study the effect of external disturbance on the control response for the servomechanism system, a square wave with amplitude 2 is used as the reference signal, and a time-varying sinusoidal wave $d = 0.1 \sin(0.5\pi t)$ is simulated as the external disturbance injected into the system measurements. Simulation result is shown in Fig.5. From this result, one can find that the developed control method can guarantee satisfactory control response even in the presence of external disturbances.

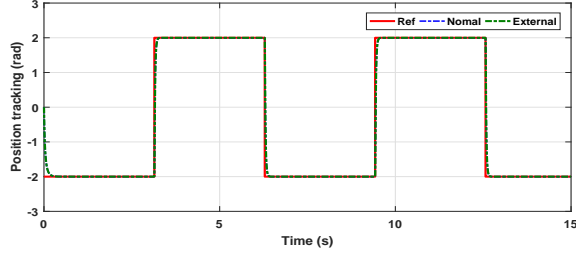


Fig. 5. Parameter estimation under external disturbance.

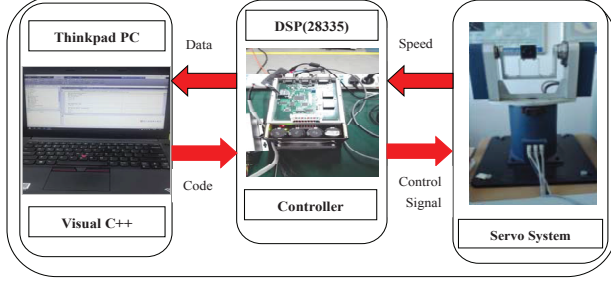


Fig. 6. Diagram of the servo mechanism.

VII. EXPERIMENTS

A. Experiment Setup

In this section, experimental results are given to demonstrate the effectiveness of the proposed control method. The experimental setup is given in Fig.6. The turntable servo system comprises of a PMSM (HC-UFS13), which is driven by a PWM amplifiers in the motor card (MR-J2S-10A). The control algorithms are implemented by the digital signal processor (DSP) with the sampling time 0.01s. A Pentium 3.0 GHz industrial control computer by running C++ program in CSS3.0 developing environment. The output position is measured by means of an encoder with a resolution of 800 divisions. A gear transmission system with a gear ration of 80 is included; then, the encoder output signals have a resolution of 64000 per rotation. In real-time experiments, a digital signal processor (DSP, TMS3202812) is adopted to implement the proposed control algorithm, and running the required multiply and addition operations involved in the proposed adaptive control methods within 10 ms is straightforward in terms of computational costs.

B. Controller Design

In this section, four control schemes are compared: adaptive control with a time-varying gain parameter updating law (AOPE), adaptive control with a constant gain parameter updating law (APE), adaptive control with gradient based updating law (Gradient), and a standard PID control, respectively. The controller parameters are set as $k_1 = 11$, $k_2 = 1.5$, $\lambda_1 = 40$, $\lambda_2 = 10$, $\nu = 17/12$, $\sigma_2 = 0.1$. For the parameter estimation, the initial conditions are set as $\hat{\Theta}(0) = [0 \ 6 \ 0 \ 0]$. A standard PID control is also implemented for comparison. The parameters are chosen as: 1) AOPE: $l = 1$, $k = 0.5$, $\rho = 20$ and $\Gamma(0) = 10 * \text{diag}([0.1 \ 1.0 \ 0.1 \ 1.0 \ 0.1])$. 2) APE: $\Gamma_1 =$

$\text{diag}([2 \ 1.1 \ 1 \ 2.0])$. 3) Gradient: $\Gamma_1 = \text{diag}([2 \ 1.1 \ 1 \ 2.0])$ is used. 4) PID control gains are $K_p = 30$, $K_i = 0.05$ and $K_d = 5$. The PID control gains were set for a certain reference $x_d = 0.4 \sin(2\pi t/5.5)$ by first using the genetic algorithm (GA) reported in [39], which can be derived based on the nominal system model. Then, to address the effect of modeling uncertainties, the obtained PID gains are slightly modified by using a trial-and-error method to achieve better control response. All the control gains are fixed for other references to show/compare their ability of adapting different operation scenarios. It is true that it might be possible to obtain better tracking performance for specific reference if we retune some control gains for each reference. However, the returning process is time-consuming and thus not preferable in practice.

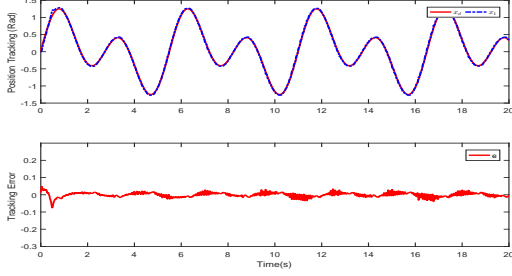
To quantitatively compare the performance of different controllers, the following error indices are adopted: 1) Integrated absolute error $IAE = \int |e(t)|dt$; 2) Integrated square error $ISDE = \int (e(t) - e_0)^2$; 3) Integrated absolute control $IAU = \int |u(t)|dt$, and 4) Integrated square control $ISDU = \int (u(t) - u(0))^2$, where $u(0)$ is the mean value of control action.

C. Experiment Results

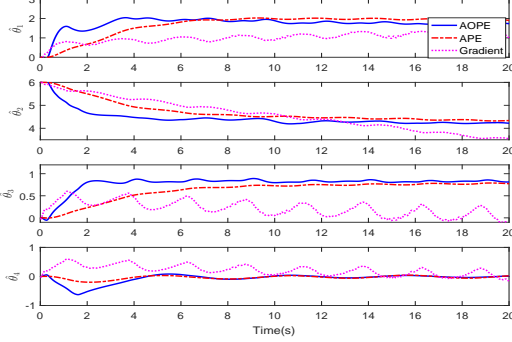
The control performance of the proposed adaptive control with parameter estimation is experimentally evaluated by comparing their responses under different position references. To evaluate the control and parameter estimation performances, experiments are first conducted to track a compound reference signal $x_d = 0.6 \sin(2\pi t) + 0.8 \sin(4\pi t)$. The experimental results are shown in Fig.7.

Fig.7 (a) show the output position versus the desired trajectories and tracking error. Fig.7 (b) depicts the parameter estimation for different updating laws (e.g., AOPE, APE, and Gradient). From Fig.7 (a), one can see that the position tracking can achieve relatively satisfactory performance (the tracking error in the steady-state is around 0.01 rad, equivalent to 0.6 degree). Moreover, we can find in Fig.7 (b) that the proposed AOPE and APE methods can guarantee that the estimated parameters to converge to the actual values. However, the transient response of the proposed AOPE is better than APE. This is attributed to the time-varying gain used in the proposed AOPE, which can address the effect of the regressor. Among three parameter estimation methods, the gradient algorithm produces the worst estimation performance since it is only driven by the tracking error s .

Moreover, to further demonstrate the effectiveness of the presented algorithm, comparative experiments have been carried out for two different reference signals (low speed, and high speed). Fig.8 (a)-(b) shows the position tracking response and tracking error for two sinusoidal trajectories x_d with different amplitudes and frequencies, i.e., $x_d = 0.4 \sin(2\pi t/5.5)$, and $x_d = 0.8 \sin(2\pi t/4)$. As depicted in Fig.8 (a), the tracking errors of the proposed control with AOPE, APE, gradient and PID methods are around 0.01, 0.02, 0.03, and 0.032 rad in the steady-state, respectively, though the transient tracking error reaches 0.1 rad before the adaptive laws achieve convergence. Moreover, Fig.8 (b) shows the tracking errors of three controllers for the reference $x_d = 0.8 \sin(2\pi t/4)$ with a



(a)



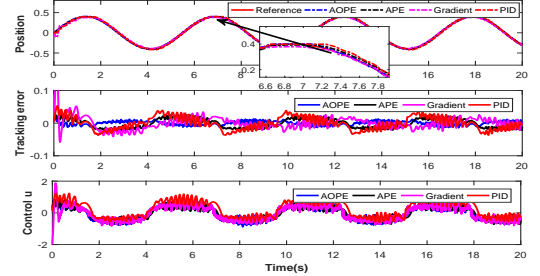
(b)

Fig. 7. Experiment results (a) Position tracking and tracking error (b) Parameters estimates for θ_1 , θ_2 , θ_3 and θ_4

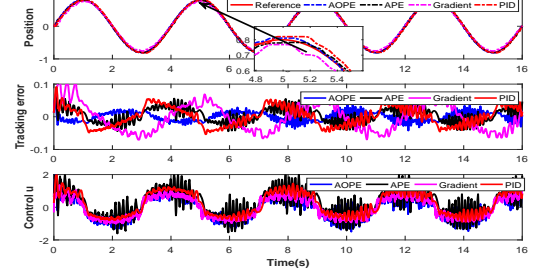
larger amplitude and a smaller period (implying fast motion dynamics), the tracking error for fast varying trajectories shown in Fig. 8(b) is slightly larger than Fig.8(a). Again, as it is shown, the tracking performance of the proposed adaptive controller with AOPE is superior over the other three control methods. It is also noted that for some case studies, the configured hardware (e.g., encoder) may lead to unavoidable lag in the control response, which also contributes to the anti-phase issue as shown in Fig.8. It is also noted that the test-rig is not build to operate in nano-scale precision. Hence, the ultimate control precision is determined by the hardware configurations. However, the provided experimental results all clearly demonstrate the advantages and better response of the proposed control scheme over other methods in terms of both the control response and estimation performance.

It should also be noted that since all parameters in the test-rig are unknown, these experimental results are indeed obtained in the presence of fully unknown parameters, and dedicated to verify the effectiveness of handling the parameter uncertainties. In fact, this is also a benefit for using the adaptive control scheme. Nevertheless, the collected measurements (e.g., x , \dot{x}) via the encoder suffer from measurement noise, so that the obtained experimental results also show the effects of the disturbance. Hence, the tracking errors converge to a small set around zero in the experiments.

Table I summarizes the performance indices for all different experimental results. One can see from Table I that the proposed adaptive control (39) with new adaptive law (40) obtains smaller IAE and ISDE in all cases and thus better control performance, i.e., it can obtain smaller tracking error. The



(a)



(b)

Fig. 8. Experiment results : (a) Low speed tracking and error for $x_d = 0.4 \sin(2\pi t/5.5)$; (b) High speed tracking and error for $x_d = 0.8 \sin(2\pi t/4)$

adaptive controller (39) with APE algorithm achieves fair control performance in terms of IAE and ISDE, while the gradient control method leads to the largest tracking errors. This exactly illustrates how the addition of the estimation error $\hat{\Theta}$ and the time-varying gain Γ allows for the compensation of time-varying dynamics to improve the overall estimation and control performance. With respect to the required control actions, it is interesting to note that all these three controllers require very similar control efforts (i.e., IAU). In addition, among these four control algorithms, PID control produces the worst control performance. Nevertheless, in terms of computational costs, PID control with 3 multiply and 3 addition operations within sampling interval, 10 ms, which clearly performs superior than the proposed AOPE that requires 32 multiply and 28 addition operations. However, as we explained, the adopted DSP can run these computations within 10 ms.

TABLE I
COMPARISON RESULTS OF PERFORMANCE INDICES.

	$x_d = 0.4 \sin(2\pi t/5.5)$				$x_d = 0.8 \sin(2\pi t/4)$			
	PID	Gradient	APE	AOPE	PID	Gradient	APE	AOPE
IAE	0.0161	0.0139	0.0129	0.0046	0.0244	0.0361	0.0193	0.0118
ISDE	0.0082	0.0086	0.0043	0.0032	0.1640	0.0339	0.0101	0.0039
IAU	10.7713	8.8217	8.5839	8.6910	13.1120	11.7714	14.1843	11.7925
ISDU	5.5950	4.4431	4.1557	4.1351	9.9513	7.8721	13.0298	9.2363

VIII. CONCLUSION

This paper developed a novel adaptive optimal parameter estimation and the associated control scheme for servo mechanisms with unknown parameters. Auxiliary filter variables

are first developed to extract the information of parameter estimation error. Then, a cost function of the extracted estimation error is constructed to derive a time-varying gain in the adaptive law to achieve optimal parameter estimation. Moreover, an adaptive NTSMC is designed by combining the terminal sliding manifold with the proposed learning algorithm to achieve convergence of both the tracking error and estimation error simultaneously. Simulation and experimental results demonstrated that the proposed method can obtain superior parameter estimation and tracking performance over classical methods. It is noted that the proposed learning algorithm can be incorporated into other adaptive control synthesis for other systems, e.g., robotics, aerospace, which will be further addressed in our future work.

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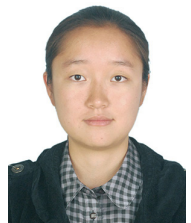
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