Automated Off-line Generation of Stable Variable Impedance Controllers According to Performance Specifications

Alberto San-Miguel, Guillem Alenyà and Vicenç Puig

Abstract—**In this paper, we propose a novel methodology for off-line generating stable Variable Impedance Controllers considering any parameter modulation law in function of exogenous signals to the robot, as e.g. the exerted force by the human in a collaborative task. The aim is to find the optimal controller according to a desired trade-off between accuracy and control effort. Each controller is formulated as a polytopic Linear Parameter Varying system consisting in a set of vertex systems at the limit operation points. Then, the stability and operating properties can be assessed through Linear Matrix Inequalities, from which an optimality index can be obtained. This index is used by a genetic optimisation algorithm to iteratively generate new controller solutions towards the best one. To exemplify our method we choose a case study of modulation laws for tasks that require a physical interaction between human and robot. Generated solutions for different trade-offs are evaluated on a object handover scenario using a 7-DoF WAM robotic manipulator.**

Index Terms—**Physical Human-Robot Interaction, Compliance and Impedance Control, Optimization and Optimal Control**

I. Introduction

ROBOTS are being increasingly introduced in conventional anthropic domains for the sake of further enhancing our lives through the interaction with them. Particularly, industrial applications are shifting towards the use of collaborative robotic platforms to work side by side or even together with humans. Many of them require a physical Human-Robot Interaction (pHRI), where the paramount concern must be to ensure user safety [1]. As a consequence, in these scenarios, the paradigm for robot control has turned from position-based control to the regulation over its compliance.

Robot compliance can be actively regulated through a force controller in a feedback-loop fashion, known as force tracking (or direct force control), or through motion control (or indirect force control). In this last group, the most common technique for pHRI is impedance tracking [2], where a relationship between motion and force is imposed. Standard Impedance Control (IC) imposes a constant relationship, which might not fit complex tasks as e.g. those involving phases of free motion, where accuracy is more relevant, and contact with a human, where the focus is on exchanged forces. In this scenario, modifying over the parameters that define the relationship, which is known as Variable Impedance Control (VIC), is preferred. But, this technique presents many challenges, mainly in terms of stability assessment.

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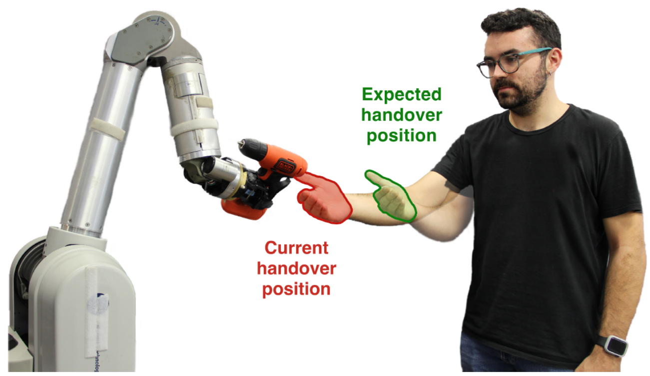

Fig. 1. In collaborative environments, contacts between human and robot might occur unexpectedly, being crucial to control over the exchanged force. In this paper, we put the spotlight on a handover task where an early contact happens.

In this paper, we propose a novel off-line tuning method for VIC through the formulation of the VIC as a polytopic Linear Parameter Varying (LPV) system, which allows to assess controller stability if a generic Lyapunov candidate function fulfilling a set of Linear Matrix Inequalities (LMI) is found. It can be applied to any VIC if parameter modulation depends on a continuous exogenous signal. Moreover, apart from stability, we use the LMI paradigm to (I) add operational constraints to control effort and (II) obtain a optimality index for each controller according to a quadratic trade-off criterion, defined by the user. This is used to generate controller solutions through an iterative optimisation process (a genetic algorithm in this case) to obtain the best one.

To validate our method, we use a case study VIC for an object handover task between a human and a robot. This task has been successfully solved through planning techniques relying on human pose estimation [3] or even considering a measure of mutual trust [4]. But only few solutions exist using IC [5] and, up to the best of our knowledge, none using VIC with stability assessment. Therefore, we propose VIC as a control level solution to complement existing planning an decision making techniques. Our approach focuses on the exchanged force when the human reaches the robot before the expected handover position (Fig. 1), only relying on the measured force.

The paper is organised as follows: Section II introduces VIC and state-of-art in stability assessment. In Section III, the generation of the polytopic LPV model for the VIC is described and detailed for the case study. The LMI problem that assesses VIC properties is presented in Section IV, and its application for the generation of controller solutions in Section V. The presented methodology is evaluated for the case study through simulations and real experiments in Section VI. Conclusions and future work are discussed in Section VII.

II. Background on Variable Impedance Control

Considering a reference trajectory of position, velocity, and acceleration $\{p^r, \dot{p}^r, \ddot{p}^r\}_{t=0}^{T}$, IC aims at imposing the following dynamic relationship between force $\mathbf{F}$ and position error $e := p^r - p(t)$ dynamics. This relationship (in 1-DoF) is characterised by a desired inertia H, damping D and stiffness K, which are time-varying in the VIC case:

$$H(t)\,\ddot{e} + D(t)\,\dot{e} + K(t)\,e = F(t) \tag{1}$$

As aforementioned, imposing a variant behaviour calls for a much more intricate analysis of its stability and operating properties. Classical approaches focus on finding *ad-hoc* Lyapunov candidate functions to derive conditions for stability regarding parameter modulation. In this direction, relaxed conditions are obtained through a modified storage function in [6] for parameter variation profiles with constant inertia, to be evaluated off-line, consisting in a constraint relating the parameter modulation. Recently, authors in [7] provide a particularisation of this method by constructing an on-line filter that maintains stability on-line through the modification of the parameter modulation profile. An energy-based approach is taken in [8] where stability is enforced through passivity by "storing" all the dissipative effects (energy-wise) for performing non-dissipative movements. Although effective, it is state-dependant and requires a fine tuning of the settings for its implementation on a real platform. Our stability and properties assessment, w.r.t. state-of-art approaches, (I) can be applied off-line, (II) considers the complete range of operation of the VIC and (III) does not impose any constraint on the impedance parameter modulations laws, except for being continuous and function of exogenous signals to the robot.

III. Polytopic LPV formulation for VIC

A. State-space formulation

We can formulate the closed-loop impedance relationship from (1) into a state-space form if $\mathbf{x}(t) := [e(t)\ \dot{e}(t)]^T$:

$$\dot{\mathbf{x}}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{K(t)}{H(t)} & -\frac{D(t)}{H(t)} \end{bmatrix}}_{\mathbf{A}(t)} \mathbf{x}(t) + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{H(t)} \end{bmatrix}}_{\mathbf{B_F}(t)} F(t) \tag{2}$$

where $\mathbf{A}(t)$ is the state matrix and $\mathbf{B}_F(t)$ is the force input matrix. Product $\mathbf{A}(t)\,\mathbf{x}(t)$ can be further divided into a state-feedback form, with a constant value $\mathbf{A}_0$ and the variant gain $\mathbf{W}$(t), conforming the controller effort $\mathbf{u}(t)$:

$$\mathbf{A}(t)\mathbf{x}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{A_0}} \mathbf{x}(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\mathbf{B_W}} \overbrace{\underbrace{\begin{bmatrix} -\frac{K(t)}{H(t)} & -\frac{D(t)}{H(t)} \end{bmatrix}}_{\mathbf{W}(t)} \mathbf{x}(t)}^{\mathbf{u}(t)}. \tag{3}$$

Time-varying $H(t)$, $D(t)$ and $K(t)$ make (2) a nonlinear closed-loop system.

B. LPV model for VIC

The Linear Parameter Varying (LPV) paradigm considers time-varying systems whose state-space representation depends on a set of varying parameters $\boldsymbol{\theta}(t) = [\theta_1, ..., \theta_j] \in \mathbb{R}^{n_{\boldsymbol{\theta}}}$ [9]. In the form of system (2):

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\boldsymbol{\theta}(t))\mathbf{x}(t) + \mathbf{B}_F(\boldsymbol{\theta}(t))F(t) \tag{4}$$

each varying parameter $\theta_j(t)$ is assumed to be a-priori known, measured or estimated on-line, and must be function of a set of exogenous (i.e. independent of $\mathbf{x}(t)$) signals to the system $\boldsymbol{\zeta}(t) \in \mathbb{R}^{\boldsymbol{\zeta}}$. To find this description for (2), we can apply the nonlinear embedding method [10], which leads to the following varying parameters:

$$\theta_1(t) = -\frac{K(t)}{H(t)}; \quad \theta_2(t) = -\frac{D(t)}{H(t)}; \quad \theta_3(t) = \frac{1}{H(t)}. \tag{5a-c}$$

The only constraint imposed by LPV formulation on parameter modulation laws is that each $\theta_j(t)$ must be continuous and defined in the operation range of the system.

C. Polytopic LPV Model

Using the LPV formulation, VIC can be described through three varying parameters. But assessing properties for this system considering all their (infinite) reachable values is not tractable. To tackle this issue, a polytopic description of LPV model can be obtained such that we handle only a set of vertex systems at the operating limits. First, all the trajectories of $\boldsymbol{\theta}(t)$ have to be contained within the polytope defined by a convex hull with vertices θ_{v_i}:

$$\Theta = \mathrm{Co}\{\theta_{v_1}, \theta_{v_2}, ..., \theta_{v_N}\} \tag{6}$$

where each vertex is the combination of $\underline{\theta_j}$ and $\overline{\theta_j}$, the lower and upper bounds of each varying parameter, i.e. $N = 2^{n_{\boldsymbol{\theta}}}$. Thus, by defining $\mathbf{A}(t)$ and $\mathbf{B}_F(t)$ in each vertex of Θ to obtain the so-called vertex systems defined by the pair $\mathbf{A}_i, \mathbf{B}_{F,i}$, a polytopic definition of the LPV system (4) can be obtained considering the non-negative coefficients $\pi_i(\boldsymbol{\theta}(t))$:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{N} [\pi_i(\boldsymbol{\theta}(t))\, \mathbf{A}_i]\mathbf{x}(t) + \sum_{i=1}^{N} [\pi_i(\boldsymbol{\theta}(t))\, \mathbf{B}_{F,i}]F(t), \tag{7}$$

In other words, system (4) can be represented at any point of operation $\boldsymbol{\theta}(t)$ through the linear combination of all $\mathbf{A}_i, \mathbf{B}_{F,i}$ weighted by $\pi_i(\boldsymbol{\theta}(t))$. To obtain θ_{v_i} of Θ, the so-called *bounding box* method [11] can be applied, where maximum and minimum values of each θ_j are determined according to the values of ζ_q in the operation range:

$$\underline{\theta_j} = \min_{\zeta_q \in \boldsymbol{\zeta}} \theta_j\,; \qquad \overline{\theta_j} = \max_{\zeta_q \in \boldsymbol{\zeta}} \theta_j\,.$$

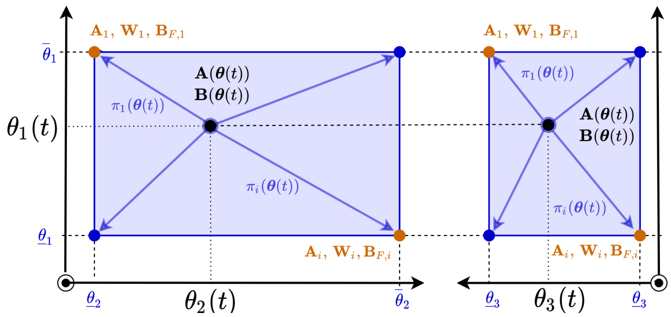

Fig. 2. Representation in $\boldsymbol{\theta}(t)$ space (through two views) of the polytopic LPV model for VIC, corresponding to the vertex of the bounding-box (shaded area) that contains all the values of $\mathbf{A}(\boldsymbol{\theta}(t))$ and $\mathbf{B}_F(\boldsymbol{\theta}(t))$.

D. Case Study

Obtaining the polytopic description of LPV model for VIC requires the chosen set of modulation laws. Considering the handover task, we make use of a case study VIC, adapted from [12], where modulation laws are function of exchanged force (exogenous signal):

$$H(t) = H_0 + \rho\, D(t)\,, \tag{8a}$$

$$D(t) = D_0\,[1 - \delta\,|F(t)|]\,, \tag{8b}$$

$$K(t) = K_0^{1-[|F(t)|/F_{max}]}\,; \tag{8c}$$

To provide a compliant and intuitive cooperation, damping and inertia are modulated according to the exchanged force $F(t)$ with the human, such that when a large motion is intended, i.e. a large force is applied, damping is reduced (and so inertia) to provide a compliant behaviour. From nominal values H_0 and D_0, the magnitude of this effect depends on the incremental gains δ (damping) and ρ (inertia). We also include for this case study a variant non-zero stiffness, following a decreasing exponential function according to $F(t)$ and the maximum possible force exerted by the human in the task F_{max}. Thus, stiffness is equal to its nominal value ($K(t) = K_0$) when no contact with the human is detected ($F(t) = 0$) to accurately follow the trajectory, and decreases its value as force approaches F_{max} to reduce contact force.

As modulation laws (8) only depend on $|F(t)|$, we can consider that $|F(t)| \in [0, F_{max}]$ to obtain $\overline{\theta_j}$ and $\underline{\theta_j}$ for θ_{v_i}, according to definitions in (5). From them, vertex systems $\mathbf{A}_i, \mathbf{B}_{F,i}$ in (7) can be determined, and similarly the vertex variant gain $\mathbf{W}_i$ using (3). Fig. 2 graphically depicts the polytopic LPV model for VIC.

IV. Design Procedure using LMI

Linear Matrix Inequalities (LMI) were introduced to obtain solutions for Lyapunov-based stability controller design involving nonlinear systems, with the key benefit that they can be formulated as convex optimisation problems. As a consequence their use was popularised and therefore there exist many design specifications formulated as LMIs [13]. LMIs can be used to assess properties for polytopic LPV systems by exploiting their main advantage: a LMI is applied for every vertex system $\mathbf{A}_i, \mathbf{B}_{F,i}$ relating them through a common Lyapunov function in order to assess properties for all the possible operation points in Θ. In other words, properties for the complete range of operation of a system can be assessed only by using a set of limit constant ones linked through a common solution.

A. Stability-H$_2$MI Conditions

The paramount concern when designing a controller to be safe is to preserve stability. Moreover, we want to choose the best controller among all the stable controllers according to a task performance criterion. For the VIC case, we should also take into account the effect of the exogenous force $F(t)$ on the controller behaviour. Thus, we use the LMI formulation for H$_2$ index control according to an optimal criterion, adapted from [14]. As VIC will be implemented on a real platform with a control cycle T_c ($t = kT_c$), the problem is defined for the discrete-time form of system (2).

Proposition 1: Stability-H$_2$ LMI Conditions for VIC polytopic LPV model. Considering the discrete-time polytopic LPV model (7), the effect of F through $\mathbf{B}_F$ can be defined by the quadratic criterion J, upper bounded by $\gamma > 0$, through the trade-off constant $\eta \in [0,1]$, and defined by symmetric matrices $\mathbf{Q}$ and $\mathbf{R}$:

$$J = \sum_{k=0}^{\infty} [(1-\eta)^2 \cdot \mathbf{x}_k^T \mathbf{Q}\, \mathbf{x}_k + \eta^2 \cdot \mathbf{u}_k^T \mathbf{R}\, \mathbf{u}_k] < \gamma. \tag{9}$$

where $\mathbf{u}_k = \sum_{i=1}^{N} \pi_i(\boldsymbol{\theta}_k) \mathbf{W}_i \mathbf{x}_k$. Thus, the equilibrium $\mathbf{x} = 0$ is stable in the sense of Lyapunov for $k = [0, \infty) \in \mathbb{N}$ if there exist a solution matrix $\mathbf{P} > \mathbf{0} \,|\, \mathbf{P} = \mathbf{P}^T$ that fulfils the following LMIs $\forall i = 1, \ldots, N$:

$$\mathbf{A}_i \mathbf{P} \mathbf{A}_i{}^T - \mathbf{P} + \mathbf{B}_{F,i} \mathbf{B}_{F,i}{}^T < 0; \tag{10a}$$

$$\text{trace}\left\{ \begin{bmatrix} \mathbf{Q}_\eta{}^{1/2} \\ \mathbf{R}_\eta{}^{1/2} \mathbf{W}_i \end{bmatrix} \mathbf{P} \begin{bmatrix} \mathbf{Q}_\eta{}^{1/2} \\ \mathbf{R}_\eta{}^{1/2} \mathbf{W}_i \end{bmatrix}^T \right\} < \gamma \tag{10b}$$

being $\mathbf{Q}_\eta = (1-\eta)^2 \mathbf{Q}$ and $\mathbf{R}_\eta = \eta^2 \mathbf{R}$.

Proof: The proof is given in Appendix A. ■

Through this formulation, we are able to consider the effect of the exogenous force into the design while fulfilling a performance criterion. Trade-off constant η has to be determined according to the target behaviour of the controller. Higher values of η will shift the criterion towards the minimisation of the controller effort u_k, determined by $\mathbf{R}$, over the minimisation of the states. On the other hand, lower values of η will make the optimisation consider the tracking accuracy (as states are velocity and position errors), determined by $\mathbf{Q}$, over the control effort. Accordingly, we define $\mathbf{Q}$ and $\mathbf{R}$ as follows:

$$\mathbf{Q} = \text{diag}\left\{1/e_d^2,\, 1/\dot{e}_d^2\right\}, \qquad \mathbf{R} = 1/u_d^2\,;$$

denoting $(\cdot)_d$ the maximum allowable values (SI units). These values should be defined according to the task to be performed, considering that those variables with lower values will have a greater impact on the quadratic criterion.

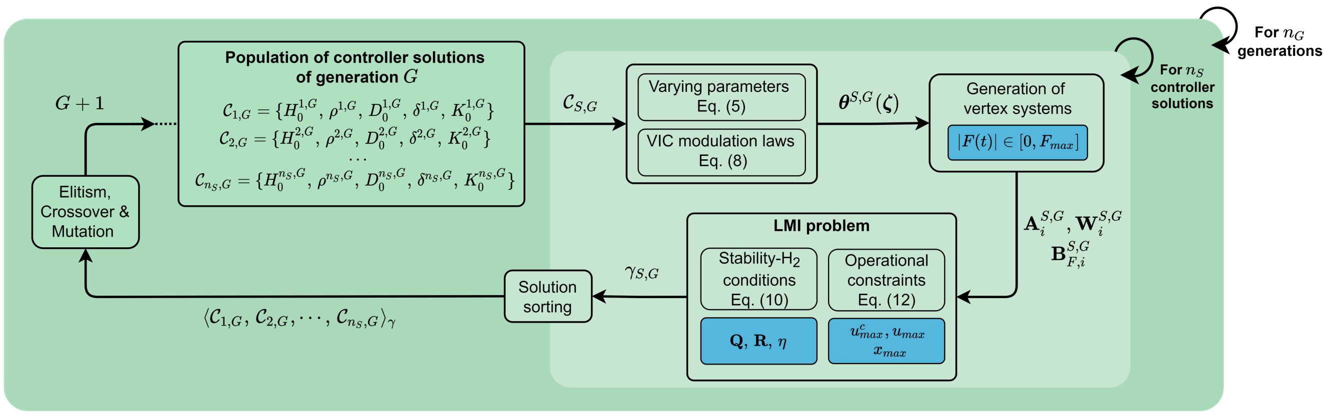

Fig. 3. Scheme for the automated generation of VIC solutions through a LMI problem, using a genetic algorithm implementation. In darker green, processes repeated for each generation G; in lighter green, steps executed for each controller solution $\mathcal{C}_S$ in G; and, in blue, the settings that determine solution generation.

B. Operational LMI Constraints

LMI paradigm allows conforming a design problem involving different conditions. In this paper, we consider a set of constraints to ensure the maximum control effort considering the maximum state values. To consider that the control effort must be reduced when contacting the human, we propose an adaptation from condition in [15] to impose different limits according to the operation point.

Proposition 2: Operational LMI Constraints for VIC polytopic LPV model. On the problem in Prop. 1, constraints

$$\parallel \mathbf{u}_k \parallel_2 \leq u_{max}\ , \quad \parallel \mathbf{x}_k \parallel_2 \leq x_{max}\ ; \tag{11a,b}$$

are satisfied for $k = [0, \infty) \in \mathbb{N}$ for an initial state $\mathbf{x}_0 = \mathbf{0}$ if the following LMIs are fulfilled $\forall i = 1, ..., N$, considering that $u_{max,i} \leq u_{max}$:

$$\begin{bmatrix} u^2_{max,i}\mathbf{I_W} & \mathbf{W}_i\mathbf{P} \\ \mathbf{P}\mathbf{W}_i^T & \mathbf{P} \end{bmatrix} \geq \mathbf{0}\,, \quad \begin{bmatrix} \mathbf{P} & \mathbf{P}\mathbf{A}_i^T \\ \mathbf{A}_i\mathbf{P} & x^2_{max}\mathbf{I_A} \end{bmatrix} \geq \mathbf{0}. \tag{12a,b}$$

where $\mathbf{I_W}, \mathbf{I_A}$ are identity matrices of the appropriate dimensions.
Proof: The proof is given in Appendix B. ■

According to the definition in Eq. (3), as $\parallel \mathbf{u_k} \parallel_2 = \mathbf{u_k}$ the value of u_{max} can be directly interpreted as the maximum control effort. For the selection of a different one for each system vertex, the interpretation of each $\mathbf{A}_i$ is required. For our case study, u^c_{max} will be chosen for those vertex evaluated at the maximum force $\overline{F}$, and u_{max} for the remaining ones, such that $u^c_{max} < u_{max}$. Note that the state value limitation is for the norm of $e(t)$ and $\dot{e}(t)$, and therefore also depends on the reference trajectory. This limitation has been included to adjust the control effort constraint to a bounded set of states and, if an on-line trajectory generation is used, x_{max} can be used to focus the planning of next reference points considering the current state of the system.

V. Automated VIC Solution Generation

First, we have to provide a description of the VIC that unequivocally determines its characteristics. Looking at the modulation laws (8), we can define a controller solution C_s as

$$C_s = \{H^s_0,\, \rho^s,\, D^s_0,\, \delta^s,\, K^s_0\}. \tag{13}$$

Generally, these parameters have to be *manually* determined until a solution that matches desired behaviour is found. We propose to automatically obtain a solution that fulfils a set of operational constraints, is stable and minimises the effect of the exogenous input while being optimal w.r.t. to a performance criterion that trades-off accuracy and control effort, assessed as a LMI problem. Most optimal solution is iteratively found through an optimisation process, in our case, provided by a genetic algorithm.

The process of obtaining a VIC solution has been summarised in Fig. 3. It starts with the selection (generation G) of an initial set (population of size n_s) of solutions, within some defined bounds for each parameter. For every solution C_s, the values of each $\theta^{s,G}_j$ as defined in (5) are obtained from the desired modulated laws ((8) for our case study VIC), as function of the exogenous variable $\boldsymbol{\zeta}$ ($|F(t)|$). Upper and lower bounds of the varying parameters can be obtained to generate vertex variables $\mathbf{A}^{s,G}_i$,$\mathbf{B}^{s,G}_{F,i}$ and $\mathbf{W}^{s,G}_i$, considering the domain of $\boldsymbol{\zeta}$ during the task. Thus, the LMI problem can be stated, involving stability and optimality conditions from (10), defined through $\mathbf{Q}$, $\mathbf{R}$ and η; and operational constraints from (12), according to desired u^c_{max},u_{max} and x_{max}. The minimum feasible upper bound $\gamma_{s,G}$ for the optimality criterion J defined in (9) is used to sort all the n_S solutions within generation G, which determines the creatation of the next generation of controllers $G+1$. For genetic algorithms, this is performed through three main processes: elitism, crossover and mutation. As this topic is out of the scope of the article, reader is referred to [16] for an in-depth description.

Note that if VIC modulation laws provide in the operation range a desired inertia $H(t) = 0$, varying parameters (5) take an indeterminate form. In the automated solution generation, this is evaluated for the domain of exogenous variables within the generation of vertex systems. If so, a high γ is given to the solution C_S, as genetic algorithm is driven towards objective minimisation. The same procedure is applied to those solutions that do not fulfil conditions of the LMI problem.

TABLE I
PARAMETERS FOR LMI DESIGN PROBLEM

$\boldsymbol{e}_d$	$\dot{e}_d$	$\boldsymbol{u}_d$	$\boldsymbol{x}_{max}$	$\boldsymbol{u}^c_{max}$	$\boldsymbol{u}_{max}$
$2.5\cdot10^{-2}$[m]	0.1[m/s]	u^c_{max}	$(e_d^2+\dot{e}_d^2)^{1/2}$	50[N/kg]	100[N/kg]

TABLE II
PARAMETER BOUNDS FOR PROPOSED VIC SOLUTIONS.

Bound	H_0^s [kg]	$\rho^s\cdot10^{-3}$	D_0^s [N·s/m]	$\delta^s\cdot10^{-3}$	K_0^s [N/m]
Upper	2	10	500	10	2500
Lower	0.5	1	10	1	100

TABLE III
BEST VIC SOLUTIONS FOR DIFFERENT TRADE-OFF VALUES

η	H_0[kg]	$\rho\cdot10^{-3}$	D_0 [N·s/m]	$\delta\cdot10^{-3}$	K_0 [N/m]	γ
0.01	1.32	9.2	498.25	9.1	1900	19.75
0.5	1.71	9.1	401.33	7	1349	5.31
0.99	1.86	9.1	398.1	5.2	402	$2.76\cdot10^{-2}$

TABLE IV
PERFORMANCE OF BEST VIC SOLUTIONS FOR DIFFERENT TRADE-OFF VALUES IN SIMULATED SCENARIO.

η	Pos. RMSE [m] $\cdot10^{-3}$	Max. $\lvert\mathbf{u}\rvert$[N/Kg] (with $F=0$)	Max. $\lvert\mathbf{u}\rvert$[N/Kg] (with $F=F_{max}$)
0.01	0.44	16.65	15.36
0.5	0.46	15.22	13.84
0.99	0.48	14.51	13.03

VI. VALIDATION

A. *Design and VIC solutions*

We validate the method presented in this paper through a design problem. According to empirical experience on handover tasks, we set $F_{max} = 50$ [N] to define the operation limits for the VIC. For the automated generation of VIC solutions, we have selected three different trade-off values η: 0.01 to focus on trajectory tracking, 0.99 to prioritise control effort minimisation and 0.5 to obtain a solution that compromises both objectives. Remaining parameters that define the LMI problem are in Table I, and bounds used for each parameter of C_s in the genetic algorithm implementation in Table II. Comments on how these specifications have been determined are given in Appendix C. Details on method implementation and execution are given in Appendix D.

Table III has the best VIC solutions for each η, and Figure 4 graphically depicts the family of best VIC solutions in the (normalised) parameter space and the standard deviation of the top ten solutions. For all the solutions, ρ is the parameter with the greater variance, which suggests that its not decisive on determining the optimality of a solution.

B. *Fulfilment of Operational Constraints*

To evaluate the fulfilment of the operational constraints in the LMI problem, we have implemented the VIC solutions in a simulated scenario to control a double integrator system with a constant nominal mass $M = 2$[kg], and evaluate their behaviour with ($t = [0, 0.5]$[s]) and without any force ($t = (0.5, 1]$[s]). Therefore, we have designed a trajectory such that $\| \mathbf{x} \|_2$ reaches x_{max} and focus the analysis on the behaviour of the control effort $\mathbf{u}$. As Figure 5 shows, control effort remains inside the bounds for all the controllers. Notice also that control efforts show the behaviour of the controllers related to their trade-off, having the one for $\eta = 0.01$ the greater values, as for this case trajectory tracking is prioritised over control effort. Values for RMSE in position and mamimum control effort have been included in Table IV.

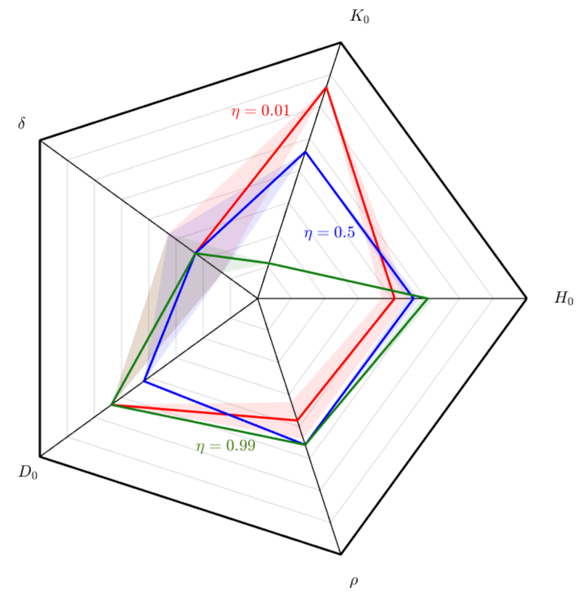

Fig. 4. Polygonal plot with the best VIC solutions for different η (solid lines) and the standard deviation of each parameter for the top ten solutions (shaded areas).

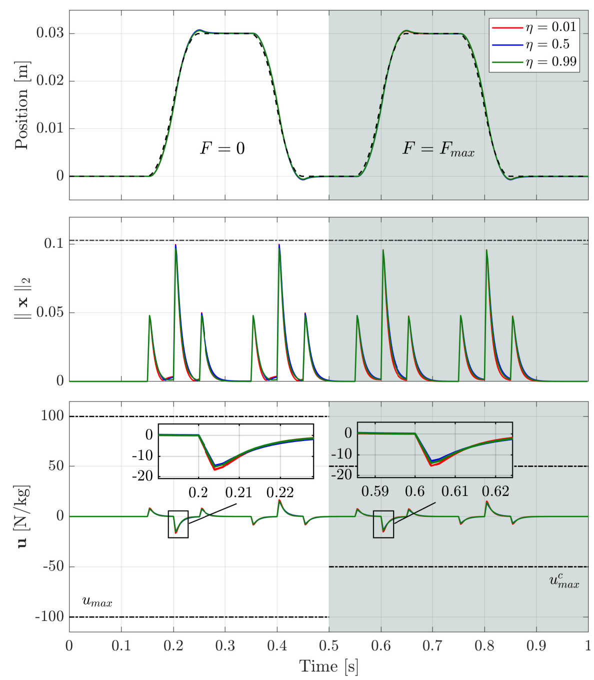

Fig. 5. Evolution of position, state norm and control effort for the simulated scenario, under no force ($t = [0, 0.5]$[s]) and with the force equal to the maximum ($t = (0.5, 1)$[s])

TABLE V
PERFORMANCE OF BEST VIC SOLUTIONS FOR DIFFERENT TRADE-OFF VALUES IN HANDOVER TASK.

η	Pos. RMSE $[\mathrm{m}] \cdot 10^{-3}$ (Free Space)	Max. Force [N] (Contact)
0.01	22.4	14.07
0.5	22.6	13.1
0.99	27.1	11.16

C. Handover experiments

Finally, each controller has been implemented on a 7-DoF WAM robotic manipulator[1] to perform a handover task over one linear direction. To impose VIC relationship in (1) to the closed-loop behaviour, we have followed the customary inverse dynamic approach [17], which only requires force sensing capabilities at the robot end-effector and an on-line available dynamic mode. Robot's control cycle is 500 [Hz] ($T_c = 2 \cdot 10^{-3}$[s]) and signal discretisation corresponds to a Zero-Order Hold. The handover scenario consists on the robot performing a linear movement to an expected handover position, where at $\approx 1/2$ of the trajectory the human (a fixed platform in the experiments) contacts the robot, retaining its movement. Therefore, we can focus on the evolution of the exchanged force for the VIC solutions according to different trade-offs. Note that, as we want to evaluate the performance of the VIC strategy we rely solely on the controller, even after the contact is made, and no re-planning strategy is invoked.

Position and contact force evolution are presented in Figure 6, and Table V includes some quantitative performance metrics. Regarding the trajectory tracking performance, looking at the free motion section (until $t \approx 5.4$[s]), it can be seen (zoom-in graph) that VIC solution for $\eta = 0.01$ has the best accuracy, followed by $\eta = 0.5$ (with quite similar performance) and $\eta = 0.99$. Accordingly, on the contact force evolution, the highest contact force is reached with the controller for $\eta = 0.01$; and the lowest for $\eta = 0.99$. Again, these results reflect that the automated generation provides solutions according to the desired behaviour defined by trade-off η. It should be mentioned that controller $\eta = 0.99$ showed a noticeable "stumbling" behaviour as, due to its lower gains, is not able to overcome minor non-compensated friction effects. As a consequence robot impacted the surface at a high speed producing the peak seen on force evolution. For the same experiments, the impedance parameter modulations are depicted in Figure 7. A video including the experiments for each VIC solution is available in the dedicated webpage[2]. This video also contains an example of the use of generated VIC solutions in a handover task with a human.

VII. CONCLUSIONS AND PERSPECTIVES

In this paper, we have presented a novel method to off-line generate VIC controllers, assessing their stability through its LPV formulation and a LMI design procedure. Additionally, this framework allows to include operational constraints for the control effort according to a maximum state norm. Best VIC solutions are found through an automated iterative process, a genetic algorithm in our case, considering their fitness according to a criterion that trade-offs between minimising trajectory tracking error or control effort that at the same time corresponds to the effect of the exogenous force into the input. Results show that all VIC solutions fulfil operational constraints and behave according to desired trade-off. Particularly, it is shown to be related with the exchanged force during contact in a set of experiments on a handover task. Thus, those solutions generated prioritising tracking accuracy provide higher contact forces; and, conversely, if trade-off is defined towards the minimisation of the control effort, contact forces are reduced, but in exchange for higher tracking errors.

As future work, we are aiming towards including other conditions in the LMI problem, related with energetic properties, as e.g. passivity and dissipativity [18], or oscillatory phenomena [19]. We are also working to embed interaction

[1]WAM by Barrett: https://advanced.barrett.com/wam-arm-1
[2]http://www.iri.upc.edu/groups/perception/#AutomatedGenVIC

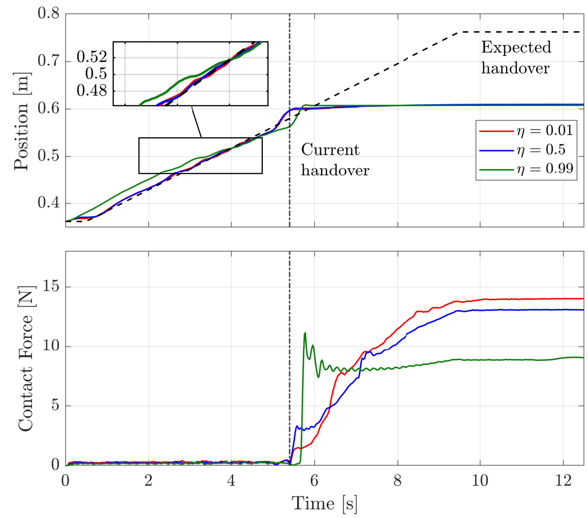

Fig. 6. Evolution of position and force for the handover scenario with the WAM Robot.

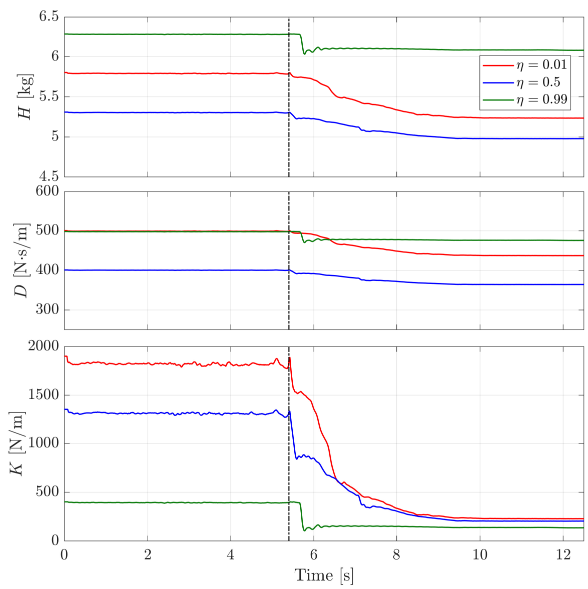

Fig. 7. Evolution of VIC parameter for the handover scenario with the WAM Robot.

directly within the VIC model, as e.g. by modeling human behaviour as in [20]. Other issues to be addressed in the future could be the use of state-of-art solution generation as e.g. based in Bayesian optimisation, or LMI conmtraint relaxation techniques as e.g. the one presented in [21].

Appendix [3]

A. Proof of Proposition 1

1) LMI conditions for Lyapunov stability for polytopic LPV models: The LMI Lyapunov stability condition for the equilibrium point $\mathbf{x}_k = 0$ as stated in [13] applied to the discrete-time for of the VIC polytopic LPV model in Eq. (7) leads to the following inequality for $\mathbf{P} = \mathbf{P}^T > 0$.:

$$\mathbf{A}_i \mathbf{P} {\mathbf{A}_i}^T - \mathbf{P} < 0 \tag{14}$$

2) Equivalence between H_2ndex and quadratic criterion: Introducing an auxiliary output $\mathbf{y}_k$ to the discrete-time form of system (2) and considering definition in (3):

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_F F_k; \quad \mathbf{y}_k = (\mathbf{C} + \mathbf{D}\mathbf{W}_k)\mathbf{x}_k \tag{15}$$

where we can define $\mathbf{S} = \mathbf{C} + \mathbf{D}\mathbf{W}_k$. The effect of the exogenous input F_k in $\mathbf{y}_k$ (which is a modulation of $\mathbf{x}_k$) can be determined through the H_2 norm over the infinite horizon of the transfer function $G_{(y,F),k}$:

$$\|G_{(y,F)}\|_2^2 = \sum_{k=0}^{\infty} {G_{(y,F)}}^T G_{(y,F)} = \sum_{k=0}^{\infty} {\mathbf{x}_k}^T \mathbf{S}^T \mathbf{S} \mathbf{x}_k$$

Thus, choosing $\mathbf{C} = [{\mathbf{Q}_\eta}^{1/2}\ 0]^T$ and $\mathbf{D} = [0\ {\mathbf{R}_\eta}^{1/2}]^T$ we obtain the following equivalence with criterion in Eq. (9):

$$J = \|G_{(y,F)}\|_2^2 < \gamma \tag{16}$$

3) Stability-H_2MI Conditions for VIC polytopic LPV model: From the formulation of the transfer function for each of the vertex systems $i = 1, ..., N$, considering $\mathbf{S}_i$ as the images of $\mathbf{S}$ in Θ and equivalence (16):

$$\begin{aligned} \|G_{i,(y,F)}\|_2^2 &= \textstyle\sum_{k=0}^{\infty} \mathrm{trace}\{G_{i,(y,F)} G_{i,(y,F)}^T\} \\ &= \mathrm{trace}\{\mathbf{S}_i (\textstyle\sum_{k=0}^{\infty} {\mathbf{A}_i}^{k-1} \mathbf{B}_{F,i} {\mathbf{B}_{F,i}}^T ({\mathbf{A}_i}^{k-1})^T) {\mathbf{S}_i}^T\} \\ &= \mathrm{trace}\{\mathbf{S_i} \mathbf{X}_{C,i} {\mathbf{S_i}}^T\} < \gamma \end{aligned} \tag{17}$$

Term $\mathbf{X}_{C,i}$ is the controllability gramian of (15), which happens to be the solution of the following Lyapunov equality:

$$\mathbf{A}_i \mathbf{X}_{C,i} {\mathbf{A}_i}^T - \mathbf{X}_{C,i} + \mathbf{B}_{F,i} {\mathbf{B}_{F,i}}^T = 0 \tag{18}$$

To generalise this solution to the LMI framework we substitute $\mathbf{X}_{C,i}$ by a common $\mathbf{P} = \mathbf{P}^T > 0$ for all the vertex systems. Thus equation (18) turns into (10a) and condition (17) into (10b). Stability is also assessed in (10a) as it is equivalent to (14) considering that $\mathbf{B}_{F,i} {\mathbf{B}_{F,i}}^T > 0$.

[3]The extended theoretical proofs from Appendices A and B are available in http://www.iri.upc.edu/groups/perception/#AutomatedGenVIC/extra/AutomatedGenVIC_ExtendedAppendix.pdf

B. Proof of Proposition 2

Following [13], considering solution matrix $\mathbf{P} = \mathbf{P}^T > 0$ and for an initial state $\mathbf{x}_0 = \mathbf{0}$, fulfilling conditions (10) and introducing intermediate variable $\mathbf{F} = \sum_{i=1}^{N} [\pi_i(\boldsymbol{\theta}(t))\mathbf{W}_i]\mathbf{P}$, condition (11a) can stated through the maximum norm of the control effort can be defined as follows:

$$\begin{aligned} \| \mathbf{u}_k \|_2^2 &\le \max_{k\ge0} \| \mathbf{u}_k \|_2^2 = \max_{k\ge0} \| \mathbf{F}\mathbf{P}^{-1}\mathbf{x}_k \|_2^2 \\ &\le \max_{\mathbf{x}} \| \mathbf{F}\mathbf{P}^{-1}\mathbf{x} \|_2^2 \le \overline{\sigma}(\mathbf{P}^{-1/2}\mathbf{F}^T\mathbf{F}\mathbf{P}^{-1/2}) \le u_{max}^2 \end{aligned}$$

Applying Schur lemma [14] for the polytopic formulation leads to (12a), where u_{max} can be substituted by $u_{max,i}$.

Similarly, for operational constraint (11b):

$$\begin{aligned} \| \mathbf{x}_k \|_2^2 &\le \max_{k\ge0} \| \mathbf{x}_k \|_2^2 = \max_{k\ge0} \| \mathbf{A}\mathbf{x}_{k-1} \|_2^2 \\ &\le \max_{\mathbf{x}} \| \mathbf{A}\mathbf{x} \|_2^2 \le \overline{\sigma}(\mathbf{P}^{1/2}\mathbf{A}^T\mathbf{A}\mathbf{P}^{1/2}) \le x_{max}^2 \end{aligned}$$

Again applying Schur lemma [14] for the polytopic formulation leads to (12b).

C. Specifications for Automated Solution Generation

Regarding optimal criterion, $\dot{e}_d$ and e_d have been determined according to the handover task. Thus, we consider position tracking more relevant than velocity tracking so $e_d > \dot{e}_d$, and their values can be determined according to the maximum allowable values on the trajectory. For operational constraints, u_{max}^c has been set to F_{max} (for 1[kg]) to avoid control forces that might surpass the limit, and u_{max} considering experience on WAM robot joint torque limits. Term x_{max} has been defined to tie in with the optimal criterion. Parameter bounds for the automated generation of solutions have been determined to ensure an initial set of feasible solutions following these steps:

1. Lower bounds $\underline{H_0}, \underline{D_0}, \underline{K_0}, \underline{\rho}$ and $\underline{\delta}$ have been defined as an initial solution that fulfils all the LMI conditions.
2. Considering $\underline{\rho}$ and $\underline{\delta}$, upper bounds $\overline{H_0}$, $\overline{D_0}$ and $\overline{K_0}$ have been found such that only conditions (10) are fulfilled.
3. Finally, $\overline{\rho}$ and $\overline{\delta}$ are iteratively found until conditions (10) are not satisfied.

Together with this procedure it should be also taken into account the pararamter effect in the impedance relationship (1) and the real platform, for example, considering signal noise.

D. Method implementation

All processes involving the off-line automated generation have been carried out within MATLAB (R2109b). The available genetic algorithm implementation has been used with a population of 500 controller solutions over 15 generations (for convergence) with the default configuration. LMI problems have been solved using the semi-definite programming algorithms provided by MOSEK [4] (8.1.0.75) through YALMIP toolbox [5] (release 12-10-18). Using an Intel Core i7-8700K CPU @3.70GHz×12 the execution time for each controller assessment has been 0.5 ± 0.15[s], from which (on average) the generation of the vertex systems represents the 33%, problem formulation using YALMIP the 64%, and solving the LMI problem with MOSEK SDP the 2%.

[4]MOSEKsoftware: https://www.mosek.com

[5]YALMIP toolbox : https://yalmip.github.io/

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