Passivation-based Control Reconfiguration with Virtual Actuators ⋆

Iury Bessa [*,**] **Vicenç Puig** [***] **Reinaldo Martínez Palhares** [****]

[*] *Federal University of Amazonas, Department of Electricity, Manaus, Brazil (email: iurybessa@ufam.edu.br)*
[**] *Federal University of Minas Gerais, Graduate Program in Electrical Engineering, Brazil*
[***] *Supervision, Safety and Automatic Control Research Center (CS2AC), Institut de Robótica i Informática Industrial (CSIC-UPC), Universitat Politécnica de Catalunya, Barcelona, Spain (e-mail: vicenc.puig@upc.edu)*
[****] *Federal University of Minas Gerais, Department of Electronics Engineering, Belo Horizonte, Brazil (e-mail: rpalhares@ufmg.br)*

Abstract: This paper presents a novel approach for designing reconfiguration blocks for fault hiding of linear systems subject to actuator faults based on the passivity/dissipativity theory. For this purpose, the concept of passivation block is used to design virtual actuators (VAs) which guarantee that the faulty plant achieves the desired passivity indices and consequently the stability. Linear matrix inequalities (LMI)-based conditions are provided for designing the proposed VAs for ensuring the stability recovery for linear systems. Finally, a numerical example is used for assessing the proposed approach.

Keywords: Fault tolerant control, Reconfiguration Block, Fault hiding, Dissipativity, Passivity

1. INTRODUCTION

Fault-tolerant control (FTC) techniques are used to mitigate the effects of faults in the control systems dynamics and to provide some guarantees under faulty operations (as e.g., stability and robustness). In general, the FTC techniques can be classified into active and passive. On the one hand, passive FTC deals with the fault occurrence as an uncertainty and provides tolerance by means of robust or adaptive strategies which do not require the fault diagnosis. On the other hand, active FTC (AFTC) requires the inclusion of real-time diagnosis in the control loop and/or law to compensate for its effects. For this reason, AFTC approaches present improved performance, but they require fault modeling and the existence of an accurate fault diagnosis system that operates in real-time. AFTC approaches can be divided into three main groups: the fault accommodation which adapts the controller parameters based on the fault estimation and without changes on the control loop; the control reconfiguration which modifies the control loop after a fault detection by redesigning a novel controller that deals only with the healthy part of the system; and the fault hiding which modifies the control loop by inserting a reconfiguration block between the faulty plant and the nominal controller that remains unchanged (Blanke et al., 2016). This paper focuses on the last class of FTC: the fault hiding approach. This approach provides minimum-invasive changes since all the control/sensor signals and loop changes are virtually performed by modifying the gains of a reconfiguration block (RB), that is inserted between the plant and the controller.

In general, the RBs structures can be virtual sensors (VSs) or virtual actuators (VAs) for dealing with sensors and actuators faults, respectively. The VSs is similar to a Luenberger observer and is used to estimate the correct output values given the knowledge about the faults which affect the system sensors. For actuator faults, the VAs are used for hiding the actuator faults from the controller while injecting a reconfigured control signal which mitigates the actuator fault effects. It is already shown that the VAs are realizations of the dual Luenberger observer (Richter et al., 2010). Due to its efficacy and flexibility, the fault hiding approach has been applied to a number of classes of systems, such as, linear time-invariant (LTI) (Yadegar et al., 2019), descriptor (Wang et al., 2020), piecewise-affine (Richter et al., 2011), linear parameter-varying (Quadros et al., 2020; Rotondo et al., 2014), switched (Rotondo et al., 2015a), networked linear systems (Schenk and Lunze, 2017), Takagi-Sugeno fuzzy (Rotondo et al., 2016), Lipschitz (Yadegar and Meskin, 2022), Lur'e (Pedersen et al., 2016), Hammerstein-Wiener (Richter, 2011), multi-agent (Yadegar and Meskin, 2021b,a), and generic input-affine nonlinear systems (Vey et al., 2015; Bessa et al., 2020a, 2021).

Recently, general purpose reconfiguration block structures were proposed (Bessa et al., 2020b,a, 2021). Those struc-

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tures can be reciprocally employed for both sensor and actuator faults. Bessa et al. (2020b) propose the use of static RBs for fault-hiding of Takagi-Sugeno fuzzy systems with nonlinear consequent, which are designed based on Lyapunov theory for ensuring the asymptotic stability recovery. A similar static RB structure is used in (Bessa et al., 2020a), but the design is based on the dissipativity theory. In this case, the static RB are designed to recover (Q,S,R)-dissipativity and passivity after the fault occurrence. In (Bessa et al., 2021), the concept of passivation blocks for fault hiding is presented. The passivation block is a RB which can be used to mitigate both sensors and actuator faults by passivation of the faulty system, i.e., the RB is designed for compensating for the lack of passivity of the faulty system. Based on the passivation block concept, this paper provides the following contributions: *(i)* it is shown that the passivation block generalizes the traditional VAs used in the literature; *(ii)* an LMI-based dissipativity analysis is proposed to compute the passivity indices which are required from a VA to compensate for the fault effects; and *(iii)* novel LMI-based design conditions are provided to ensure the passivation of the faulty plant.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1 Dissipativity and Stability

Consider the continuous-time LTI system Σ:

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^m$ are, respectively, the state, input and output vectors, and A, B, C, and D are real state-space matrices with adequate dimensions.

Dissipativity is a useful concept for dynamic system analysis that allows to investigate their stability by means of the energy balance, *i.e.*, the difference between the stored and supplied energy. A system is said to be dissipative if its stored energy, represented by the temporal derivative of a positive semidefinite continuously differentiable storage function, is always less than or equal to the supplied energy, represented by a supply rate function.

Different supply functions can be used for dissipativity analysis. An important case is the (Q,S,R)-dissipativity that assumes the following supply rate function

$$\mathcal{S}\left(u(t), y(t)\right) = y(t)^\top Q y(t) + 2u(t)^\top S y(t) + u(t)^\top R u(t), \tag{2}$$

where $u(t)$ and $y(t)$ are the input and output signals of the system and $Q = Q^\top$, S, and $R = R^\top$ are the parameters of the supply rate function. For continuous-time LTI systems, the next lemma borrowed from Kottenstette et al. (2014) provides an LMI-based test for the (Q,S,R)-dissipativity.

Lemma 1. ((Kottenstette et al., 2014)). The system Σ (1) is (Q,S,R)-dissipative if and only if there exist a matrix $P = P^\top \succ 0$ such that the following inequality is satisfied:

$$\begin{bmatrix} \mathrm{He}\left\{PA\right\} - C^\top Q C & PB - C^\top S - C^\top Q D \\ \star & -\mathrm{He}\left\{D^\top S\right\} - D^\top Q D - R \end{bmatrix} \preceq 0. \tag{3}$$

The (Q,S,R)-dissipativity eases the asymptotic stability analysis, since that a (Q,S,R)-dissipative system is asymptotically stable if $Q \prec 0$ (Bessa et al., 2021).

Passivity is a special case of dissipativity, i.e., a system is said to be passive if it is dissipative with respect to the supply rate $S(u(t), y(t)) = u(t)^\top y(t)$. In particular, the framework of passivity indices is useful to obtain an indication of the passivity degree (Kottenstette et al., 2014) as shown in Definition 1.

Definition 1. **Passivity indices** (Xia et al., 2018).

The system Σ described in (1) is

- output feedback passive (OFP) if for any $\rho \in \mathbb{R}$ it is dissipative with respect to the supply rate
$$S(u(t), y(t)) = u(t)^\top y(t) - \rho y(t)^\top y(t); \tag{4}$$
- input feed-forward output feedback passive (IF-OFP) if for any $\nu, \rho \in \mathbb{R}$ it is dissipative with respect to the supply function
$$S\left(u(t), y(t)\right) = u(t)^\top y(t) - \nu u(t)^\top u(t) - \rho y(t)^\top y(t). \tag{5}$$

In these cases, Σ is denominated OFP(ρ) and IF-OFP(ν,ρ), respectively, where ν and ρ are called passivity indices that correspond to the excess of passivity of Σ.

The passivity indices provide intuitiveness to the dissipativity framework because they are related to the concept of lack (or excess) of passivity of a system. This means that ν indicates how much feedfoward gain can be inserted preserving Σ stability given the excess of input passivity $\nu u(t)^\top u(t)$. Similarly, ρ indicates how much feedback gain can be inserted to preserve Σ stability given the excess of output passivity $\rho y(t)^\top y(t)$. In this sense, every system is IF-OFP(ν, ρ) for some ν and ρ, but the maximum value that ν and ρ can assume are indicators of its stability margins. For instance, if the maximum ρ that can be assigned for a system Σ is negative, then Σ is open-loop unstable and requires a negative feedback action to compensate for such lack of passivity that results in the instability.

Lemma 2. (Kottenstette et al., 2014). Assume that the system Σ (1) is (Q,S,R)-dissipative. This system is

- passive if $Q = 0$, $S = \frac{1}{2}I$, and $R = 0$;
- OFP(ρ) if there exists a $\rho \in \mathbb{R}$ such that $Q = -\rho I$, $S = \frac{1}{2}I$, and $R = 0$;
- IF-OFP(ν,ρ) if there exist $\nu \in \mathbb{R}$ and $\rho \in \mathbb{R}$ such that $Q = -\rho I$, $S = \frac{1}{2}I$, and $R = -\nu I$;
- $\mathcal{L}_2$-stable with the gain less than or equal to γ if and only if there is a $\gamma \in \mathbb{R}_{\geq 0}$ such that $Q = -I$, $S = 0$, and $R = \gamma^2 I$.

Moreover, it also possible to establish the relation between (Q,S,R)-dissipativity and asymptotic stability of Σ. For $u = 0$, $Q \preceq 0$ implies the asymptotic stability of Σ. Similar conclusions can be drawn for passivity indices, i.e., Σ is asymptotically stable if $\rho > 0$.

2.2 Problem statement

Based on the dissipativity theory and the concepts discussed in Subsection 2.1, this paper aims to solve the problem of passivation of LTI systems under fault occurrences on actuators by using VAs respectively to guarantee the asymptotic stability of the reconfigured system.

In this sense, consider the plant subject to actuator faults which is nominally represented by the following LTI model:

$$\Sigma_P : \begin{cases} \dot{x} = Ax + Bu_p, \\ y_p = Cx, \end{cases} \tag{6}$$

where A, B, and C are state-space matrices with appropriate dimensions, $x \in \mathbb{R}^n$ is the plant states, $u_p \in \mathbb{R}^m$ is the vector of control inputs injected into the plant actuators, $y_p \in \mathbb{R}^m$ is the vector of outputs with the sensors' measurements. The same plant is represented by the following model when it is under faulty operation:

$$\Sigma_P : \begin{cases} \dot{x} = Ax + B_f u_p, \\ y_p = Cx, \end{cases} \tag{7}$$

where B_f is a matrix with appropriate dimensions which represents the occurrence of multiplicative actuator faults. In this sense, the fault effect is represented as follows:

$$B_f = B\left(I_m - \operatorname{diag}\{f_a\}\right), \tag{8}$$

where the $f_{a,i} \in [0, 1]$ denotes the i-th entry of the vector f_a which indicates the efficiency loss of the i-th actuator, such that $f_{a,i} = 1$ and $f_{a,i} = 0$ indicate, respectively, the complete loss and the fault absence in the i-th actuator.

Assumption 1. It is assumed that there exists an FDI system that provides the correct values of f_a without delay.

Nominally, the plant is controlled by a controller Σ_C,

$$\Sigma_C : \begin{cases} \dot{x}_c = A_c x_c + B_c y_c, \\ u_c = C_c x_c + D_c y_c, \end{cases} \tag{9}$$

where u_c is the control signal, and y_c is the sensor measurement which is injected into Σ_C. The fault hiding inserts an RB denoted by Σ_R between the faulty plant and the controller to recover the nominal performance or stability. In this work, we propose designing Σ_R based on the passivation of the controller Σ_C to ensure that the closed-loop system presents desired dissipativity properties.

In this paper, we propose using passivation-based VAs for fault hiding of systems under actuator faults. In (Bessa et al., 2021), the design of generic passivation blocks is already discussed. Inspired by that proposal, this paper extends the concept for designing the usual reconfiguration block structures, in particular, the VA. In this context, the following fault hiding problem is stated.

Problem 1. Let Σ_P be a nominal system (6), with fault model Σ_{P_f} (7), connected to a controller Σ_C. Considering Assumption 1, find a VA such that the origin of the reconfigured system is asymptotically stable.

3. PASSIVATION-BASED FAULT HIDING

3.1 Passivation Blocks

The idea of dynamic passivation block (DPB) is proposed in Bessa et al. (2021). The DPB is described as follows:

$$\Sigma_R : \begin{cases} \dot{x}_r = A_r x_r + B_{r,y} y_f + B_{r,u} u_c \\ y_r = C_{r,y} x_r + R_1 y_f + R_2 u_c \\ u_r = C_{r,u} x_r + R_3 y_f + R_4 u_c \end{cases} \tag{10}$$

with $x_r(t) \in \mathbb{R}^n$, $y(t), u_c(t), y_r(t), u_r(t) \in \mathbb{R}^m$ and gain matrices A_r, $B_{r,y}$, $B_{r,u}$, $C_{r,y}$, $C_{r,u}$, R_1, R_2, R_3, and R_4 with appropriate dimensions. The DPB described as (10) generalizes the existing linear dynamic RBs found in the literature (i.e., VAs and VSs). In particular, the VA is the RB which is usually employed to handle actuator faults. The VAs is denoted here as Σ_A and it is described as (Richter, 2011):

$$\Sigma_A : \begin{cases} \dot{x}_\Delta = (A - B_f M)x_\Delta + (B - B_f N)u_c \\ y_r = C x_\Delta + y_f \\ u_r = M x_\Delta + N u_c \end{cases} \tag{11}$$

The VA (11) is a particular case of the DPB in (10), obtained by defining

$$A_r = A - B_f M, \quad B_{r,y} = 0_{n_r \times p}, \quad B_{r,u} = B - B_f N, \quad C_{r,y} = C,$$
$$C_{r,u} = M, \quad R_1 = I_m, \quad R_2 = 0_{p \times m}, \quad R_3 = 0_{m \times p}, \quad R_4 = N,$$

and $x_r(0) \triangleq 0_{n_r \times 1}$. The insertion of a VA for FTC of systems with actuator faults is shown in Fig. 1.

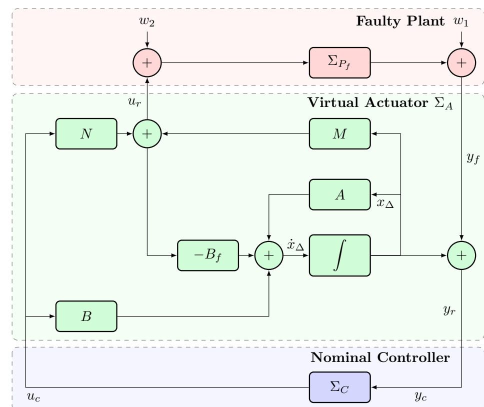


Fig. 1. Fault hiding by using a VA Σ_A.

3.2 Dissipativity analysis of control reconfiguration

In this section, LMI-based conditions for computing the DPB are presented to ensure the asymptotic stability recovery by fault hiding. For this purpose, it is worth to obtain conditions for dissipativity analysis of DPB. In particular, Lemma 3 is borrowed from Bessa et al. (2021) and provides conditions for Σ_R in (10) to be IF-OFP(ν_r,ρ_r).

Lemma 3. (Bessa et al., 2021). Σ_R is IF-OFP(ν_r,ρ_r) if there exist a matrix $P = P^\top$, and scalars ν_r and ρ_r such that the following inequalities are satisfied:

$$P \succ 0, \qquad \Theta \preceq 0 \tag{12}$$

with

$$\Theta \triangleq \begin{bmatrix} \mathrm{He}\{PA_r\} + W_{11} & PB_{r,y} - \frac{1}{2}C_{r,y}^\top + W_{12} & PB_{r,u} - \frac{1}{2}C_{r,u}^\top + W_{13} \\ \star & \nu_r I_m - \frac{1}{2}\mathrm{He}\{R_1\} + W_{22} & -\frac{1}{2}R_2 - \frac{1}{2}R_3^\top + W_{23} \\ \star & \star & \nu_r I_m - \frac{1}{2}\mathrm{He}\{R_4\} + W_{33} \end{bmatrix}$$

$$W_{11} \triangleq \rho_r C_{r,y}^\top C_{r,y} + \rho_r C_{r,u}^\top C_{r,u},\ W_{12} \triangleq \rho_r C_{r,y}^\top R_1 + \rho_r C_{r,u}^\top R_3,$$
$$W_{13} \triangleq \rho_r C_{r,y}^\top R_2 + \rho_r C_{r,u}^\top R_4,\ W_{22} \triangleq \rho_r R_1^\top R_1 + \rho_r R_3^\top R_3,$$
$$W_{23} \triangleq \rho_r R_1^\top R_2 + \rho_r R_3^\top R_4,\ W_{33} \triangleq \rho_r R_2^\top R_2 + \rho_r R_4^\top R_4.$$

The next Lemma presents conditions for the (Q,S,R)-dissipativity of the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$.

Lemma 4. If Σ_{P_f} is OFP(ρ_f), Σ_C is finite-gain $\mathcal{L}_2$ stable with $\mathcal{L}_2$ gain less than or equal to γ, and Σ_R is IF-OFP(ν_r,ρ_r), then $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is (Q,S,R)-dissipative with:

$$Q = \begin{bmatrix} -\left(\nu_r + \rho_f\right) I_m & 0 & \frac{1}{2}I_m & \frac{1}{2}I_m \\ \star & -\left(\nu_r + 1\right) I_m & 0 & \frac{1}{2}I_m \\ \star & \star & -\left(\rho_r - \gamma^2\right) I_m & 0 \\ \star & \star & \star & -\rho_r I_m \end{bmatrix}, \tag{13}$$

$$S = \begin{bmatrix} -\nu_r I_m & \frac{1}{2}I_m \\ 0 & 0 \\ \frac{1}{2}I_m & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} -\nu_r I_m & 0 \\ 0 & 0 \end{bmatrix}. \tag{14}$$

Proof. Define the following vectors of inputs and outputs of the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ and of the RB:

$$\bar{y} = \begin{bmatrix} y_p^\top \; u_c^\top \; y_r^\top \; u_r^\top \end{bmatrix}^\top, \quad \bar{u} = \begin{bmatrix} w_1^\top \; w_2^\top \end{bmatrix}^\top, \tag{15}$$

$$\bar{y}_r = \begin{bmatrix} y_r^\top \; u_r^\top \end{bmatrix}^\top, \quad \bar{u}_r = \begin{bmatrix} y_f^\top \; u_c^\top \end{bmatrix}^\top, \tag{16}$$

In this sense, the internal signals u_p, y_f, y_p, u_c, y_r, and u_r:

$$u_p = u_r + w_2 = I_{4,(4)}\bar{y} + I_{2,(2)}\bar{u}, \tag{17}$$

$$y_f = y_p + w_1 = I_{4,(1)}\bar{y} + I_{2,(1)}\bar{u}, \tag{18}$$

$$y_p = I_{4,(1)}\bar{y}, \quad u_c = I_{4,(2)}\bar{y}, \quad y_r = I_{4,(3)}\bar{y}, \quad u_r = I_{4,(4)}\bar{y}. \tag{19}$$

Based on $\bar{y}$ and $\bar{u}$ defined in (15) and the internal signals defined in (17)–(19), the dissipation equations for Σ_{P_f} be OFP(ρ_f), Σ_C be finite-gain $\mathcal{L}_2$ stable with $\mathcal{L}_2$ gain less than or equal to γ, and Σ_R be IF-OFP(ν_r,ρ_r) can be represented respectively as:

$$\dot{V}_p(x) \leq u_p^\top y_p - \rho_f y_p^\top y_p = \bar{y}^\top Q_p \bar{y} + 2\bar{y}^\top S_p \bar{u} + \bar{u}^\top R_p \bar{u}, \tag{20}$$

$$\dot{V}_c(x_c) \leq -u_c^\top y_r + \gamma^2 y_r^\top y_r = \bar{y}^\top Q_c \bar{y} + 2\bar{y}^\top S_c \bar{u} + \bar{u}^\top R_c \bar{u}, \tag{21}$$

$$\begin{aligned} \dot{V}_r(x_r) &\leq \bar{u}_r^\top \bar{y}_r - \nu_r \bar{u}_r^\top \bar{u}_r - \rho_r \bar{y}_r^\top \bar{y}_r \\ &= \bar{y}^\top Q_r \bar{y} + 2\bar{y}^\top S_r \bar{u} + \bar{u}^\top R_r \bar{u}, \end{aligned} \tag{22}$$

where $V_p(x)$, $V_c(x_c)$, and $V_r(x_r)$ are valid storage functions for Σ_{P_f}, Σ_C, and Σ_R respectively, and

$$Q_p = \begin{bmatrix} -\rho_f I_m & 0 & 0 & \frac{1}{2}I_m \\ \star & 0 & 0 & 0 \\ \star & \star & 0 & 0 \\ \star & \star & \star & 0 \end{bmatrix}, \; Q_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \star & -I_m & 0 & 0 \\ \star & \star & \gamma^2 I_m & 0 \\ \star & \star & \star & 0 \end{bmatrix},$$

$$Q_r = \begin{bmatrix} -\nu_r I_{2m} & \frac{1}{2}I_{2m} \\ \star & -\rho_r I_{2m} \end{bmatrix}, \; R_p = R_c = 0, \; R_r = \begin{bmatrix} -\nu_r I_m & 0 \\ 0 & 0 \end{bmatrix},$$

$$S_p = \begin{bmatrix} 0 & \frac{1}{2}I_m \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; S_c = 0, \; S_r = \begin{bmatrix} -\nu_r I_m & 0 \\ 0 & 0 \\ \frac{1}{2}I_m & 0 \\ 0 & 0 \end{bmatrix}.$$

Thus, adopting the storage function for $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$:

$$\bar{V}(x, x_r, x_c) \triangleq V_p(x) + V_c(x_c) + V_r(x_r),$$

the following dissipation inequality can be obtained by summing the inequalities (20), (21) and (22)

$$\begin{aligned} \dot{\bar{V}}(x, x_r, x_c) \leq &\bar{y}^\top (Q_p + Q_c + Q_r)\bar{y} + 2\bar{y}^\top (S_p + S_c + S_r)\bar{u} \\ &+ \bar{u}^\top \left(R_f + R_c + R_r\right)\bar{u}. \end{aligned} \tag{23}$$

Therefore, $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is (Q,S,R)-dissipative with

$$Q = Q_p + Q_c + Q_r, \; S = S_p + S_c + S_r, \; R = R_p + R_c + R_r. \; \blacksquare$$

3.3 Passivation Blocks as Virtual Actuators

In this section, novel conditions for designing dynamic VAs for linear systems based on the dissipativity theory and the passivity indices framework are developed. In particular, such conditions are derived from Lemma 4 which indicates the (Q,S,R)-dissipativity of the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ by considering that the VAs are particular cases of the DPBs described as (10). The next lemma provides LMI-based conditions to compute the required passivity indices of the DPBs to ensure the passivation and stabilization of the faulty plant connected to a finite-gain $\mathcal{L}_2$-stable controller Σ_C.

Lemma 5. Let Σ_{P_f} be a linear model (7) for a system interconnected to a controller Σ_C (9) and to a DPB (10). If there exist scalars δ, $\bar{\nu}_r$, $\bar{\rho}_r$, and ρ_f, and symmetric positive definite matrices P_f and P_c which satisfy the following inequality

$$\text{He}\left\{P_f A\right\} + \rho_f C^\top C \preceq 0, \tag{24}$$

$$\begin{bmatrix} \text{He}\left\{P_c A\right\} + C_c^\top C_c & P_c B_c + C_c^\top D_c \\ \star & D_c^\top D_c - \delta I_m \end{bmatrix} \preceq 0, \; \delta \geq 0 \tag{25}$$

$$\begin{bmatrix} -\left(\bar{\nu}_r + \rho_f\right) & 0 & \frac{1}{2} & \frac{1}{2} \\ \star & -\left(\bar{\nu}_r + 1\right) & 0 & \frac{1}{2} \\ \star & \star & -\left(\bar{\rho}_r - \delta\right) & 0 \\ \star & \star & \star & -\bar{\rho}_r \end{bmatrix} \prec 0, \tag{26}$$

where

$$P_f B_f - \tfrac{1}{2}C^\top = 0, \tag{27}$$

then any DPB (10) which is IF-OFP(ν_r, ρ_r) for scalars $\nu_r \geq \bar{\nu}_r$ and $\rho_r \geq \bar{\rho}_r$ ensures the asymptotic stability of the origin of the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$.

Proof. If (24) and (27) are satisfied for some symmetric positive-definite matrix P_f and scalar ρ_f, then the faulty plant Σ_{P_f} is (Q,S,R)-dissipative with $Q = -\rho_f I_m$, $S = \frac{1}{2}I_m$, and $R = 0_{m\times m}$ according to Lemma 1. Thus, according to Lemma 2, Σ_{P_f} is OFP(ρ_f). Similarly, if (25) is satisfied for some symmetric positive-definite matrix P_c and positive scalar δ, then the controller Σ_C is (Q,S,R)-dissipative with $Q = -I_m$, $S = 0_{m\times m}$, and $R = \delta I_m$ according to Lemma 1. Thus, according to Lemma 2, Σ_C is $\mathcal{L}_2$-stable with the gain less than or equal to $\gamma = \sqrt{\delta}$. It is shown in Lemma 4 that the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is (Q,S,R)-dissipative with Q, S, and R given by (13) and (14). In this sense, if the inequality (26) is satisfied for some scalars $\bar{\nu}_r$ and $\bar{\rho}_r$, then $Q \preceq 0$. Thus, a DPB which is IF-OFP($\bar{\nu}_r$, $\bar{\rho}_r$) is enough to ensure the passivation of the faulty system and the asymptotic stability of the origin of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$. Notice that the Q given by (13) is still negative definite for any $\nu_r \geq \bar{\nu}_r$ and $\rho_r \geq \bar{\rho}_r$, therefore the asymptotic stability of the origin of the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is guaranteed by any DPB (10) which is IF-OFP(ν_r, ρ_r). $\blacksquare$

The next theorem provides novel sufficient conditions for designing the VAs (11) based on the passivation of a faulty plant which is OFP(ρ_f) and a finite-gain $\mathcal{L}_2$ stable controller as presented in Lemma 5.

Theorem 1. Let Σ_{P_f} be a linear model described as (7) for a system under actuator faults interconnected as Fig. 1 to a feedback controller Σ_C and to a VA Σ_A described as (11). Assume that Σ_{P_f} be OFP(ρ_f), Σ_C be $\mathcal{L}_2$ stable with gain less than or equal to $\gamma = \sqrt{\delta}$, and that there exist scalars $\bar{\rho}$ and $\bar{\nu}$ for which the inequality (26) is satisfied. Given the scalars δ, $\bar{\nu}_r$, $\bar{\rho}_r$, and ρ_f, the origin of reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable if there exist some scalars μ_r and ν_r, and matrices $Y = Y^\top \succ 0$, $\bar{M}$, and N that satisfy the inequalities

$$\mu_r \leq \tfrac{1}{\bar{\rho}_r}, \tag{28}$$

$$\begin{bmatrix} -\left(\nu_r + \rho_f\right) & 0 & \frac{1}{2} & \frac{1}{2} \\ \star & -\left(\nu_r + 1\right) & 0 & \frac{1}{2} \\ \star & \star & -\left(\bar{\rho}_r - \gamma^2\right) & 0 \\ \star & \star & \star & -\bar{\rho}_r \end{bmatrix} \prec 0 \tag{29}$$

$$\Phi\left(\nu_r, \mu_r\right) = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \star & \Phi_3 \end{bmatrix} \preceq 0 \tag{30}$$

where

$$\Phi_1 = \begin{bmatrix} \mathrm{He}\left\{AY - B_f\bar{M}\right\} & -\frac{1}{2}YC^\top & B - B_fN - \frac{1}{2}\bar{M}^\top \\ \star & -(1-\nu_r)I_m & 0_{m\times m} \\ \star & \star & -\frac{1}{2}\mathrm{He}\left\{N\right\} + \nu_r I_m \end{bmatrix}, \quad (31)$$

$$\Phi_2 = \begin{bmatrix} YC^\top & \bar{M}^\top \\ I_m & 0_{m\times m} \\ 0_{m\times m} & N^\top \end{bmatrix}, \qquad \Phi_3 = -\mu_r I_{2m}. \quad (32)$$

In this case, the gain M is given by

$$M = \bar{M}Y^{-1}. \quad (33)$$

Proof. By the Schur's complement Lemma, (30) implies $\Phi_1 - \Phi_2\Phi_3\Phi_2^\top \preceq 0$. Pre- and post multiplying it by $\mathrm{diag}\left\{Y^{-1}, I_m, I_m\right\}$, given (33), it follows that if (30) is satisfied for some scalars μ_r and ν_r, and matrices $Y = Y^\top \succ 0$, $\bar{M}$, and N, then (12) is also satisfied for $P = Y^{-1}$, $\rho_r = \frac{1}{\mu_r}$, and the VA (11), which implies the VA is IF-OFP(ν_r,ρ_r).

Assuming that Σ_{P_f} be OFP(ρ_f), Σ_C be $\mathcal{L}_2$ stable with gain less than or equal to $\gamma = \sqrt{\delta}$, and that the given scalars δ, $\bar{\nu}_r$, $\bar{\rho}_r$, and ρ_f satisfy the inequality (26), Lemma 5 guarantees that the origin of reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable for any VA (11) which is IF-OFP(ν_r, ρ_r) for scalars $\rho_r \geq \bar{\rho}_r$ and $\nu_r \geq \bar{\rho}_r$. Notice that the positiveness of $\rho_r = \frac{1}{\mu_r}$ and $\bar{\rho}_r$ is ensured by (30) and (26), respectively, thus (28) ensures $\rho_r > \bar{\rho}_r$. Finally, using the arguments presented in Lemma 5 for (26), the inequality (30) ensures that the origin of reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable for any VA (11) which is IF-OFP(ν_r, ρ_r) for $\rho_r \geq \bar{\rho}_r$. ■

3.4 Passivation-based VA Design procedure

The proposed passivation-based design methodology for VAs is performed in two steps based on Lemma 5, and Theorem 1. The idea is: firstly, to compute the passivation indices of the VA (related to $\bar{\mu}_r$ and $\bar{\nu}_r$), which are sufficient to compensate for the lack of passivity due to the fault occurrence; secondly, to compute the VA gains which are able to meet the sufficient passivity indices. In principle, any feasible scalars ρ_f, δ, $\bar{\nu}_r$, and $\bar{\rho}_r$ satisfying the conditions of Lemma 5 are able to provide valid supply rate functions for Σ_C, and Σ_{P_f}, and compute bounds for the passivity indices of Σ_R to guarantee the asymptotic stability of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$. However, one can notice that minimizing $\bar{\rho}_r$ and δ, and maximizing ρ_f reduce the conservativeness of the conditions of Theorem 1. In this regard, the following optimization problem is solved in the first step to compute the scalars δ and ρ_f which describes the dissipativity property of the plant Σ_{P_f} which is OFP(ρ_f), and of the controller Σ_C which is $\mathcal{L}_2$-stable with gain $\sqrt{\delta}$.

$$\min_{\rho_f,\, \delta,\, \bar{\nu}_r,\, \bar{\rho}_r} \quad \bar{\rho}_r + \delta - \rho_f \quad (34)$$
$$\text{s.t.} \quad P_f \succ 0,\ P_c \succ 0, (24), (25), (26), (27).$$

The solution of (34) is combined to the result of Theorem 1 to design the VA for guaranteeing the asymptotic stability of the reconfigured system. The following inequalities are included to avoid high gains for the VA:

$$-\lambda_N I_m \preceq \mathrm{He}\left\{N\right\} \preceq \lambda_N I_m, \quad \lambda_N \in \mathbb{R}_{\geq 0}. \quad (35)$$

Thus, the following optimization problem is solved to compute the gains of the VA:

$$\min_{Y,\, \bar{M},\, N,\, \mu_r,\, \nu_r} \quad \lambda_N \quad (36)$$
$$\text{s.t.} \quad (28), (29), (30), (35).$$

Finally, the gain M is computed by using (33).

4. NUMERICAL EXAMPLE

Consider the open-loop unstable system presented in Rotondo et al. (2015b) described as (6) and interconnected to the controller (9) with $B = C = I_2$,

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 1.5 \end{bmatrix}, \ A_c = \begin{bmatrix} -16.6325 & -0.2556 \\ -0.313 & -15.9459 \end{bmatrix}, \ C_c = \begin{bmatrix} 0.0894 & 0.0255 \\ 0.0218 & 0.0563 \end{bmatrix},$$
$$B_c = \begin{bmatrix} -16.0585 & 0.1392 \\ -0.8227 & -15.6573 \end{bmatrix}, \ D_c = \begin{bmatrix} -2.4119 & -0.353 \\ -0.4528 & -1.6959 \end{bmatrix} \quad (37)$$

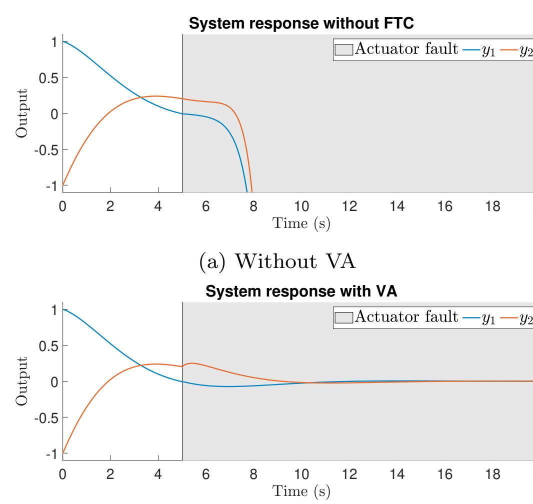
Two scenarios are considered where actuator fault occurs. For both scenarios, actuator faults are included in B_f as in (8). Then, Theorem 1 and the procedure described in Subsection 3.4 are used to compute the VA (11) which guarantees the passivation and stabilization of the reconfigured system. [1]

In the first scenario, a partial loss of 75% in the first actuator is simulated after $t = 5$ s, i.e., $f_a = [0.75\ 0]^\top$. In this case, the VA with the following gains is obtained:

$$M = \begin{bmatrix} 75.1843 & 0.7029 \\ 1.1544 & 4.8732 \end{bmatrix}, \ N = \begin{bmatrix} 0 & 0.0039 \\ -0.0039 & 0 \end{bmatrix}. \quad (38)$$

In the second scenario, an actuator fault of $f_a = [0\ 1]^\top$ occurs, i.e., the second actuator is fully lost after $t = 8$ s. In this case, the VA with the following gains is obtained:

$$M = \begin{bmatrix} 18.8467 & 54.8181 \\ 7.7980 & 28.8474 \end{bmatrix}, \ N = \begin{bmatrix} 0 & -0.0262 \\ 0.0262 & 0 \end{bmatrix}. \quad (39)$$



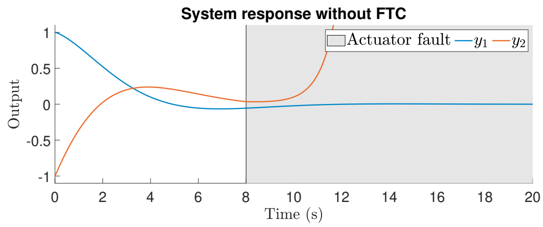
(a) Without VA

(b) With VA

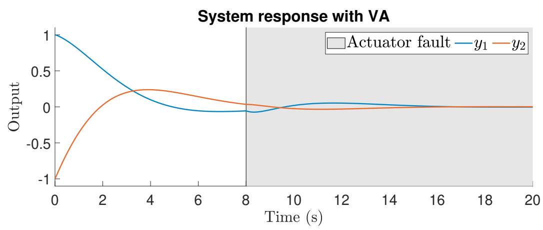
Fig. 2. Response of the system (7) with matrices (37) and actuator fault $f_a = [0.75\ 0]^\top$ at $t = 5$ s (first scenario).

The results of the first and second scenario with and without VA are depicted respectively in Figs. 2 and 3. Notice that the passivation-based VAs, designed based on Theorem 1, are able to guarantee the stability of the

[1] In particular, the LMI-based optimization problems are solved in Matlab using the parser Yalmip and the solver Mosek.



(a) Without VA



(b) With VA

Fig. 3. Response of the system (7) with matrices (37) and actuator fault $f_a = [0\ 1]^\top$ at $t = 8$ s (second scenario).

reconfigured systems for both partial fault (cf. Fig. 2b) and a complete actuator loss (cf. Fig. 3b). A small overshooting can be noticed just after the fault occurrence and the VA insertion, but the trajectories keep converging to the origin. Otherwise, without the proposed VA, the trajectories become unstable in both scenarios, as depicted in Figs. 2a and 3a.

5. CONCLUSION

This paper has presented a novel approach, based on the passivity/dissipativity theory, for designing RBs for fault hiding of linear systems subject to actuator faults. It is shown that the Virtual Actuator (VA) is a particular case of passivation block and novel conditions for designing VAs to guarantee that the faulty plant achieves the desired passivity indices are also provided. Moreover, LMI-based conditions were provided for designing the proposed VAs for ensuring the stability recovery for linear systems after the passivation. A numerical example has been used for illustrating the proposed approach.

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