Economic Reliability-Aware MPC for Operational Management of Flow-Based Networks using Bayesian Networks

Javier Pedrosa[1,2], Vicenç Puig[1,2] and Fatiha Nejjari[1]

Abstract— This paper presents an economic reliability-aware Model Predictive Control (MPC) approach for the Prognostics and Health Management (PHM) of generalized flow-based networks. The main enhancement with respect to some existing approaches relies on the integration of the network reliability model obtained from a Bayesian Network. In this case, the controller is able to optimally manage the supply taking into consideration the distribution of the control effort. The life of the actuators is extended by delaying as much as possible the network reliability decay. The proposed methodology also considers an optimal inventory replenishment policy based on a desired risk acceptability level, leading to the availability of safety stocks for unexpected excess demand in networks. The proposed implementation is illustrated with a real case study corresponding to an aggregate model of the Drinking Water transport Network (DWN) of Barcelona.

I. INTRODUCTION

The management of flow-based networks is an interesting research topic owing to the fact that they are uncertain and complex systems with an important economic, environmental, and social impact. The random behaviour of the network demands and the variability on the prices of the electricity (which directly influences the actuators operation costs) are the critical characteristics when trying to control this type of systems in real-time. To deal with this problem, several approaches have been proposed from the research community [9]. Most of them employ an MPC strategy, which solves a finite time-horizon optimization problem given the future predictions, that suitably fits with the need to take into account the demand forecasts to sufficiently fill the different reservoirs on time. Different approaches were suggested to enhance this concept, such as in [1], where an LPV-MPC controller based on a single-layer economic optimization problem with dynamic constraints has been implemented to manage a pre-established risk acceptability levels to cope with the uncertainty of the demand forecast. In [2], an MPC based on a single-layer economic optimization problem with dynamic constraints to cope with the components degradation awareness and safety stock availability to satisfy non-

stationary flow demands, has been proposed. In [3], a health-aware LPV-MPC by using a chance-constraints approach of the reliability model has been developped.

The main contribution of this paper is to integrate a Bayesian Network reliability model of a generalized flow-based network into the common economic MPC used in these cases; and evaluate it together with the optimal inventory replenishment conditions. Thus, the system reliability obtained from the Bayesian inference is an event-oriented performance criterion that measures the probability that all customer demands will be completely served within a given time interval from the stock on hand without delay, under normal and emergency conditions. In [10], a first attempt of such integration has already been proposed using tracking MPC and reliability obtained using Bayesian Networks combined with reliability importance method to include an additional term in the objective function that takes into account the actuators' reliability.

In this paper, the main enhancement with respect to some existing approaches relies on augmenting the DWN model with the network reliability model obtained from a Bayesian Network. Moreover, this paper includes the development of the whole approach to achieve a network flow optimisation considering both economic and reliability criteria. Another contribution of the work is the application of the proposed technique to a real case of Barcelona DWN.

The structure of this work is as follows. Section II presents the economic MPC of drinking water transport networks. Section III describes the inclusion of the health-aware objective in the economic MPC. Section IV illustrates the proposed approach in an aggregate model of the Barcelona DWN. Finally, Section V draws the main conclusions and suggest future research paths.

II. ECONOMIC MPC OF DRINKING WATER TRANSPORT NETWORKS

A. Control-Oriented Model

The control-oriented model to implement the MPC is a discrete-time system (1) that corresponds to the dynamics of the storage devices for all time instant $k \in \mathbb{Z}_{\geq 0}$.

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k), \qquad (1)$$

being $x \in \mathbb{R}^{n_x}$ the states of the model, and n_x the number of storage devices on the network; they represent the tanks' levels. The vector $u \in \mathbb{R}^{n_u}$ represent the control inputs associated to the flow rates through the actuators of the network, being n_u the total number of them. The vector $d \in \mathbb{R}^{n_d}$ represent the disturbances corresponding to the

[1] V. Puig, J. Pedrosa and F.Nejjari are with the Advanced Control Systems Group, Universitat Politècnica de Catalunya (UPC), Rambla Sant Nebridi 22, 08222 Terrassa, Spain

[2] V. Puig and J. Pedrosa are with the Institut de Robòtica i Informàtica Industrial (CSIC-UPC), Carrer Llorens Artigas, 4-6, 08028 Barcelona, Spain. e-mail: javier.pedrosavicenc.puig@upc.edu

network demands quantities, being n_d the total number of them. And $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$ and $B_d \in \mathbb{R}^{n_x \times n_d}$ are the system time-invariant matrices that depends on the network configuration.

Furthermore, the system is subject to some constraints. First of all, it is subject to the flow-mass balance relations in the nodes (being n_n the total number of them), leading one equation for each node formulated in a matrix form like:

$$0 = E_u u(k) + E_d d(k), \tag{2}$$

being $E_u \in \mathbb{R}^{n_n \times n_u}$ and $E_d \in \mathbb{R}^{n_n \times n_d}$ the time-invariant matrices that depends on the network's junctions.

Besides, a DWN is subject to the physical inputs and states constraints, provided by convex and closed polytopic sets defined as:

$$x(k) \in \mathbb{X} := \{x \in \mathbb{R}^{n_x} | Gx \leq g\}, \tag{3a}$$
$$u(k) \in \mathbb{U} := \{u \in \mathbb{R}^{n_u} | Hu \leq h\}, \tag{3b}$$

where $G \in \mathbb{R}^{n_x \times n_x}$, $g \in \mathbb{R}^{n_x}$, $H \in \mathbb{R}^{n_u \times n_u}$, and $h \in \mathbb{R}^{n_u}$ are matrices collecting the system constraints.

Concerning the operation of the considered flow-based networks, it is assumed that the demands in $d(k)$ and the states in $x(k)$ are measurable at each time instant $k \in \mathbb{Z}_{\geq 0}$; while the pair (A, B) is stabilizable.

B. Optimization Problem Formulation

As the MPC requires some criteria to obtain the control actions, an optimization problem must be defined. Then, the control goal is to minimize a convex stage cost function $\ell : \mathbb{Z}_{\geq 0} \times \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}_{\geq 0}$, which might carry any functional relationship with the system operation. Therefore, the control aim can be expressed for minimization of a convex multi-objective cost function

$$J = \sum_{i=0}^{N_p} (\ell_e(i|k) + \ell_s(i|k) + \ell_{\Delta u}(i|k)) \tag{4}$$

where:

- **Economic objective**: the economical costs that involve the flow transport while providing the demanded volume should be minimized

$$\ell_e(k) \triangleq \alpha(k)^T W_e u(k) \tag{5}$$

where $\alpha(k)$ is the price per volume unit, and W_e is a diagonal positive definite matrix that is used as a weight to prioritize the terms in the complete objective function.

- **Safety objective**: the storage devices should guarantee some safety supply level to deal with unexpected variations in the demand

$$\ell_s(k) \triangleq \begin{cases} \|x(k) - x_s\|_2 & if \ x(k) \leq x_s \\ 0 & otherwise \end{cases} \tag{6}$$

where x_s indicates the storages' safety levels. However, this piecewise linear formulation can be avoided by considering that the safety cost function can be expressed through a soft constraint by using a slack variable ξ like:

$$x(k) \geq x_s - \xi(k) \tag{7}$$

and also being introduced as an objective term to retain feasibility of the optimization problem:

$$\ell_s(k) \triangleq \xi^T(k) W_s \xi(k) \tag{8}$$

where W_s is a diagonal positive definite matrix that is used as a weight to prioritize the terms in the complete objective function.

- **Smoothness objective**: in order to avoid overloads on the pipes, and to preserve the network's components lifetime, the actuators are managed based on smooth control actions. To achieve this smoothing effect, the variation of the control actions among two consecutive time instants is penalized as follows:

$$\ell_{\Delta u}(k) \triangleq \Delta u(k)^T W_{\Delta u} \Delta u(k) \tag{9}$$

where $\Delta u(k) \triangleq u(k) - u(k-1)$, and $W_{\Delta u}$ is a diagonal positive definite matrix that is used as a weight to prioritize the terms in the complete objective function.

III. RELIABILITY-AWARE ECONOMIC MPC USING BAYESIAN NETWORK APPROACH

A. Bayesian Model

Before introducing the Bayesian Model (BM), it would be convenient to remind what Bayesian probability and Bayes' theorem stands for. The first basically consists on an interpretation of the probability, in such, instead of frequency or propensity of some phenomenon, probability is interpreted as a degree of belief in an event, like quantifying a reasonable expectation. While the second is a probability theory used to deal with Bayesian statistics, which describes a conditional probability for an event based on some data, such as prior information or beliefs about the event. Thus, a Bayesian Model is a very useful statistical model to manage the inference of complex systems with conditional dependencies; and to compute the reliability of any event on them. In addition, they are usually represented by an acyclic directed graph, which easily shows the parental relationships between the relevant system components.

In this project, a BM is aimed to be used for the reliability evaluation of the flow-based network, but only considering the actuators as components with considerable reliability influence. In order to lighten the computation of the proposed approach, and consequently, make it able to compute larger systems, a BM is evaluated for each demand independently. And once the reliabilities of all the demands are obtained,

the global system reliability is calculated. Figure 1 shows a graph example of the BM for a single demand of a simple DWN. Then, to integrate the BM in the MPC, an equivalent interpretation of the implied inference is required. This reinterpretation starts finding the minimal paths from all the available sources, and as indicated, only taking into account the actuators involved in them. The pure parents are considered to be these actuators, corresponding to the first light blue nodes column of Figure 1. These are observable evidences, required to perform the conditional probabilities for the subsequent children (next graph columns of Figure 1), which are, respectively:

Step 1: All paths leading to the relevant demand, whose reliabilities lies on the series multiplication of the actuators' reliabilties involved in each one, which means the direct product of these probabilities: $P = \prod_{i=0}^{n} P_i$, implying that all of them must be operative to make the path feasible. In the network example of the Figure 1, this is represented with the nodes corresponding to the available paths in the second purple nodes column. The parents for each one are the actuators involved in each path, and they are the same ones whose reliabilities will be multiplied to obtain each path's one.

Step 2: All sources with access to provide the relevant demand. In this case, the reliability for each one is the parallel multiplication of the reliabilities of those paths that provide supply from the relevant source, which means the complementary of the product of all the complementaries of these probabilities: $P = 1 - \prod_{i=0}^{n} 1 - P_i$, implying that one feasible path is enough to make the source available. In the network example of the Figure 1, this is represented with the nodes corresponding to the available sources in the third light green nodes column. The parents for each one are the paths supplying from each source, and they are the same ones whose reliabilities will be computed to obtain each source's one.

Step 3: Finally, there is the relevant demand, whose reliability is also a parallel multiplication of the available sources, implying that one available source is enough to provide the supply. In this case, it corresponds to the last column with a single red node of the graph example of Figure 1, and all the sources are the parents to perform the relevant reliability calculation.
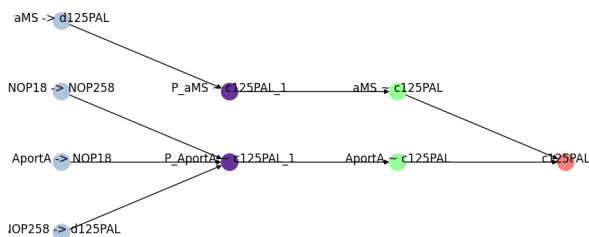

Fig. 1. Single demand Bayesian Model graph.

Afterwards, once the reliabilities of all the demands are computed separately, the reliability for the global system is evaluated as well as a series multiplication of all the demands, implying that all of them must be provided to satisfy the system.

B. Augmenting Network Model with the Bayesian Model

As discussed in the introduction, the main contribution of this work is to integrate the information about system health in the MPC controller by using a BM. This way, and considering the actuators the only components which the degradation affects to, a continuous evaluation of their states can be performed to check the overall system reliability.

1) Individual Reliability: In the literature, different types of distributions have been considered to characterize the evolution of the reliability with time. The most commonly used are exponential, normal, log-normal, and Weibull distributions [5]. In this case, the exponential function is considered.

First of all, it is important to define the concept of failure rate, which is crucial to obtain reliability. The general explanation of failure rate, indicated by λ, is presented as the fraction of the density of the stochastic lifetime to the remainder function (i.e., conditional probability). Particularly, systems are designed to work under different load values. According to [5], the load firmly affects the component failure rate. Therefore, for presenting system reliability evaluation, the load versus failure rate relationship should be considered.

In this paper, actuator failure rates under various load levels are considering in function of the applied control input. The following exponential law is the most widely used relationship to characterize the variation of the actuator fault rates with the load

$$\lambda_i(k) = \lambda_i^0 exp\Big(\beta_i u_i(k)\Big), \qquad i = 1, ..., n \qquad (10)$$

where λ_i^0 represents the baseline failure rate (nominal failure rate) and $u_i(k)$ is the control action at a time instant k for the i-th actuator, being n. β_i is a constant parameter that depends on the actuator characteristics.

Then, the reliability of a component $R_i(t)$, in the useful life period, can be specified at a certain time t by exploiting the exponential function as follows

$$R_i(t) = exp\Big(-\int_0^t \lambda_i(\tau)\, d\tau\Big), \quad i = 1, ..., n \qquad (11)$$

In discrete-time, Equation (11) can be rewritten as

$$R_i(k+1) = R_i(k)\cdot exp\Big(-T_s\cdot\lambda_i(k)\Big), \quad i = 1, ..., n \quad (12)$$

where $\lambda_i(k)$ is the failure rate at a time instant k that is acquired from the i-th component under varying load levels u_i; and T_s is the sampling time.

2) Overall System Reliability Modeling: The system lifespan can be determined by the reliability of the overall system, which is denoted as $R_g(k)$. This reliability is obtained based on the computation of the previous presented reliabilities of elementary components (or subsystems). Therefore, $R_g(k)$ is influenced by the configuration of the actuators, that can be computed from the combination of parallel and/or series of the network components [7].

To add the Bayesian inference in the MPC, the developed reinterpretation of the Bayesian model introduced in Section III-A must be performed, starting computing the reliabilties for the multiple paths P_j subsystems as

$$R_{pj}(k) = \prod_{i \in P_j} R_i(k), \quad P_j \subset I \quad j = 1, 2, ..., n_p \quad (13)$$

being n_p the total number of paths; and P_j the subset of the i-indices corresponding to the actuators involved in each path.

Consecutively, the paths are subdivided in new subsystems. First, according the demand which provide to; and then according the source from where the supply is provided. So, to obtain the reliabilities for each source, the following computation is performed

$$R_{sl}(k) = 1 - \prod_{j \in S_l} (1 - R_{pj}(k)), \quad S_l \subset J \quad l = 1, 2, ..., n_s$$
$$(14)$$

being n_s the total number of sources; and S_l the subset of the j-indices corresponding to the paths providing from each source. And, to obtain the reliabilities for each demand

$$R_{dh}(k) = 1 - \prod_{l \in D_h} (1 - R_{sl}(k)), \quad D_h \subset L \quad h = 1, 2, ..., n_d$$
$$(15)$$

being n_d the total number of sources; and D_h the subset of the l-indices corresponding to the available sources for each demand.

Finally, to infer the overall system reliability, the reliabilities of all the demands are evaluated in series; since, as mentioned above, they all must be supplied for the system feasibility.

$$R_g(k) = \prod_{h=1}^{n_d} R_{dh}(k) \quad (16)$$

3) Inclusion in the Economic MPC Problem Formulation: The economic MPC formulation presented in Section II must be modified to include the preservation of the actuators lifetime. This is achieved by adding a new term in the MPC objective function that aims the reliability maximization, and by augmenting the system model according to the reliability model obtained using the Bayesian modeling approach presented above. For this purpose, the reliabilities evolution over time is included to the model in such a way a conversion is needed that allows computing them in a linear-like form. This conversion is based on applying logarithms, starting

from rewriting the individual actuators reliability evolution from Equation (12) as follows

$$\log R_i(k + 1) = \log R_i(k) - T_s \cdot \lambda_i(k), \quad i = 1, ..., n \quad (17)$$

This expression allows to the system to actualize the parent reliabilites, but, for computation simplicity, the extra objective term should only include the whole system reliability, computed from (16). So, in order to obtain the demands reliabilities carrying on the use of logarithms to preserve the linear computation convenience, the Equation (15) is expanded while taking the unreliability ($F(k) = 1 - R(k)$) leading to

$$F_{dh}(k) = \prod_{j \in S_l \ \& \ l \in D_h} \left(1 - \prod_{i \in P_j} R_i(k)\right), \quad (18)$$

Then, in order to relate (17) with (18), the following variable changing is introduced

$$z_j(k) = 1 - \prod_{i \in P_j} R_i(k), \quad (19)$$

which applying logarithms drives to

$$\log z_j(k) = \frac{\log z_j(k)}{\log (1 - z_j(k))} \cdot \sum_{i \in P_j} \log R_i(k), \quad (20)$$

and according to (18),

$$\log F_{dh}(k) = \sum_{j \in S_l \ \& \ l \in D_h} \log z_j(k), \quad (21)$$

which is equivalent to the following expression with $\beta_j = \dfrac{\log z_j(k)}{\log (1 - z_j(k))}$

$$\log F_{dh}(k) = \sum_{j \in S_l \ \& \ l \in D_h} \left(\beta_j \cdot \sum_{i \in P_j} \log R_i(k)\right), \quad (22)$$

All in all, the structure of the augmented system is the following

$$x_r(k + 1) = A_r x_r(k) + B_r u(k) + B_{dr} d(k), \quad (23)$$

where the state vector will be also augmented by including the logarithms of the demands unreliabilities and the logarithms of the actuators reliabilities; in order to actualize them properly in each iteration, as follows

$$x_r(k) = [x_1(k), ..., x_{nx}(k), \log F_{d1}(k), ... , \log F_{dnd}(k), \log R_1(k), ..., \log R_n(k)]^T \quad (24)$$

and the system matrices corresponding to

$$A_r = \begin{bmatrix} A & \vdots & 0_{nx \times nd+n} \\ \hline & \vdots & \sum_{i \in p_{s,l}}^{s} \beta_j(k) \\ 0_{nd \times nx} & \vdots\ I_{nd \times nd} & \vdots \\ & \vdots & \sum_{i \in p_{s,l}}^{s} \beta_j(k) \\ \hline 0_{n \times nx} & \vdots\ 0_{n \times nd} & I_{n \times n} \end{bmatrix}, \quad (25)$$

$$B_r = \begin{bmatrix} B \\ \hline 0_{nd \times n} \\ \hline -\lambda_i \times I_{n \times n} \end{bmatrix}, B_{dr} = \begin{bmatrix} B_d \\ \hline 0_{nd \times nd} \\ \hline 0_{n \times nd} \end{bmatrix}.$$

However, to be able to implement the Equation (17) in the augmented system, we should remove the exponential of the failure rate function (10); since it has the control input in its exponent, and it does not comply with the linear computation of (23). To achieve the required linearity, a good approach would be to approximate the exponential law to its corresponding first order Taylor series, as follows

$$exp\Big(\beta_i \cdot u_i(k)\Big) \approx 1 + \beta_i \cdot u_i(k), \quad (26)$$

which fits quite well for values $<< 1$; and taking into account that the real values of λ_i^0, β_i and $u_i(k)$ will have this order, it meets the need. Then, in terms of (10), the expression would end up being

$$\lambda_i(k) = \lambda_i^0 + \lambda_i^0 \cdot \beta_i \cdot u_i(k), \qquad i = 1, ..., n \quad (27)$$

The influence of the failure decay can be interpreted with the resulting terms of (27), which corresponds to a typical linear function: with a constant independent term, that would represent the unavoidable constant degradation over time; and a proportional term as a function of the control input, that would represent the relative degradation to the applied control. Assuming that the constant degradation computation does not provide us with any further advantage, owing to it only would imply the addition of a constant term in (23); it is removed as well to simplify the computation, (27) resulting in

$$\lambda_i(k) = \lambda_i^0 \cdot \beta_i \cdot u_i(k), \qquad i = 1, ..., n \quad (28)$$

Regarding the objective function, the economic MPC formulation presented in (4) is modified by including the next term:

$$\ell_r(k) \triangleq W_r \cdot \log F_g(k), \quad (29)$$

where $F_g(k)$ stands for the unreliability of the overall system, which is computed like Equation (16) but considering the unrealibility; and W_r is a positive scalar used as a weight to prioritize the terms in the complete objective function.

Leading the following new multi-objective cost function to be minimized in the prediction horizon N_p:

$$J_r = \sum_{i=1}^{N_p} (\ell_e(i|k) + \ell_s(i|k) + \ell_{\Delta u}(i|k) + \ell_r(i|k)) \quad (30)$$

To assure the feasibility of the obtained control actions, the cost function must be subjected to the system constraints introduced in the Section II-A, as well as to the one related with the safety objective. So, at each time instant, the following optimization problem is solved online:

$$\min_{u(k), x_r(k), \xi(k)} J_r(u(k), x_r(k), \xi(k)) \quad (31)$$

subject to:

$$x_r(i+1|k) = A_r x_r(i|k) + B_r u(i|k) + B_{dr} d(i|k)$$
$$0 = E_u u(i|k) + E_d d(k), \ i = 0, \cdots, N_p - 1$$
$$u(i|k) \in \mathbb{U}, \ i = 0, \cdots, N_p - 1$$
$$\xi(i|k) \geq 0, \ i = 0, \cdots, N_p$$
$$x_r(0|k) = x_r(k),$$
$$x(i|k) \in \mathbb{X}, \ i = 1, \cdots, N_p$$
$$x(i|k) \geq x_s - \xi(i|k), \ i = 1, \cdots, N_p$$

IV. APPLICATION

A. Case Study

In order to evaluate the approach proposed in Section III, a part of the Barcelona DWN, presented in [2], is used as the case study. This network, corresponding to the one in Figure 2, includes 9 sources, consisting of 5 underground and 4 surface sources, which currently provide an inflow of about 2 m^3/s. And it is composed of 17 tanks, 12 nodes, 25 demands and 61 actuators (valves and pumps).
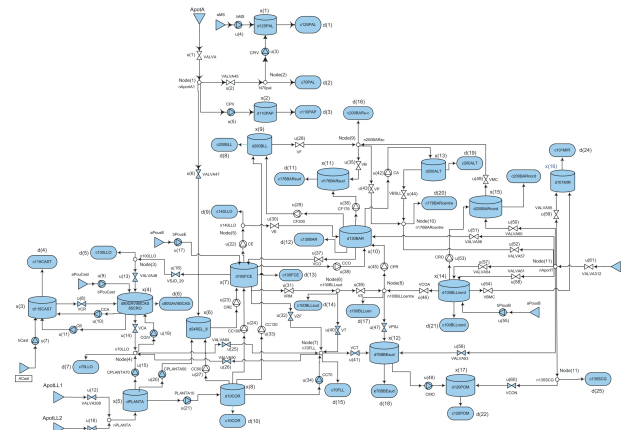

Fig. 2. Barcelona drinking water transport network.

B. Results And Discussion

In order to show and assess the effect of including the reliabilities awareness to the MPC optimization, its related weight of the relevant term of objective function (W_r) can

be set greater than 0; otherwise, the controller would not take them into account. Besides, the total economic cost and the final reliabilities of the actuators, as well as the whole system one, could be good indicators to evaluate their contributions. Then, a four days horizon simulation was performed with some demand and cost values collected from the same paper as the case study [2]. Starting with an overall system reliability of 100%, the results of this simulation taking the reliabilities into account ($W_r = 100$) gave a final overall system reliability of 83%, with a cost of 52.32 e.u.; and the results of the same mentioned simulation not taking the reliabilities into account ($W_r = 0$) gave a final overall system reliability of 79%, with a cost of 52.34 e.u.. The prices and the initial failure probabilities values are not realistic, but they were selected in order to better appreciate the influence of the implementation.

Before the main discussion, it is important to highlight all the weights used in this particular simulation for the objective function of the optimization problem (introduced in Section II-B), which are: $W_e = 100; W_{\Delta u} = 50; W_s = 10$; and $W_r = \{100, 0\}$. In addition, to support the importance of the objective function terms' weights, their influence can be properly noted when analysing the same case study with different weight combinations, as proceeds in the Figure 3. In its simulations, only two of the weights were changing (W_e and W_r); the other two were fixed: $W_{\Delta u} = 50; W_s = 10$, because they are not the main object of this work.
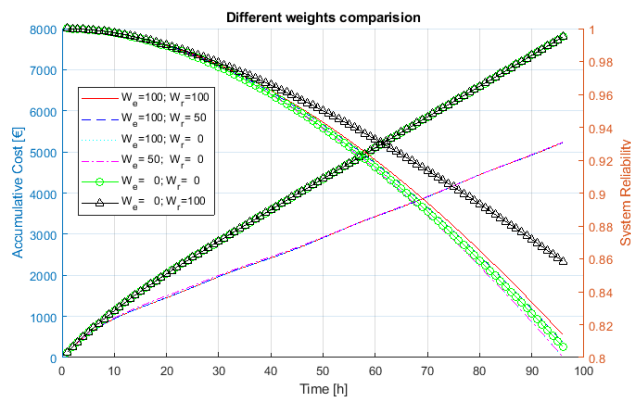

Fig. 3. System reliability and accumulative cost for different weights cases.

Then, the first observable evidence from these results, and an expected outcome, is that, taking reliabilities into account, makes the whole system reliability remain closer to 1 (which corresponds to the functions starting from the upper-left figure corner and the right axis); while, otherwise, it is getting worse. However, it was expected that the reliabilities integration would affect negatively to the total operational costs; but surprisingly, if not modifying the economic term weight of the objective function, the cost remains practically the same and keeping the same trend.

Regarding the two weights modified to see their influence on the system, the economic term weight seems to not offer a variability on the results, but rather the choice of whether or not to optimize the production; owing to the fact that, until it was disabled by a null weight, the trend was being the same; and once disabled, the trend was held. On the other hand, the reliability term weight seems to yield a minimal variance, but it looks like restricted by the economic term, since until it was not disabled by a null weight, the reliability is not optimized in a considerable better way.

V. CONCLUSIONS

This paper has presented a new approach of an economic reliability-aware model predictive control (MPC) for the management of generalized flow-based networks. The main enhancement with respect to some existing approaches relies in the dynamic integration of the Bayesian model of the whole system in the controller to manage the supply taking into consideration the distribution of the control effort, in order to extend the life of the network by delaying as much as possible the reliability decay. Besides, it also considers an optimal inventory replenishment policy based on a desired risk acceptability level, leading to the availability of safety stocks for unexpected excess demand in networks. The proposed implementation has been illustrated with a real case study corresponding to a sector of the water transport network of Barcelona.

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