Leak Detection in Water Distribution Networks Based on Water Demand Analysis

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Abstract:
This paper deals with the problem of leak detection in Water Distribution Networks (WDN). A leak detection method based on the water demand analysis of District Metered Areas (DMAs) is proposed. Historical leak-free data of water demand flow is used to extract minimum, maximum values, and statistical distributions of differences (errors) between demand flow and predicted values at different time hours of the day. The concept of sensor fusion is applied to reduce measurement uncertainties. For this, a virtual measurement is generated that considers each hour of the day a feature and, combined, develops a more accurate error analysis capable of detecting leaks and estimating the leak size magnitude. Furthermore, to increase the accuracy of the leak detection method, prediction errors are analyzed in a moving time window. Finally, the performance of the proposed leak detection method is assessed by using actual data of different real DMAs of the Barcelona WDN.

Keywords: Water Distribution Network, Leak Detection, Flow Analysis

1. INTRODUCTION

Pipeline systems are essential in modern society because of the substance they distribute, such as oil, gas, refined products, etc. One system, in particular, is crucial for the survival of society, which is the water distribution systems in cities. Water leaks are a significant problem in these systems that cause substantial economic losses and environmental issues. Several factors can cause a system leak, such as weak joints, water hammers, construction or excavation of utilities, seasonal temperature changes, heavy traffic, and other things. When a leak occurs, it can cause several difficulties with contamination (Xu et al., 2014) and health problems (Ali and Choi, 2020) and the loss of water at a time when the world's demand for water is only increasing (Leflaive, 2012). Due to the importance of detecting and locating leaks in the system, different methods are studied.

It is possible to divide the methods into three categories: acoustic instrumentation, transient-based, and hydraulic sensors data. The first one is based on collecting data from acoustic sensors (Shimanskiy et al., 2003), cameras (Fahmy and Moselhi, 2010), ground-penetrating radar (GPR) (Stampolidis et al., 2003), and fiber optic (Sadeghioon et al., 2014). The main problem with this category is installation and maintenance costs and high power consumption.

The second category is related to the information of the transient hydraulic flow parameters collected from the sensor and compared with the steady-state equation, an example of the method is shown in (Covas and Ramos, 2010). Thus allowing the detection of anomalies in the system.

The last category is based on hydraulic sensor data and can be referred to as real-time leak detection methods. The main idea is to use information from different sensors, like flow, temperature, pressure measurements, or qualitative parameters such as turbid, to build historical data to predict future parameter values by data mining models. Some hydraulic sensor data analysis examples can be found on (Verde et al., 2016) that can detect and isolate single leaks in a pipeline with a branch junction by measuring flow and pressure at the ends of the line. In (Shekofteh et al., 2020) that apply the artificial neural network (ANN) technique, graph theory combined with pressure sensors. In (Soldevila et al., 2021) uses flow sensor analyses integrated with an ad hoc statistical test to validate the leak detection, and (Laucelli et al., 2016) uses evolutionary polynomial regression (EPR) online data recorded by low-cost pressure/flow devices. Among those methods, a classification based on the analysis of the Minimum Night Flow (MNF) that uses the minimal inlet flow information that usually happens during the night- between 2:00 and 6:00- (Marzola et al., 2021), (Cantos et al., 2020).

A critical circumstance of leak detection on WDN is that many are not equipped to monitor these networks' flow/pressure measurements. Therefore, the only reliable information comes from water inlet points, such as tanks and reservoirs volumes. Therefore, usually the company applies an annual water balance that evaluates the amount of lost water in the network established in the International Water Association (IWA) (Farley, 2003). (Lambert, 2002) the information of the water inflow and the meter readings collected, usually in four or six months.

Another critical factor is the knowledge of estimating the current volume of leakage in the WDN. This aspect is vital for the management of the system, allowing the water distribution company to have the necessary control to keep the leakage at a specific level. In addition, the leak estimation in the system can be used in future investigations to identify the location of the leakage (Alves et al., 2021), (Laucelli et al., 2016), (Soldevila et al., 2016).

This work presents a new leak detection and leak estimation methodology using flow measurement historical data and real-time sensor data available on the inlet network. The method uses sensor fusion calculations with a demand forecast model to identify a leak in the system and provide an estimated leak size.

The remaining of the paper is outlined as follows: In Section 2, a discussion of the proposed leak detection method is detailed. Then, the performance of the proposed method is evaluated using a real DMA of Barcelona WDN in Section 3. Finally, Section 4 concludes the paper.

2. PROPOSED METHOD

The proposed method to leak detection descends from the base of sensor fusion theory using the inlet flow measurement of the WDN to generate a virtual measurement. The technique can analyze if there is a leak in the system during all-day hours. Because of that, the leak detection will be faster than a method based on MNF that uses flow during night hours. Figure 1 shows the schema of the proposed method, which can be divided into two phases. First, the offline phase is the calibration of the parameters. Furthermore, the second online phase is where the evaluation of leak presence in the network is analyzed. We will explain the methodology in detail in the following.

2.1 Methodology description

The fundamental aspect of offline and online phases represents the WDN inlet flow, approximating the current and historical input flow. Therefore, the demand forecast in WDN is out of the scope of this work. However, it can be assumed that a demand forecast method calibrated being historical data of the WDN (Donkor et al., 2014) is available with the WDN input flow $y(k)$ at the instantLI-1 k:

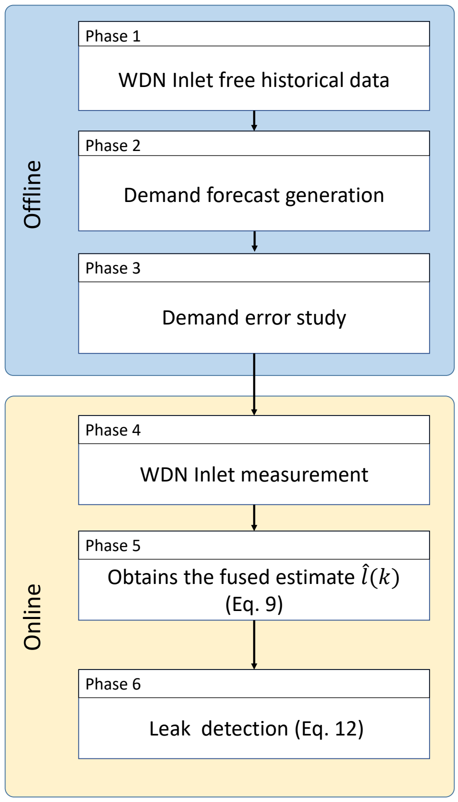

Fig. 1. Overview of the proposed method

$$y(k) = \hat{y}(k) + e(k) \tag{1}$$

where $k = 0, 1, 2, 3, ...$ denotes the discrete time corresponding to time $0, T_s, 2T_s, 3T_s, ...$, being T_s the sample time of demand forecasting model, $\hat{y}(k)$ is the demand forecast and $e(k)$ is the error that for this study is considered adjusted by a normal distribution (Gaussian) (Malik, 2016) represented by the notation $\mathcal{N}(\mu, \sigma^2)$ with mean μ and standard deviation σ. The incoming demand is more accurate in some periods of the days, thus a periodic variation in time T will be considered:

$$e(k) \sim \mathcal{N}(0, \sigma^2) \quad \text{with} \quad \sigma^2 = \sigma^2(k+T) = \sigma^2(k) \tag{2}$$

Let us consider $l(k)$ as the leak indicator, in the presence of a leak, $l(k) > 0$. Thus, Equation (1) can be rewritten as

$$y(k) = \hat{y}(k) + e(k) + l(k) \tag{3}$$

An approximation of the leak size $\hat{l}(k)$ can be given by the difference between the actual and the estimated inlet flow, with a leak estimation error equal to the demand forecasting error.

$$\hat{l}(k) = y(k) - \hat{y}(k) = l(k) + e(k) \tag{4}$$

It is possible to generate different leak estimations using a time window, W, taking into account the current inlet flow value and the previous values using the following equations:

$$
\begin{aligned}
&\hat{l}(k) = y(k) - \hat{y}(k) \\
&\hat{l}(k+1) = y(k+1) - \hat{y}(k+1) \\
&... \\
&\hat{l}(k-W+1) = y(k-W+1) - \hat{y}(k-W+1)
\end{aligned} \tag{5}
$$

Notice that leak estimations $\hat{l}(k-i)$ with $i = 0, ..., W-1$ have zero mean Gaussian errors with variance $\sigma^2(k-i)$. Considering slow leak variation in time window W, we get:

$$
l(k) \approx \bar{l}(k) = \sum_{i=0}^{W-1} \frac{l(k-i)}{W} \tag{6}
$$

an average leak estimation $\hat{\bar{l}}(k)$ can be computed at instant k applying the maximum Likelihood estimation method to the joint probability distribution of the W estimations fused in $\bar{l}(k)$. This joint probability distribution function will be denoted as $p(\hat{l}(k), ..., \hat{l}(k-W+1)|\bar{l}(k), \sigma_W^2)$ and can be expressed as

$$
p(\hat{l}(k), ..., \hat{l}(k-W+1)|\bar{l}(k), \sigma_W^2) = \prod_{i=0}^{W-1} \frac{1}{\sigma(k-i)\sqrt{2\pi}} e^{\frac{-(\hat{l}(k-i)-\bar{l}(k))^2}{2\sigma^2(k-i)}} \tag{7}
$$

where σ_W^2 is the variance of the fused value $\bar{l}(k)$).

The likelihood function L is defined as the logarithm of $p(\hat{l}(k), ..., \hat{l}(k-W+1)|\bar{l}(k), \sigma_W^2)$, given by:

$$
L(\hat{l}(k), ..., \hat{l}(k-W+1)|\bar{l}(k), \sigma_W^2) = -\frac{W}{2}\log(2\pi) - W\sum_{i=0}^{W-1}\log\sigma(k-i) - \sum_{i=0}^{W-1}\frac{(\hat{l}(k-i)-\bar{l}(k))^2}{2\sigma^2(k-i)} \tag{8}
$$

Maximizing the value of $L(\hat{l}_1, \hat{l}_2, ..., \hat{l}_W|\bar{l}(k), \sigma_W^2)$, equaling to zero the derivative of $p(\hat{l}_1, \hat{l}_2, ..., \hat{l}_W|\bar{l}(k), \sigma_W^2)$ with respect to $\bar{l}(k)$, obtains the new virtual fused estimate measurement $\hat{\bar{l}}(k)$:

$$
\hat{\bar{l}}(k) = \frac{\sum_{i=0}^{W-1} \frac{\hat{l}(k-i)}{\sigma^2(k-i)}}{\sum_{i=0}^{W-1} \frac{1}{\sigma^2(k-i)}} \tag{9}
$$

that presents a zero mean estimation error

$$
e_W(k) = \bar{l}(k) - \hat{\bar{l}}(k) \tag{10}
$$

with variance

$$
\sigma_W^2 = \frac{1}{\sum_{i=0}^{W-1} \frac{1}{\sigma^2(k-i)}} \tag{11}
$$

The leak detection problem can be formulated as a change detection problem because, in a non-leak scenario, $\hat{\bar{l}}(k)$ will lead to small values but different from zero due to demand

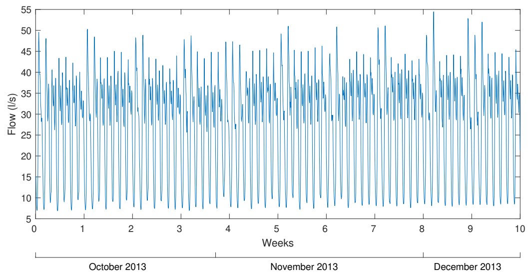

Fig. 2. Historical leak-free inlet flow in Barcelona

estimation errors, and in a leak scenario, its value will increase.

In the offline phase, the computation of the threshold Δ_W using historical free leak data will determine a value of $\hat{\bar{l}}(k)$ above which we can assume that a leak is present in the system. This threshold can be computed applying Equation (9) to historical free leak data, considering the worst-case scenario Δ_W will be equal to the maximum value of $\hat{\bar{l}}(k)$ computed for the whole historical non-leak data.

Given Δ_W, to reduce the number of false alarms, a n_d value can be stipulated, being the several following leak estimations bigger than the threshold that is necessary to trigger the leak detection.

In the online phase, the process of fused estimate leak size can be done, and the leak detection method can be computed by:

$$
\begin{cases} \hat{\bar{l}}(k-i) > \Delta_W \Rightarrow \text{Leak} & , \quad \forall \text{ i=1,..,}n_d \\ \text{Otherwise} \Rightarrow \text{No Leak} \end{cases} \tag{12}
$$

3. CASE STUDY

The method presented was applied to a DMA of the Barcelona WDN. In particular, the set of historical inlet flows free-leak data depicted in Figure 2 was available. For the leak detection analysis, different leak scenarios have been created considering the constant size of the leaks. In all studies, the sample time, T_s, is one hour, and the period T equals 24 (1 day).

The demand forecast was considered only the periodicity of the demand extracted from the historical data to construct an estimate of the current water demand, as proposed in (Donkor et al., 2014). So, given a set of historical inlet flow free-leak data of N_d days sampled at T_s =1 hour, the demand forecast model will consist of 24 values (features) $\hat{y}_h$ with $h = 1, .., 24$ organized by the 1st feature equal to demand forecast at 1 AM, and the 24th is equal to 00 AM. These values are computed from historical data as follows.

$$
\hat{y}_h = \frac{1}{N_d}\sum_{d=0}^{N_d-1} y(h+24d) \qquad h = 1, ..., 24 \tag{13}
$$

The first analysis made was concerning the election of the best amount of features, the leak detection proposed in the previous section considers inlet flow values from all the hours of the day (i.e., all the features), while most leak detection methods are based on Minimum Flow Analysis that only consider the flow at some hours during the night. So a general analysis was made considering time window $W = T = 24$ (i.e., one day) and maximum error threshold for fault detection (12) using a different number of features. In addition, simple thresholds $\delta_h \quad h = 1, ..., 24$ are computed as the maximum error of hourly demand estimations $\hat{y}_h$ computed by (13) considering the historical leak-free data. These thresholds show the lowest leak value detectable only considering hourly measurements. The following equation was used to generate δ_h:

$$\delta_h = \max_{d=0,\dots,N_d-1} |y_h(h + 24d) - \hat{y}_h| \tag{14}$$

This analysis is represented in upper Figure 3, which represents the hourly demand estimation (13) in blue and the upper red line the prediction $\pm\delta_h$ threshold. On the other hand, in the lower Figure 3 represents hourly error variance σ_h^2. Note that the biggest variance σ_h^2 happens between 8 AM to 3 PM; this is already expected because they are the times of the biggest water consumption in cities. Consequently, we have a larger δ_h in those hours. With the same analysis, the smallest variance occurs during the night between 2 AM and 5 AM, as it is the last water consumption period, having the smaller δ_h. Remark that in the worst case of leak detection, a leak is produced in the hours with high σ_h^2.

Continuing the analysis to know how the data is affected by the number of features selected, they were stocked by the best variations in ascending order, i.e., the feature with the smallest variance is now the 1st feature, and the biggest is the 24th feature, showing in upper Figure 4 . In order to know how it would affect the threshold Δ_W, lower Figure 4 depicts the twenty-four errors that can be computed $e_{f|24}(k)$ with $f = 1, ..., 24$ using error Equation (10) but considering leak estimation in Equation (9) only with f features, being $e_{24|24}(k) = e_w(k)$. With every error obtained applying to leak-free data it can be computed a maximum error $\Delta_{f|24}$, being the $\Delta_{24|24} = \Delta_W$. In the analysis of the $\Delta_{f|24}$, it is noted that this value decreases smoothly when the number of features is bigger than five.

The second analyzes were regarding the performance of the leak detection method. Then, artificial data considering observed demand variance and introducing different leak sizes at different time instants have been generated. To analyze this, the following parameters, were considered:

- True Positive Rate (TPR) is the percentage of leaks that are correctly identified as such.
- False Positive Rate (FPR) is the percentage of leak-free data that triggered the leak detection method.
- Difference Time Detection (DTD) is the time (in hours) from the leak appearance to the leak detection.

Tables 1, 2, 3 and 4 were created for the analysis of these parameters. The first choice of the features was 4, which are the first four features with the lower variance that included in the hours of the Minimum Night Flow Analysis. The second resource option was to use all available resources, which means the number of Features f equals

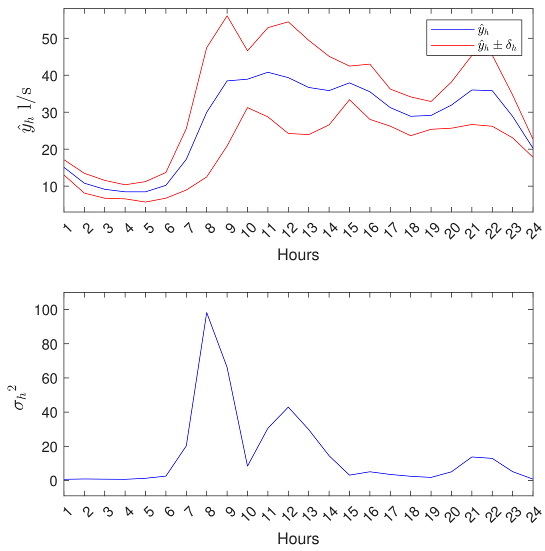

Fig. 3. Hourly demand estimations $\hat{y}_h$ with the respect δ_h and variance values σ_h^2

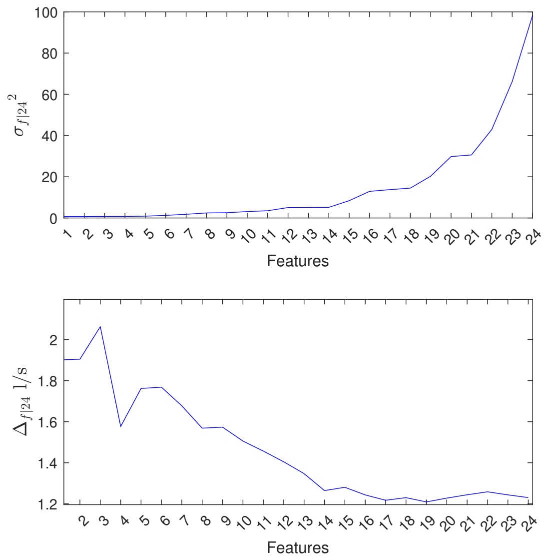

Fig. 4. Sorted σ_h^2 by feature and $\Delta_{f|24}$ with respect to the number of Features f

24. The importance of n_d was also examined, with Table 1 and Table 2 considering n_d equal to 1 and Table 3 and Table 4 considering n_d equal to 3.

Table 1. Leak detection performance considering $n_d = 1$. Considering $f = 4$.

Leak magnitude (l/s)	FPR= 0.009	
	TPR (%)	DTD(hour)
0.5	17.7	162.872
1	90.3	111.374
1.5	100	36.888
2	100	18.998

Regarding n_d, a small increase in the detection time is noted when using $n_d = 3$ because more measures are needed to activate the detection. On the other hand, the

Table 2. Leak detection performance considering $n_d = 1$. Considering $f = 24$.

Leak magnitude (l/s)	FPR= 0.004	
	TPR (%)	DTD(hour)
0.5	36.4	154.212
1	99.5	52.608
1.5	100	18.997
2	100	14.599

Table 3. Leak detection performance considering $n_d = 3$. Considering $f = 4$.

Leak magnitude (l/s)	FPR= 0.005	
	TPR (%)	DTD(hour)
0.5	8.7	160.332
1	78.3	125.795
1.5	100	44.439
2	100	21.243

Table 4. Leak detection performance considering $n_d = 3$. Considering $f = 24$.

Leak magnitude (l/s)	FPR= 0.001	
	TPR (%)	DTD(hour)
0.5	21.6	162.781
1	99.5	69.084
1.5	100	21.904
2	100	16.640

number of FPR is significantly lower: more than 50% reduction when $f = 4$. In addition, it can be noted that it is already possible to detect leaks with a size of $0.5l/s$ but with a small TPR. Besides, it has 100% detection when the leak size is greater than or equal to $1.5l/s$.

For each network, it is necessary to do this type of analysis to know the value of the parameters to choose because each one has different behavior. The parameters must be selected according to each water distribution company's priorities. For example, it is also possible to manipulate data by dividing it into working and unworked days. However, it is necessary to have a wide range of data without leakage for this type of manipulation. For this data presented in Figure 2, it is not worth separating them, as the amount of information is not sufficient.

Table 5 was developed to compare the proposed method's efficiency with a method using only MNF. As already mentioned, this study is used more frequently for leak detection because, at this time of the day has the lowest water consumption and, consequently, the smallest variance. The same case study of the previous analysis was applied, only using the measurements from 2 am to 6 am. The leak estimation error (4) was applied in the leak detection method (12) with $n_d = 1$. The FPR index was a reference to the threshold selection to improve the comparison. In this case, the FPR index of Table 1 was picked because it is an equivalent analysis that uses the four best features during the day that occurs during the night.

The study using only the MNF measurement shows that for an FPR result equal to 0.010%, the TPR is inferior to the result in Table 1 and the time detection is more significant because of the time delay of monitoring, that if a leak is not detected during the night or the leak started during the day, it is necessary to wait 24 hours to obtain new measurement information.

Table 5. Leak detection performance using MNF measurement

Leak magnitude (l/s)	FPR= 0.010	
	TPR (%)	DTD(hour)
0.5	12.6	153.465
1	41.1	145.955
1.5	82.5	115.881
2	99.3	64.333

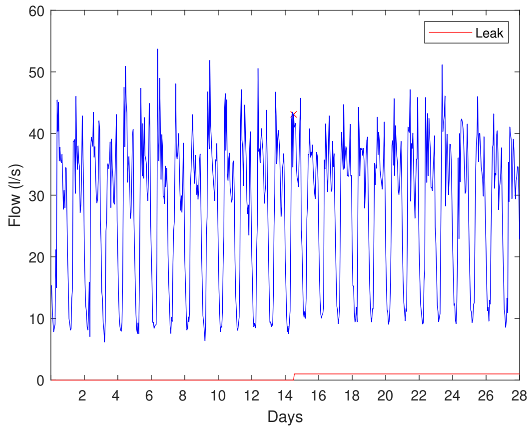

Fig. 5. Inlet flow with a leak,start on the 14th day at 12PM

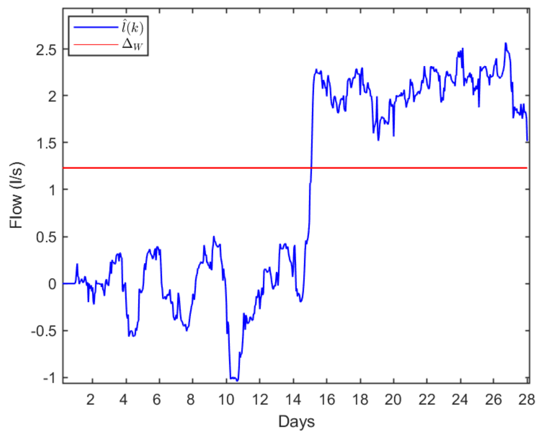

Fig. 6. Error analysis

Figure 5 shows the model of artificial data created with 14 days without a leak, and 14 days with a leak of $2l/s$, the marker "x" and the red line show the exact instant that the leak was produced, in this case at 12 PM, remark that visually it is difficult to identify the leak. Figure 6 shows the error calculated with Equation (9) to the case simulation in Figure 5 . The analysis shows the leak detection when the error crosses the threshold. A second study can be done regarding the average error after detection, which is around $2l/s$ that is the leak magnitude.

4. CONCLUSION

The detection of leaks in WDNs is an essential issue for modern society to prevent losses of hydraulic resources and protect the healthy population from the water contamination that can occur in a leak situation. This paper demonstrated a new method of detection and estimation of leak size magnitude only using the information of inlet flow measurement, which is obtained with the flow sensors usually installed in the system, making possible the method implementation in most WDNs.

The method uses historical free leak data of the network to calculate a demand forecast combined with the sensor fusion theory to develop a leak detection that can analyze all measurements collected during the day, being an improvement due to most leak detection methods only using the analysis of the night flow. The leak detection method can be divided into two-phase, online and offline. The offline phase uses the historical data to generate a threshold that defines the regular operation of the system. In the online phase, the information of inlet flow measurement is processed and classified into regular or fault operations.

The case study presented was the real system in Barcelona, where three months of free historic data were provided. First, several analyzes were carried out with them: the study of the demand forecast and its variance in the respective hours of the day the study of the evolution of the threshold regarding the number of features. Then, different scenarios were generated with different magnitudes of leakage, ranging from 0.5 to 2 l/s, and the True Positive Rate, False Positive Rate, and Difference Time Detection were analyzed. In two scenarios, one using only nighttime measurements and the other using all available features, it is noted that when the leak is small, with the leak size value being similar to the threshold, it is not possible to detect 100%, still having a better result when it is used all available measurements.

As stated in the result analysis, a study to determine how many features are necessary must be done for all WDN implementing this method. With that in mind, future work is studying the best method to define the number of features automatically. Furthermore, following the method improvement that can be performed, systems with multiple leaks can be investigated.

ACKNOWLEDGEMENTS

This work has been partially funded by L-BEST project (Ref. PID2020- 115905RB-C21) from the Spanish Ministry of Science and by AGAUR ACCIO RIS3CAT UTILITIES 4.0–P1 ACTIV 4.0. ref. COMRDI-16-1-0054-03.

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