An Inextensible Model for the Robotic Manipulation of Textiles

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Abstract: We introduce a new cloth model which models the dynamics of textiles as inextensible surfaces. This assumption challenges most models in literature where elasticity is allowed, sometimes by necessity (Textile Engineering) or in the pursuit of spectacularity (Computer Graphics). In [1], the inextensibility assumption is shown to be realistic by comparing simulations to experimental data, finding that the difference between the simulated and the real position of each point in garments is lower than 1cm on average.

Inextensibility is modeled as follows: we assume that our cloth S is a surface (with boundary) moving through space whose metric (first fundamental form) is preserved. In order to implement these conditions (which are in fact PDE's) on a computer, we assume that S has been triangulated (or quadrangulated) and then apply a novel and non-trivial Finite Element discretization to the inextensibility constraints. If we denote by $\boldsymbol{\varphi}(t)$ the position of the nodes of the polyhedron, this gives raise to a smooth (actually quadratic) constraint function $\mathbf{C}(\boldsymbol{\varphi}) = 0$ which must be preserved at all times. Making use of Signorini's contact model, the dynamic equations of motion then would be:

$$\begin{cases} \mathbf{M}\ddot{\boldsymbol{\varphi}} = \mathbf{F}(\boldsymbol{\varphi}, \dot{\boldsymbol{\varphi}}) - \nabla \mathbf{C}(\boldsymbol{\varphi})^\intercal \boldsymbol{\lambda} + \nabla \mathbf{H}(\boldsymbol{\varphi})^\intercal \gamma, \\ \mathbf{C}(\boldsymbol{\varphi}) = 0, \\ \mathbf{H}(\boldsymbol{\varphi}) \geq 0, \quad \boldsymbol{\gamma} \geq 0, \quad \boldsymbol{\gamma}^\intercal \cdot \mathbf{H}(\varphi) = 0, \end{cases} \tag{1}$$

where we have grouped in the force term $\mathbf{F}$ damping, gravity, stiffness, aerodynamics, friction, etc. On the other hand, $\mathbf{H}(\boldsymbol{\varphi}) \geq 0$ contains the implicit equation of a given obstacle (e.g. a table) and in addition self-collision constraints. Since the collision and inextensibility forces turn out to be very stiff, the system must be integrated implicitly. This is done by solving a sequence of quadratic problems with linear constraints (basically by linearizing all non-linearities in (1)). In order to solve these quadratic programs efficiently, we develop a novel active-set numerical algorithm which takes into account which constraints were active from one iteration to the next. To our knowledge, our method is the first that results in a non-decoupled resolution of contacts, friction and inextensibility for cloth simulation in a single pass.

References:

[1] Franco Coltraro, Jaume Amorós, Maria Alberich-Carramiñana, Carme Torras, *An inextensible model for the robotic manipulation of textiles*, Applied Mathematical Modelling, Volume 101, 2022, Pages 832-858.