Adaptive Nonlinear Parameter Estimation for a Proton Exchange Membrane Fuel Cell

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Abstract

Parameter estimation is vital for modeling and control of fuel cell systems. However, the nonlinear parameterization is an intrinsic characteristic in the fuel cell models such that classical parameter estimation schemes developed for linearly parameterized systems cannot be applied. In this paper, an alternative framework of adaptive parameter estimation is designed to address the real-time parameter estimation for fuel cell systems. The parameter estimation can be divided into two cascaded components. First, the dynamics with the unknown parameters are estimated by a new unknown system dynamics estimator (USDE). Inspired by an invariant manifold, this USDE is designed by applying simple filter operations such that the information of the state derivative is not required. Secondly, an adaptive law driven by the function approximation error is proposed for recovering unknown model parameters. Exponential convergence of the estimated parameters to the true values can be proved under the monotonicity condition. Finally, experimental results on a practical proton exchange membrane fuel cell system are given to verify the effectiveness of the proposed schemes.

Index Terms

Adaptive parameter estimation, proton exchange membrane fuel cell, nonlinear parameters, uncertain system.

I. Introduction

Proton exchange membrane fuel cell (PEMFC) is considered as one of the most attractive renewable energy conversion systems due to its low or zero pollution emission and approximately 40-60% conversion efficiency [1], [2]. Since PEMFC operates at low temperature with fast start-up properties, it has been applied in vehicle power plants and residential power systems [1], [3]. However, the relatively fast rate of mechanical degradation limits the practical applications of PEMFC. To address this issue, many control algorithms have been developed to improve the system performance and ensure the PEMFC operation at safe conditions. In this line, most existing controllers

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for PEMFC systems have been designed based on accurate mathematical models. Thus, it is emerging to carry out modeling work for PEMFC, which is attracting increasing attentions in both academic and industrial sectors [4], [5].

For modeling fuel cell systems, the unknown parameters in the mathematical models stemming from chemical mechanisms should be considered, since they could inevitably affect the model accuracy and even the stability of the model-based control systems. Therefore, adaptive parameter estimation is of considerable interest in the field. The typical adaptive estimation methods, such as gradient-descent algorithm and least-square (LS) algorithm, have been used for specific fuel cell plants in [6]–[8]. These classical methods are limited to linearly parameterized systems only, where the unknown parameters are in a linear form with respect to the regressors. Recently, the adaptive recursive LS algorithm was also applied for hybrid electric trams or vehicles with energy management strategy in [9], [10]. In [11], comparative results between the gradient-descent method and the Kalman filter for a fuel cell stack were presented. However, all these results focus on linearly parameterized models only. In fact, the nonlinearly parameterized function is an intrinsic characteristic in the PEMFC system [1], which makes the existing adaptive parameter estimation unsuitable.

On the other hand, some intelligent optimization methods have been used to determine model parameters for fuel cell systems and lithium-ion battery, such as genetic algorithm (GA) [12], particle swarm optimization (PSO) [13] and harmony search (HS) algorithm [14]. More recently, a modified Monarch butterfly optimization algorithm [15] was adopted to achieve the parameter identification for PEMFC. Sultan *et al.* [16] proposed an improved salp swarm algorithm for PEMFC, where the search performance of the classic salp swarm method is enhanced. In [17], an Elman neural network was used to build the PEMFC model, and a combination of the world cup optimization and the fluid search method was applied to estimate the unknown parameters. In [18], a support vector regression with GA was proposed to estimate the state-of-health for a battery. However, these optimization methods are all based on the *offline* fitting procedures, which are dedicated to minimizing the error based on the whole batch of collected data. Thus, they cannot be used to directly estimate the unknown model parameters in real-time.

In fact, only a few results have been reported for adaptive parameter estimation of nonlinearly parameterized systems. To ensure the solvability of this problem, some specific conditions should be imposed on the nonlinear functions with unknown nonlinearly embedded parameters, e.g., convex or concave constraint, monotonic condition. It is shown that the convexity or concavity condition can ensure the parameter convergence in a local region of the parameter set, and then a min-max algorithm was proposed for a nonlinearly parameterized system [19], [20]. To relax the convex or concave condition, a two-step adaptive estimation was presented in [21], where the unknown dynamics in the nonlinearly parameterized form was first obtained by a high-gain observer and then a parameter estimation was adopted as [22]. In [23], the nonlinearity was approximated based on the Hadamard's lemma to obtain a linearly parametric form. Then a set-based estimation method was used to estimate the uncertainty set and update the nominal parameter values. Recently, an adaptive parameter estimation with Taylor's expansion based local linearization was reported in [4], [24]. On the other hand, the monotonic nonlinearity is also beneficial to achieve the parameter convergence. With this fact, Tyukin *et al.* [25] proposed a gradient-based adaptive law under the monotone condition for the nonlinearly parameterized function. In [26], the immersion and invariance method was proposed by designing a free function to achieve a P-monotone condition. However, this method needs to solve a partial differential equation, which is not a trivial task. Nevertheless, the above methods have not been exploited for fuel cell systems. Hence, it is recognized that adaptive estimation for nonlinearly parameterized systems still remains as a challenging problem in theory, yet a useful topic for fuel cell applications.

The aim of this paper is to develop a constructive cascaded framework for adaptive parameter estimation of a PEMFC system with nonlinearly parameterized parameters. More precisely, the function with unknown nonlinear parameters is first estimated by a new unknown system dynamics estimator (USDE), which can be designed by applying simple low-pass filter operations. Furthermore, a new adaptive law is designed for estimating unknown parameters, which is driven by the function approximation error (difference between the estimated dynamics from USDE and the function with the online updated parameters). It is proved that the exponential convergence of the estimated parameters to the true values can be ensured under a monotonic condition of the functions with nonlinearly embedded parameters. Finally, the proposed schemes is validated via experiments on a practical PEMFC system to showcase its superior performance over the gradient-descent method and the extended Kalman filter (EKF).

To this end, the main contributions of this paper are:

1) A constructive cascaded parameter estimation framework is proposed for nonlinearly parameterized systems. A simple USDE is first used to reconstruct the functions with unknown parameters via the measurable input and output, then an adaptive law is designed to estimate the unknown parameters under a monotonic condition.
2) The proposed method is applied to solve the parameter estimation problem for a PEMFC plant, showing superior performance over the gradient-descent method and the extended Kalman filter (EKF).

The paper is organised as follows: The PEMFC model and the problem formulation are given in Section II. In Section III, the parameter estimation method is proposed together with convergence analysis. Practical experiments on a PEMFC system are presented in Section V. Several conclusions are drawn in Section VI.

II. PEMFC Model and Problem Statement

A. Description of PEMFC

In this section, a ZBT[1] closed-cathode PEMFC stack with 8 cells and 130 W rated power is considered. The practical PEMFC test-rig is depicted in Fig. 1. The input mass flows of hydrogen and synthetic air are controlled by EL-FLOW®F-201C and F-201AC, respectively. Two Cellkraft®P-10 humidifiers are used to add the water stream into the input gases such that the required humidity for the PEMFC stack can be guaranteed. Furthermore, an external fan under on/off mode is used to regulate the temperature of PEMFC stack at the low-power operation condition. Finally, the data acquisition system is built by the LabView®real-time system.

B. Mathematical Model of PEMFC

The modelling of PEMFC system includes dynamics of voltage balance, thermal energy balance and mass balance. Due to the limited space, only a brief description about the voltage balance and thermal energy balance that are

[1]Zentrum für BrennstoffzellenTechnik GmbH

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Fig. 1. The PEMFC test facility.

related to the parameter estimation is introduced in this section. More detailed modelling of mass balance for the PEMFC system can be found in [1], [27], which will not be repeated here.

In order to obtain the average temperature of whole PEMFC stack, we assume that the gases channels, anode, cathode and electrolyte layers in the stack are with the same temperature [28]. Based on this assumption, the thermal energy balance is calculated via the energy conservation principle [28], [29] as

$$\begin{aligned} m_{fc}C_{p,fc}\frac{dT}{dt} &= \sum_{ca}\frac{w_i^{in}}{M_i}\int_{T_{ref}}^{T_{ca,in}} C_{p,i}\left(T\right)dT \\ &+ \sum_{an}\frac{w_i^{in}}{M_i}\int_{T_{ref}}^{T_{an,in}} C_{p,i}\left(T\right)dT - \frac{w_{H_2}^r}{M_{H_2}}\Delta H_r^o \\ &- \sum_{ca+an}\frac{w_i^{out}}{M_i}\int_{T_{ref}}^{T} C_{p,i}\left(T\right)dT - V_{fc}\cdot I - H_l \end{aligned} \tag{1}$$

where m_{fc} and $C_{p,fc}$ are the stack's mass and average specific heat capacity of PEMFC stack, respectively; T denotes the PEMFC stack temperature; I represents the current; i represents each species of gases in the stack channel; $C_{p,i} = a_i + b_iT + c_iT^2 + d_iT^3$ is the specific heat of i gas; a_i, b_i, c_i and d_i are the heat capacity coefficients [30]; ΔH_r^o represents the specific heat of chemical reaction; $T_{an,in}$ and $T_{ca,in}$ denote the input gas temperature for anode and cathode layers; The input and output gas flow rates are w_i^{in} and w_i^{out}, respectively; And the reacted flow rate of hydrogen is $w_{H_2}^r$. For computing the flow rate of each gas, the detailed modelling is explained in [27], [31]. M_i is the molar mass of i gas and M_{H_2} denotes the molar mass of hydrogen. Moreover, there is the heat transfer phenomenon between the PEMFC stack and environment temperature through the external fan. H_l is the convection loss by the external fan under the on/off mode, which is expressed as

$$H_l = \begin{cases} K_l A(T - T_{ref}), & T > T_{set} \\ 0, & T \leq T_{set} \end{cases}$$

where K_l is the heat transfer constant; A is the surface area of PEMFC; and T_{set} is the desired temperature of the stack; T_{ref} is the ambient temperature.

The PEMFC voltage V_{fc} is modelled as [27]:

$$V_{fc} = n \cdot (V_{ner} - V_{act} - V_{ohm} - V_{con}) \tag{2}$$

where n denotes the number of cells; V_{ner} represents the theoretical voltage or Nernst voltage, which is expressed as

$$V_{ner} = \Delta V_0 + \frac{\Delta s}{2F}(T - T_{ref}) + \frac{RT}{2F}\left[\ln(P_{H_2}) + \frac{1}{2}\ln(P_{O_2})\right]$$

where R denotes the gas constant; F is the Faraday's constant; The standard cell potential is ΔV_0; P_{H_2} and P_{O_2} are the partial pressure of hydrogen and oxygen, respectively. Δs is the empirical correction factor, which is dependent on the change of reaction heat. Furthermore, there are three irreversible voltage losses during operation, such as activation losses V_{act}, ohmic losses V_{ohm} and concentration losses V_{con}, which are calculated by

$$\begin{aligned} V_{ohm} &= \frac{\delta_m I}{\eta_m A}, \quad V_{con} = \frac{I}{A}(c_2 \frac{I}{I_l})^{c_3} \\ V_{act} &= V_{act,0} + K_{act}\left(1 - e^{-c_1 \frac{I}{A}}\right) \end{aligned}$$

where the membrane thickness is denoted as δ_m; $V_{act,0}$ is the initial activation potential at zero current condition; K_{act}, c_1, c_2 and c_3 are empirical constants, which depend on gas partial pressure and temperature; I_l represents the limiting current; η_m denotes the membrane conductivity represented as

$$\eta_m = (b_{11}\lambda_m - b_{12})e^{-b_2(\frac{1}{T_0} - \frac{1}{T})}$$

where the reference temperature for membrane conductivity test is at $T_0 = 303$ K; b_{11}, b_{12} and b_2 are empirical values related to the membrane water content λ_m and fuel cell temperature T.

C. Problem Formulation

In the mathematical model of PEMFC, there are some empirical parameters, which determine the behavior of PEMFC and affect the model accuracy. It is generally difficult to determine the most proper parameters to cover different operation regions. In general, ΔH_r^o in the thermal energy model (1) is computed by the specific enthalpy of each gas under the standard temperature and pressure condition [32]. Moreover, Δs can affect the Nernst voltage V_{ner}, which is usually set as the standard value of entropy change [27]. Besides, the empirical value b_2 is related to the membrane water content λ_m while the membrane water content is determined by water transport in the practical PEMFC [33]. In the realistic operations, those parameters may not be precisely determined based on the configurations such that the derived model cannot accurately present the system performance. Hence, we choose the following empirical parameters to be estimated

$$\theta = \left[\Delta H_r^o, \Delta s, b_2\right]^{\mathrm{T}}. \tag{3}$$

It should be noted that the selection of PEMFC model and the model parameters for parameter estimation is to seek for a trade-off between the model accuracy and the required persistent excitation (PE) condition for this nonlinear parameterization systems. Hence, only three critical parameters defined in (3) are considered in this study. For simplicity of notation, the PEMFC models of (1) and (2) with nonlinearly embedded unknown parameters given in (3) are rewritten in a compact form:

$$\dot{x} = f_1(x, u, \theta_1) + g(x, u, y) \tag{4}$$

$$y = f_2(x, u, \theta_2, \theta_3) \tag{5}$$

where $x \in \mathbb{R}$ is the system state of fuel cell temperature T; $y \in \mathbb{R}$ is the system output of voltage V_{fc}; $u \in \mathbb{R}$ is the control input of current I [2]; $g(x, u, y) \in \mathbb{R}$ is a smooth nonlinear function, which can be computed through the measurable state x, input u and output y, $f_1(x, u, \theta_1) \in \mathbb{R}$ and $f_2(x, u, \theta_2, \theta_3) \in \mathbb{R}$ are continuous functions, where the unknown parameters $\theta_1 = \Delta H_r^o$, $\theta_2 = \Delta s$, $\theta_3 = b_2$ are embedded in these nonlinear functions.

The aim of this paper is to estimate the unknown parameters θ. It should be noted that as shown in (4)-(5), the unknown parameters are in a nonlinearly parameterized form, so that the classical adaptive methods [34]–[36] developed for linearly parameterized systems cannot be directly adopted to deal with this problem for PEMFC. In fact, from theoretical point of view, adaptive parameter estimation of nonlinearly parameterized systems has not been fully solved. For fuel cell application, the widely-used approaches to address this problem are the intelligent optimization methods (e.g., PSO), which are *offline* metaheuristic optimization algorithms with heavy computational costs and are dedicated to minimizing the error based on the whole batch of collected data. Hence, the time-varying behavior may not be fully described by the determined steady-state parameters. Motivated by these discussions, the main idea of this paper is to develop a new adaptive parameter estimation scheme for the nonlinearly parameterized PEMFC system (4)-(5) to estimate the unknown parameters θ_1 from system (4) and θ_2, θ_3 from system (5) separately via the measurable state x, input u and output y.

III. Adaptive Parameter Estimation

For the adaptive parameter estimation of nonlinearly parameterized PEMFC system (4)-(5), there are two difficulties to be handled: the first one is that only the input u and outputs x, y are known, while the nonlinear function $f_1(x, u, \theta_1)$ is unknown (since an unknown parameter θ_1 is involved); the second one is that the unknown parameters $\theta_i, i = 1, 2, 3$ are not in a linearly parameterized form, making the classical parameter estimation methods, e.g., [6], [7]), unsuitable.

[2] In the performed experiments, the input mass flows of air and hydrogen are determined by the stoichiometric ratios and the input current. In this case, we take the input current as the control variable.

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Fig. 2. Schematic of the proposed adaptive parameter estimation method.

To tackle these problems, inspired by [21], a two-step estimation procedure is proposed. First, we let $d = f_1(x, u, \theta_1)$ be the unknown dynamics (since the parameter θ_1 is unknown), and then suggest an USDE to reconstruct this uncertainty based on the measurable state x, output y and input u only, where the information of the state derivative $\dot{x}$ is avoided. Secondly, a new parameter estimation method is developed with respect to the nonlinearly parameterized functions $f_1(x, u, \theta_1)$, $f_2(x, u, \theta_2, \theta_3)$, which can estimate the unknown constant parameter θ_1 with the estimation $\hat{d}$ from USDE and the unknown parameters θ_2 and θ_3 from $f_2(x, u, \theta_2, \theta_3)$. The structure of the proposed adaptive parameter estimation framework is shown in Fig. 2.

Before we present the parameter estimation scheme, the following assumptions are introduced for systems (4)-(5):

Assumption 1: The system state x, input u and output y are measurable and bounded.

Assumption 2: The function f_1 with unknown parameter θ_1 is continuous and its derivative is bounded such that $|\dot{f}_1| \leq \eta$ for a constant $\eta > 0$.

Remark 1: The studied PEMFC system trivially fulfills the assumptions mentioned above. In the practical PEMFC plant with proper controllers, the stack temperature T and the input current I, input air flow w_{air}^{in}, and input hydrogen flow $w_{H_2}^{in}$ are all bounded. They can also be easily measured via the installed sensors in the PEMFC. Nevertheless, the knowledge of time derivative $\dot{f}_1$ is not required in the implementation of parameter estimation. Instead, this boundedness condition is used for the convergence analysis of USDE only.

A. Uncertain System Dynamics Estimator

In order to develop the USDE for estimating $d = f_1(x, u, \theta_1)$ using x, y, u only, the following filtered variables of x_f and g_f are defined with respect to the measured x and known g as

$$\begin{cases} \kappa\dot{x}_f + x_f = x, & x_f(0) = 0 \\ \kappa\dot{g}_f + g_f = g, & g_f(0) = 0 \end{cases} \tag{6}$$

where $\kappa > 0$ is a filter constant.

We can construct an invariant manifold with known, filtered functions and variables to motivate the design of USDE. Thus, the following lemma is presented:

Lemma 1: For the system (4) and filtered variables in (6), there is an implicit manifold defined by:

$$\lim_{t\to\infty} \left[\lim_{\kappa\to 0} (x - x_f)/\kappa - g_f - d \right] = 0 \tag{7}$$

and it is invariant and exponentially convergent for any small constant $\kappa > 0$.

Proof: We first define the off-the-manifold coordinate as

$$z = \frac{x - x_f}{\kappa} - g_f - d \tag{8}$$

Based on (4), (6) and (8), its derivative is computed as

$$\dot{z} = \frac{\dot{x} - \dot{x}_f}{\kappa} - \dot{g}_f - \dot{d} = -\frac{z}{\kappa} - \dot{d} \tag{9}$$

A Lyapunov function is chosen as $V_1 = z^2/2$, whose time derivative is

$$\dot{V}_1 = z\dot{z} = -\frac{z^2}{\kappa} - z\dot{d} \tag{10}$$

Based on the Young's inequality and Assumption 2, we can derive that

$$\dot{V}_1 \leq -\frac{1}{2\kappa}z^2 + \frac{\kappa}{2}\dot{d}^2 \leq -\frac{1}{\kappa}V_1 + \frac{\kappa}{2}\eta^2 \tag{11}$$

By integrating (11), we get that $V_1(t) \leq e^{-t/\kappa}V_1(0) + \kappa^2\eta^2/2$. Consequently, for any constant $\kappa > 0$, $z(t)$ will exponentially converge to a small compact set around zero, that is

$$|z(t)| \leq \sqrt{z^2(0)e^{-t/\kappa} + \kappa^2\eta^2} \tag{12}$$

Since $\dot{d}$ is bounded based on Assumption 2, we can derive that $\kappa\dot{d} \to 0$ and $\kappa^2\eta^2 \to 0$ as $\kappa \to 0$. Based on this fact and the boundedness of variable defined in (8), we can draw a conclusion that for $\kappa \to 0$ and/or $\eta \to 0$, such that

$$\lim_{t\to\infty} [(x - x_f)/\kappa - g_f - d] = 0 \tag{13}$$

Therefore, $z = 0$ is an invariant manifold. ■

Since the manifold in (7) provides an implicit mapping from the measurable state x and the filtered variables x_f and g_f to the unknown dynamics d, the USDE can be designed as:

$$\hat{d} = \frac{x - x_f)}{\kappa} - g_f \tag{14}$$

The above USDE can be implemented by applying the filter (6) and trivial calculation in (14). Another salient feature of this USDE is that only one scalar κ used in the filter needs to be set by the designers, which will be explained latter.

Now, we give the following theorem for the USDE (14):

Theorem 1: Consider system (4) with the time-varying dynamics d and the USDE (14), then the estimation error $\tilde{d} = d - \hat{d}$ will exponentially converge to a small compact set around zero.

Proof: We first define the filtered variable d_f of d as

$$\kappa \dot{d}_f + d_f = d, \qquad d_f(0) = 0 \tag{15}$$

Applying the filter $1/(\kappa s + 1)$ on system (4), we have

$$\frac{s}{\kappa s + 1}[x] = \frac{1}{\kappa s + 1}[g] + \frac{1}{\kappa s + 1}[d] + \xi \tag{16}$$

where ξ denotes the effect of the initial condition $x(0)$ through the low-pass filter, which can be neglected since it is exponentially vanishing. This consideration has been widely studied in the adaptive parameter estimation [23], [37].

Recalling (6), (15) and neglecting ξ, we can rewrite (16) in the time-domain form as

$$\dot{x}_f = \frac{x - x_f}{\kappa} - g_f - d_f \tag{17}$$

By comparing (8) with (17), we can derive that $\hat{d} = d_f$. Hence, the derivative of estimation error $\tilde{d}$ is computed as

$$\dot{\tilde{d}} = \dot{d} - \dot{\hat{d}} = \dot{d} - \dot{d}_f = \dot{d} - \frac{d - d_f}{\kappa} = -\frac{\tilde{d}}{\kappa} + \dot{d} \tag{18}$$

Now, a Lyapunov function is chosen as $V_2 = \tilde{d}^2/2$ and its time derivative is derived as

$$\dot{V}_2 = \tilde{d}\dot{\tilde{d}} = -\frac{\tilde{d}^2}{\kappa} + \tilde{d}\dot{d} \tag{19}$$

According to the Young's inequality, we get

$$\dot{V}_2 \leq -\frac{1}{2\kappa}\tilde{d} + \frac{\kappa}{2}\dot{d}^2 \leq -\frac{1}{\kappa}V_2 + \frac{\kappa}{2}\eta^2 \tag{20}$$

By integrating (20), we get $V_2 \leq e^{-t/\kappa}V_2(0) + \kappa^2\eta^2/2$. Consequently, the estimation error $\tilde{d}$ will exponentially converge to a small compact defined by

$$|\tilde{d}(t)| \leq \sqrt{\tilde{d}^2(0)e^{-t/\kappa} + \kappa^2\eta^2} \tag{21}$$

Clearly, one can further verify that $\tilde{d}$ will converge to zero if $\kappa \to 0$ and/or $\eta \to 0$ (when d is constant). ■

Remark 2: There are several advantages of the proposed USDE in comparison to the other estimators (i.e., [38], [39]). The information of state derivative $\dot{x}$ is not required in the USDE due to the introduced low-pass filter operations. Moreover, the discontinuities and chattering phenomena encountered in the sliding mode-based estimator can be avoided. In addition, since fast (exponential) guaranteed convergence of this estimator is rigorously proved, the estimate $\hat{d}$ can be used to replace $f_1(x, u, \theta_1)$ in system (4) for the purpose of parameter estimation.

Remark 3: As shown in the proof of Lemma 1 and Theorem 1, the filter constant κ has an impact on the convergence speed of z and the ultimate bound of estimation error $\tilde{d}$. Hence it should be set as small as possible. On the other hand, the bandwidth of the low-pass filter $1/(\kappa s+1)$ applied on the state x and nonlinear function g in (6) is also related to κ such that the robustness of USDE against external disturbance and noise also depends on κ. Thus, the choice of this filter constant needs to be considered as a trade-off between the robustness and the convergence speed.

B. Adaptive Estimation of Unknown Parameters

In what follows, the unknown parameter vector θ is estimated by using the output y in (5) and the estimated dynamics $\hat{d}$ in (14) to replace $f_1(x,u,\theta_1)$ in (4). Since the functions $f_1(x,u,\theta_1), f_2(x,u,\theta_2,\theta_3)$ are available now, we can design an adaptive law for parameter estimation. However, the nonlinearly parameterized issue of θ_j $(j=1,2,3)$ should be considered specifically, leading to difficulties in the design of adaptive laws. In this line, certain assumptions must be imposed on the functions to ensure the identifiability of these unknown parameters as [19], [26]. Inspired by recent work [25], the monotonicity of nonlinear functions is used in this paper, which can be fulfilled in the PEMFC application.

Assumption 3: [25] The system functions fulfills the following condition

$$f_k\left(x,u,\theta_j\right)=w_k\left(x,u\right)f_{m,k}\left(x,u,\phi_j\left(x,u\right)\theta_j\right) \tag{22}$$

where $k=1,2$ denotes the number of function f_k in (4) and (5); $j=1,2,3$ is the number of unknown parameters to be estimated. $w_k(x,u)\in\mathbb{R}$ and $\phi_j(x,u)\in\mathbb{R}$ are known nonlinear functions with respect to the measurable state x and input u; $f_{m,k}$ is a smooth and continuous function, which contains the linearly parametric terms $\phi_j(x,u)\theta_j$ of unknown parameter θ_j and regressor function $\phi_j(x,u)$.

It is noted that in this PEMFC application, $f_2\in\mathbb{R}$ contains two unknown parameters. Hence, the above assumption should be imposed to analyze the monotonicity of the nonlinearly parameterized function $f_{m,k}$ with respect to $\phi_j\theta_j$. Nevertheless, the above assumption can be fulfilled in the studied PEMFC model. For instance, the regressor function can be chosen as

$$\phi_1=1,\quad \phi_2=1,\quad \phi_3=\frac{1}{T_0}-\frac{1}{T_{fc}}$$

Then we will introduce the following assumption to analyze the monotonicity of the nonlinearly parameterized function and design an adaptive law:

Assumption 4: [25] There exists a function $D_j\in\mathbb{R}$ such that the nonlinearly parameterized function f_k fulfills

$$D_j\left(x,u\right)\left(\theta_j-\hat{\theta}_j\right)\Big(f_k\left(x,u,\theta_j\right)-f_k(x,u,\hat{\theta}_j)\Big)\geq 0 \tag{23}$$

and

$$\begin{aligned}\epsilon_{1j}|D_j\left(x,u\right)\left(\theta_j-\hat{\theta}_j\right)| &\leq |f_k\left(x,u,\theta_j\right)-f_k(x,u,\hat{\theta}_j)|\\ &\leq \epsilon_{2j}|D_j\left(x,u\right)\left(\theta_j-\hat{\theta}_j\right)|\end{aligned} \tag{24}$$

where $\epsilon_{1j}>0$ and $\epsilon_{2j}>0$ are positive constants and D_j is bounded such that $|D_j|\leq\sigma$ for a constant $\sigma>0$.

Assumption 4 implies the monotonicity of nonlinear functions $f_k(x, u, \theta_j)$ of unknown parameters θ_j. Similar conditions have also been used in [25]. When the system (22) satisfies Assumption 4, the nonlinearly parameterized function $f_{m,k}$ goes slower than the linearly parameterized function $\phi_j \theta_j$ such that $f_{m,k}$ fulfills a global Lipschitz condition. For the PEMFC system with the unknown parameter vector θ studied in this paper, we can choose the following function D_j such that condition (24) in Assumption 4 can be guaranteed [25]:

$$D_j(x, u) = \begin{cases} -w_k(x, u)\,\phi_j(x, u), & f_{m,k} \text{ is non-increasing} \\ w_k(x, u)\,\phi_j(x, u), & f_{m,k} \text{ is non-decreasing.} \end{cases}$$

The function D_j with the above formulation is set to ensure that the search process of the adaptive law is in the correct direction.

With this property, an adaptive estimation method will be proposed to estimate the unknown parameters in the nonlinearly parameterized system. With this choice of D_j, an adaptive law can be designed based on the function approximation error $e = \hat{d}_k - f_k(x, u, \hat{\theta}_j)$ between the measured function $\hat{d}_k$ and its estimate with the online updated parameters θ_j, that is

$$\dot{\hat{\theta}}_j = \Gamma_j D_j(x, u)\left[\hat{d}_k - f_k(x, u, \hat{\theta}_j)\right] \tag{25}$$

where $\Gamma_j > 0$ is an adaptive learning gain. Note with the proposed USDE, the estimate $\hat{d}_1 = \hat{d}$ is used to estimate the unknown parameter θ_1 and the measurement $\hat{d}_2 = y$ is used to estimate the unknown parameters θ_2, θ_3.

Before analyzing the convergence of adaptive law (25), the following definition is introduced.

Definition 1: [35] The function $D_j(x, u)$ satisfies the persistent excitation (PE) condition, if there are constants $T_1 > 0$ and $\eta_1 > 0$ such that

$$\int_t^{t+T_1} D_j(x(\tau), u(\tau)) D_j(x(\tau), u(\tau)) d\tau \geq \eta_1 I, \forall t \geq 0 \tag{26}$$

This PE condition has been recognized as a necessary condition to ensure the convergence of parameter estimation [35].

Then, the following theorem can be provided:

Theorem 2: Consider systems (4) and (5) with Assumption 3 and Assumption 4, the adaptive law (25) is applied and the function D_j satisfies the PE condition, then the estimation error $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$ will exponentially converge to zero.

Proof: Based on (23) in Assumption 4, we know that $\tilde{\theta}_j D_j(x, u) \in \mathbb{R}$ and $[\hat{d}_k - f_k(x, u, \hat{\theta}_j)] \in \mathbb{R}$ are with the same sign such that $\tilde{\theta}_j D_j(x, u)[\hat{d}_k - f_k(x, u, \hat{\theta}_j)] = |\tilde{\theta}_j D_j(x, u)||\hat{d}_k - f_k(x, u, \hat{\theta}_j)|$ is fulfilled. Then we choose a Lyapunov function as $V_2 = \frac{1}{2}\tilde{\theta}_j \Gamma_j^{-1} \tilde{\theta}_j$. By differentiating this function V_2 and recalling the adaptive law (25) with the fact $\dot{\tilde{\theta}}_j = -\dot{\hat{\theta}}_j$ (Note θ_j is constant), we can obtain

$$\begin{aligned} \dot{V}_2 &= \tilde{\theta}_j \Gamma_j^{-1} \dot{\tilde{\theta}}_j = -|\tilde{\theta}_j D_j(x, u)||\hat{d}_k - f_k(x, u, \hat{\theta}_j)| \\ &\leq -\epsilon_{1j} |\tilde{\theta}_j D_j(x, u)|^2 \leq 0 \end{aligned} \tag{27}$$

By integrating (27), then there is a constant $T_1 > 0$ such that for $t \geq 0$, it follows

$$\int_t^{t+T_1} \dot{V}_2(\tau) d\tau \leq -\epsilon_{1j} \int_t^{t+T_1} |\tilde{\theta}_j(\tau) D_j(x(\tau), u(\tau))|^2 d\tau \tag{28}$$

Recall the inequality $(a+b)^2 \geq a^2/2 - b^2$ and the fact $\tilde{\theta}_j(\tau)D_j\left(x(\tau),u(\tau)\right) = \tilde{\theta}_j(t)D_j\left(x(\tau),u(\tau)\right) + (\tilde{\theta}_j(\tau) - \tilde{\theta}_j(t))D_j\left(x(\tau),u(\tau)\right)$, we get

$$\int_t^{t+T_1} |\tilde{\theta}_j(\tau)D_j\left(x(\tau),u(\tau)\right)|^2 d\tau \geq \frac{1}{2}\int_t^{t+T_1} |\tilde{\theta}_j(t)D_j\left(x(\tau),u(\tau)\right)|^2 d\tau - \int_t^{t+T_1} |(\tilde{\theta}_j(\tau) - \tilde{\theta}_j(t))D_j\left(x(\tau),u(\tau)\right)|^2 d\tau \tag{29}$$

By using the Cauchy-Schwarz inequality, we have

$$\begin{aligned}
&\int_t^{t+T_1} |(\tilde{\theta}_j(\tau) - \tilde{\theta}_j(t))D_j\left(x(\tau),u(\tau)\right)|^2 d\tau \\
=&\int_t^{t+T_1} \left|\int_t^{\tau} \dot{\tilde{\theta}}_j(\tau_1)D_j\left(x(\tau),u(\tau)\right)d\tau_1\right|^2 d\tau \\
\leq&\Gamma_j^2\epsilon_{2j}^2\int_t^{t+T_1}\int_t^{\tau} |D_j\left(x(\tau_1),u(\tau_1)\right)D_j\left(x(\tau),u(\tau)\right)|^2 d\tau_1 \\
&\times\int_t^{\tau} |\tilde{\theta}_j(\tau_1)D_j\left(x(\tau_1),u(\tau_1)\right)|^2 d\tau_1 d\tau \\
\leq&\sigma^4\Gamma_j^2\epsilon_{2j}^2\int_t^{t+T_1} |\tilde{\theta}_j(\tau_1)D_j\left(x(\tau_1),u(\tau_1)\right)|^2 \int_{\tau_1}^{t+T_1}(\tau - t)d\tau d\tau_1 \\
\leq&\frac{\sigma^4\Gamma_j^2\epsilon_{2j}^2T_1^2}{2}\int_t^{t+T_1} |\tilde{\theta}_j(\tau_1)D_j\left(x(\tau_1),u(\tau_1)\right)|^2 d\tau_1
\end{aligned}$$

On the other hand, we can derive the following equation based on Definition 1:

$$\int_t^{t+T_1} |\tilde{\theta}_j(t)D_j\left(x(\tau),u(\tau)\right)|^2 d\tau \geq \eta_1|\tilde{\theta}_j|^2 \geq 2\eta_1\Gamma_j V_2$$

Then, Eq. (29) implies that

$$\int_t^{t+T_1} |\tilde{\theta}_j(\tau)D_j\left(x(\tau),u(\tau)\right)|^2 d\tau \geq \psi V_2 \tag{30}$$

where $\psi = 2\eta_1\Gamma_j/(2+\epsilon_{2j}^2\sigma^4\Gamma_j^2T_1^2)$ is a positive constant, which can be set smaller than 1. By substituting (30) in (28), we have $V_2(t+T) \leq (1-\epsilon_{1j}\psi)V_2(t)$. Based on Theorem 1.5.2 in [35], we can obtain that $V_2(t) \leq \psi_1 e^{-\mu_1 t}V_2(0)$, where $\psi_1 = 1/(1-\epsilon_{1j}\psi)$ and $\mu_1 = \ln(1/(1-\epsilon_{1j}\psi))/T$. Then, we can get that $|\tilde{\theta}_j(t)| \leq \sqrt{\psi_1}e^{-\mu_2 t}|\tilde{\theta}_j(0)|$, where $\mu_2 = 1/(2\mu_1)$. Thus, the estimation error $\tilde{\theta}_j$ converges to zero exponentially. ■

Remark 4: The exponential convergence of estimation error depends on the PE condition of D_j. Moreover, the boundedness of D_j can affect the convergence speed, as shown in the proof of Theorem 2. Thus, the choice of function D_j needs to be considered in corresponding to system dynamics. In the PEMFC system, the term $w_k\left(x,u\right)\phi_j\left(x,u\right)$ can be set trivially to fulfill the PE condition due to the variations of temperature and voltage, which will be shown later.

Remark 5: Since a cascaded estimation framework is suggested for the unknown parameter θ_1, the convergence of parameter error $\tilde{\theta}_1$ may be affected by the estimation error $\tilde{d}$ of the USDE for nonlinear function $f_1(x,u,\theta_1)$. It has been shown in Theorem 1 that the estimation error $\tilde{d}$ will exponentially converge to a small compact set for any time-varying function $f_1(x, u, \theta_1)$, hence the convergence of $\tilde{\theta}_1$ can be retained though its transient response may be influenced by the variation of f_1, which will be shown in the experiments.

To implement the proposed parameter estimation framework for the nonlinearly parameterized PEMFC model, we present Algorithm 1.

Algorithm 1 Implementation of the Proposed Estimation

1: **Initialization:** Set the initial value $\hat{\theta}(0)$, the filter coefficient κ, and the learning gains Γ_j. Set the sampling time Δt and the stop time t_d.

2: **Start procedure**

3: **while** $t \leq t_d$ **do**

4:
- Calculate $\hat{d}$ via the USDE (14)
- Compute the function $D_j(x, u)$
- Compute the nonlinear function $f_1(x, u, \hat{\theta}_1)$ and $f_2(x, u, \hat{\theta}_2, \hat{\theta}_3)$.
- Update the estimated parameter $\hat{\theta}(t)$ by (25).
- Set $t = t + \Delta t$

5: **return** $\{t,\, \hat{\theta}(t)\}$

6: **end while**

7: **End procedure**

IV. Comparison With Other Estimation Methods

To show the efficacy of the proposed estimation method, we will compare it with the well-known adaptive gradient-descent method [7], [35] and extended Kalman filter (EKF) [40].

A. Adaptive Gradient-descent Method

The adaptive gradient-descent method was originally developed for the linearly parameterized model [7], [35], where the unknown parameters are in a linear form with respect to the regressor. In order to achieve the parameter estimation for nonlinearly parameterized systems, we have to assume that the functions in (4) and (5) fulfill the convex condition, and thus the Taylor's expansion can be used to linearize the nonlinear functions around a local set [4], [24] as

$$f_k = f_k(x, u, \hat{\theta}_j) + \sum_j \left(\theta_j - \hat{\theta}_j\right) \frac{\partial}{\partial \theta_j} f_k(x, u, \hat{\theta}_j) + \nu_k \tag{31}$$

where ν_k denotes the higher-order residual term, which can be considered as a disturbance. We define $\Phi_j = \partial f_k(x, u, \hat{\theta}_j)/\partial \theta_j$ as the known functions since $\hat{\theta}_j$ can be updated by the adaptive law. Then, the PEMFC models (4) and (5) are rewritten as

$$\begin{aligned} \dot{x} &= \Phi_1\theta_1 + f_1(x, u, \hat{\theta}_1) - \Phi_1\hat{\theta}_1 + g\,(x, u, y) + \nu_1 \\ y &= \Phi_2\theta_2 + \Phi_3\theta_3 + f_2(x, u, \hat{\theta}_2, \hat{\theta}_3) - \Phi_2\hat{\theta}_2 - \Phi_3\hat{\theta}_3 + \nu_2 \end{aligned} \tag{32}$$

To apply the gradient-descent algorithm, the following predictor needs to be constructed

$$\begin{aligned} \dot{\hat{x}} &= f_1(x,u,\hat{\theta}_1) + g\left(x,u,y\right) + K_1(x-\hat{x}) \\ \hat{y} &= f_2(x,u,\hat{\theta}_2,\hat{\theta}_3) \end{aligned} \tag{33}$$

where $K_1 > 0$ is a predictor gain. Then, the following gradient-descent adaptive law [7], [35] can be used

$$\dot{\hat{\theta}}_j = \Upsilon_j \Phi_j \delta_j \tag{34}$$

where δ_j is the output error ($\delta_1 = x - \hat{x}$, $\delta_2 = \delta_3 = y - \hat{y}$); $\Upsilon_j > 0$ denotes as the learning gain.

In the gradient-descent adaptive law (34), the used regressors for parameter estimation are derived by the Taylor's expansion, which is sensitive to measurement noise. In this case, the nonlinear functions f_k need to be differentiable with respect to the unknown parameters θ_k. Moreover, the linearization of these nonlinear functions is valid around a local set only, while the effective range of this local set cannot be explicitly defined. Furthermore, the adopted predictor (33) leads to increased computational costs in the practical implementation as verified in the experiments.

B. Extended Kalman Filter (EKF)

For comparison, the widely known EKF given in [40] is also considered. For this purpose, the PEMFC models (4) and (5) are rewritten as

$$\begin{aligned} \dot{z} &= f(z,u) \\ s &= h(z,u) \end{aligned} \tag{35}$$

where the extend states are defined as $z = \left[x, \theta_1, \theta_2, \theta_3\right]^{\mathrm{T}}$. The measurable outputs are defined as $s = \left[x, y\right]^{\mathrm{T}}$. The nonlinear vectors are defined as $f = \left[f_1(x,u,\theta_1) + g\left(x,u,y\right), 0, 0, 0\right]^{\mathrm{T}}$ and $h = \left[x, f_2(x,u,\theta_2,\theta_3)\right]^{\mathrm{T}}$. The EKF [40] is now given as

$$\begin{aligned} \dot{\hat{z}} =& f(\hat{z},u) + P\frac{\partial h}{\partial z}(\hat{z},u)^{\mathrm{T}} R^{-1}(s - h(\hat{z},u)) \\ \dot{P} =& \frac{\partial f}{\partial z}(\hat{z},u)P + P\frac{\partial f}{\partial z}(\hat{z},u)^{\mathrm{T}} + Q \\ & - P\frac{\partial h}{\partial z}(\hat{z},u)^{\mathrm{T}} R^{-1}\frac{\partial h}{\partial z}(\hat{z},u)P \end{aligned} \tag{36}$$

where $\hat{z}$ is the estimated states; Q is the covariance matrix of the drift Gaussian noise; R is the covariance matrix of the measurement Gaussian noise.

In the EKF (36), the first term is the copy of dynamics in (35). Then, the output error with a gain $P(\partial h(\hat{z},u)/\partial z)^{\mathrm{T}} R^{-1}$ is added in (36) to develop the observer of the EKF. This gain is similar to the Kalman-Bucy filter gain used for linear system, where nonlinear functions $f(z,u)$ and $h(z,u)$ are linearized around the estimated states $\hat{z}$ such that there is a local set that can ensure the convergence of the estimation error. In this case, the EKF also requires the convex condition for nonlinear functions as the gradient-descent algorithm. Finally, the extended stated for observer design also impose the increased computational costs.

figure2/current-eps-converted-to.pdf

Fig. 3. The curve profile of the input current I.

V. Practical Validation

In this section, the mathematical model is first calibrated by the standard PSO method [13], [41], which can provide the general steady-state information of unknown parameters and further provide a guideline to evaluate the efficacy of several different estimation results. Therefore, we carried out comparative studies among the proposed parameter estimation method (14) and (25), the adaptive gradient-descent method with the Taylor's expansion (31)-(34) and the EKF (36) to showcase their performance and verify the effectiveness of those estimation methods.

In the performed experiments, the input current is set as $I = 5\ A$. The realistic current profile is depicted in Fig. 3. It is noted that the oscillations in Fig. 3 are the measurement noise, which are less than 0.1 V. In practice, it is trivial to apply such a constant current to the PEMFC. The stoichiometric ratios of air and hydrogen are respectively set as 2.5 and 2, which are used to determine the input mass flows of air and hydrogen [27]. Moreover, the temperature values of the humidifiers at the cathode and anode are set as 30^oC and 40^oC, respectively. The fuel cell temperature is regulated by the cooling fan to maintain the desired temperature. The values of parameters used in the PEMFC model are given in Table I. In order to make comparisons of different estimation methods, experimental data of input current, output voltage and temperature is first recorded in the online experiments. Then, these three parameter estimation schemes are implemented in Matlab/Simulink based on the same experimental data.

The standard PSO method is first employed to seek optimum values for the unknown parameters given in (3) for the baseline of the parameter estimation. Table II shows the search range of unknown parameters for the PSO method in [1], [27]. The sum of root-mean-square errors with respect to the voltage and temperature is considered as the objective function, which is given by

$$\min \sqrt{\sum_{t=0}^{t=N\Delta t} \left(|V_{exp}(t) - V_{fc}(\theta,t)|^2 + |T_{exp}(t) - T_{fc}(\theta,t)|^2\right)}$$

$$\text{s.t. } \theta_{min} \le \theta \le \theta_{max}$$

where U_{exp} and T_{exp} are the experimental data of voltage and temperature, respectively; Δt is the sampling interval and N is the sampling times determined in the experiment; θ_{min} and θ_{max} denote as the minimum and maximum values of θ.

Subsequently, three above mentioned parameter estimation methods: adaptive gradient-descent method, EKF and the proposed method are applied for the derived mathematical model with the experimental data. The learning parameters for the proposed method (14) and (25) are set as $\kappa = 0.7$, $\Gamma_1 = 6.75 \times 10^6$, and $\Gamma_2 = 10^6$, $\Gamma_3 = 1.45 \times 10^6$. Based on the explanations given in Section III-B, the functions D_j used in the adaptive law (25) are chosen as

$$\begin{aligned} D_1 &= -\frac{w^r_{H_2}}{M_{H_2}}, \quad D_2 = \frac{n}{2F}(T_{fc} - T_{ref}) \\ D_3 &= \frac{n\delta_m I}{(b_{11}\lambda_m - b_{12})A} \times \left(\frac{1}{T_0} - \frac{1}{T_{fc}}\right) \end{aligned}$$

such that the monotonicity condition in Assumption 4 and the PE condition in Definition 1 are satisfied. Moreover, the learning parameters for the gradient-descent method (34) are set as $K_1 = 5$, $\Upsilon_1 = 8 \times 10^8$, and $\Upsilon_2 = 10^6$, $\Upsilon_3 = 1.45 \times 10^6$. For the EKF, the learning parameters are set as $Q = \text{diag}\{[10, 10^6, 10, 15]\}$, and $R = \text{diag}\{[0.09, 0.09]\}$. The initial values of the estimated parameters are set as $\hat{\theta}_1(0) = \hat{\theta}_2(0) = \hat{\theta}_3(0) = 0$.

In this experiment, the sampling time is set as $\Delta t = 1$ ms. The computation time required by the proposed method to obtain the estimated parameters at each time step is 1.266×10^{-4} s. The computation time for the other two methods for each time step is around 1.4×10^{-4} s. Fig. 4 and Fig. 5 show the profiles of the estimated parameters by using the PSO, the gradient-descent method, the EKF, and the proposed method. It is illustrated that the estimated results by the gradient-descent method and the proposed method are similar, while the gradient-descent method provides more high-frequency oscillations. The estimation results of the EKF are smoother than other schemes due to its high anti-noise property. Moreover, the estimated parameters by the gradient-descent method and the proposed method are slowly changing around the steady-state values determined by the PSO. Moreover, the Taylor's expansion related to the convex condition is used to linearize the nonlinearity around a local value, hence the estimation performance can be affected by the initial value and the operation scenarios. However, the proposed method based on the monotonic condition can handle the nonlinearly parametric issue directly, so that it can provide a satisfactory performance. For the parameter ΔH^o_r in (3), the unknown dynamics d containing ΔH^o_r is first estimated by the USDE (14). The estimated profile of unknown dynamics d is depicted in Fig. 6. Different from the gradient-descent method, the proposed method with the USDE (14) can provide a fast convergence performance figure2/theta1_total-eps-converted-to.pdf

Fig. 4. Parameter estimation H_r^o by the proposed method ((14), (25)), the PSO and the gradient method (34).

for the parameter ΔH_r^o as shown in Fig. 4. Moreover, there are some oscillations in the estimated results with the gradient-descent method in Fig. 4, which may be induced by the temperature regulation affected by the cooling fan and the noise from the sensor. However, those oscillations can be reduced by the proposed method with the help of the USDE (14), such that those oscillations in $\hat{d}$ does not influence the overall modeling accuracy.

In order to verify the effectiveness of the proposed method and the correctness of the estimated parameters given above, the output voltage and temperature are reconstructed via the model with the estimated parameters. Fig. 7 and Fig. 8 show the reconstructed profiles of the fuel cell voltage and temperature in comparison to the collected experimental data. It is shown that there is an explicit difference between the experimental data and the model output with the estimated parameters determined by the PSO scheme. This is reasonable since the PSO scheme can obtain the steady-state parameters only based on the whole experimental data, while the unavoidable variations in the system may not be captured. Different to the PSO method, the proposed adaptive estimation methods can track the varying dynamics involved in the model parameters for practical PEMFC systems such that they can achieve a superior performance. Specifically, to compare the performance quantitatively, the mean squared error (MSE), the figure2/theta2_3-eps-converted-to.pdf

Fig. 5. Parameter estimation $\Delta s, b_2$ by the proposed method ((14), (25)), the PSO and the gradient method (34).

maximum absolute Error (MAE) and the standard deviation (SD) are used to assess the estimation performance of the Taylor's expansion-based adaptive parameter estimation method, the EKF, and the proposed method, which are defined as follows:

$$MSE = \sum_{t=t_0}^{t=N\Delta t} |e_{ss}(\hat{\theta}, t)|^2$$

$$MAE = \max\{|e_{ss}(\hat{\theta}, t)|^2\}$$

$$SD = \sqrt{\frac{1}{N\Delta t} \sum_{t=t_0}^{t=N\Delta t} |e_{sd}(\hat{\theta}, t)|^2}$$

where $e_{ss}(\hat{\theta}, t)$ is the output error between the collected data and the model output with the estimated parameters $\hat{\theta}$ (i.e., $e_{ss} = V_{exp}(t) - V_{fc}(\hat{\theta}, t)$ or $e_{ss} = T_{exp}(t) - T_{fc}(\hat{\theta}, t)$); $e_{sd}(\hat{\theta}, t)$ is the difference between the model output and the average of the collected output data (i.e., $e_{sd} = V_{fc}(\hat{\theta}, t) - \bar{V}$ or $e_{sd} = T_{fc}(\hat{\theta}, t) - \bar{T}$); $\bar{V}$ and $\bar{T}$ represents average values of collected outputs for the voltage and temperature, respectively. t_0 is the starting computing time, figure2/UIO-eps-converted-to.pdf

Fig. 6. The profile of estimated dynamics $\hat{d}$ by USDE (14).

which is set as $t_0 = 2\ h$ for this comparison. Table III shows the statistical evaluation of the estimation performance of these three methods. It is shown that the proposed method and the Taylor's expansion-based gradient method have similar performances with respect to the output voltage. For the performance of the temperature, the proposed scheme provides a superior performance over the gradient-descent method, in particular for the SD index related to the oscillations. Nevertheless, the EKF obtained worse performance than the other two schemes due to its robustness against noise, which on the other hand makes it less sensitive to the variations involved in the unknown model parameters. Hence, the estimated parameters have less variations as shown in Fig. 4 and Fig. 5. Consequently, the model outputs (voltage and temperature as shown in Fig. 7 and Fig. 8) with the estimated parameters via the EKF are also with less variations in comparison to the results obtained by the other two estimation algorithms.

VI. Conclusion

In this paper, a new adaptive parameter estimation method is proposed for a PEMFC system with unknown parameters embedded in nonlinear functions. The key idea is to first estimate the unknown dynamics containing figure2/voltage_total-eps-converted-to.pdf

Fig. 7. Comparison of the experimental data and the model output V_{fc} with the estimated parameters by the proposed method ((14), (25)), the PSO and the gradient method (34).

the unknown parameters through a USDE with one tuning parameter only. The USDE is constructed by applying simple filter operations on the system input and output measurements, so that the requirement of the derivative of system state is remedied. Then, an adaptive law is designed for estimating the unknown parameters via the function approximation errors, which stem from the difference between the estimated dynamics and the function with the online updated parameters. It is proved that when a monotonicity condition of the functions with unknown parameters is satisfied, the exponential convergence of the estimated parameters to their true values can be guaranteed. The proposed estimation method is evaluated via experiments on a practical PEMFC plant, which imply that the suggested estimation scheme can achieve better responses than the gradient-descent method with Taylor's expansion and the EKF. In our future work, we will apply the sensitivity analysis to select the dominant parameters in the PEMFC model and then extend the proposed method for the time-varying parameter estimation to cover wider operation regions of fuel cells.

figure2/temperature_total-eps-converted-to.pdf

Fig. 8. Comparison of the experimental data and the model output T with the estimated parameters by the proposed method ((14), (25)), the PSO and the gradient method (34).

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He is currently responsible for the IRI fuel cell laboratory, where he is in charge of all the test and trial stations, participating in various projects at national and European level.

TABLE I
THE PARAMETERS FOR THE PEMFC MODEL

Symbol	Description	Value
A	stack area	$50\ cm^2$
$C_{p,fc}$	specific heat capacity for the PEMFC	$5.5\ kJ/(kg \cdot K)$
F	Faraday's constant	$9.6485 \times 10^4\ C/mol$
K_l	heat transfer constant	2000
ΔV_0	standard cell potential	$1.205\ V$
M_{H_2}	molar mass of hydrogen	$2 \times 10^{-3}\ kg/mol$
M_{H_2O}	molar mass of water	$18 \times 10^{-3}\ kg/mol$
M_{N_2}	molar mass of nitrogen	$28 \times 10^{-3}\ kg/mol$
M_{O_2}	molar mass of oxygen	$32 \times 10^{-3}\ kg/mol$
R	gas constant	$8.3145\ J/(mol \cdot K)$
δ_m	membrane thickness	$1.275 \times 10^{-2}\ cm$
I_l	limiting current	$25\ A$
T_{ref}	reference temperature	$25^o C$
b_{11}	empirical value in the membrane conductivity	0.00158
b_{12}	empirical value in the membrane conductivity	-0.0052
c_1	empirical value in the activation losses	627.69
c_2	empirical value in the concentration losses	0.54
c_3	empirical value in the concentration losses	2
n	the number of cells for the PEMFC	8
m_{fc}	stack mass	$2.25\ kg$

TABLE II
PSO SEARCH RANGE OF THE UNKNOWN PARAMETERS FOR PEMFC

Parameter	ΔH_r^o	Δs	b_2
Lower	-1×10^5	-10	-100
Upper	1×10^5	1000	100

TABLE III
COMPARATIVE ESTIMATION PERFORMANCE

Output	Indices	The proposed method	The gradient method	The EKF
V_{fc}	MSE	0.07	0.07	0.31
	MAE	0.013	0.013	0.033
	SD	0.13	0.13	0.27
T	MSE	4.62×10^9	4.62×10^9	4.62×10^9
	MAE	546.6	547.0	547.9
	SD	4.86	8.28	15.49