Microalgae Production and Maintenance Optimization via Mixed-Integer Model Predictive Control ⋆

Juan Martinez-Piazuelo * Carlos Ocampo-Martinez * Nicanor Quijano ** Ari Ingimundarson ***

* *Automatic Control Department, Universitat Politècnica de Catalunya, Barcelona, Spain (e-mail: juan.pablo.martinez.piazuelo@upc.edu, carlos.ocampo@upc.edu).*
** *Electronics Engineering Department, Universidad de los Andes, Bogota, Colombia (e-mail: nquijano@uniandes.edu.co)*
*** *VAXA Technologies Ltd., Iceland (e-mail: ari@vaxa.life)*

Abstract: This paper studies the joint production and maintenance scheduling in microalgae manufacturing systems comprised of multiple machines, which are subject to coupled production demand agreements and operational maintenance constraints. Namely, there are some microalgae production demands to be met over a given horizon, and the maintenance of each microalgae manufacturing unit must be done before a given deadline. Moreover, the number of units whose maintenance can be done simultaneously over the same day is limited, and the units that undergo maintenance cannot contribute to microalgae production during their maintenance day. To solve the considered problem, we design a mixed-integer nonlinear model predictive controller, which is implemented in two optimization stages. The former regards a mixed-integer model predictive control problem, while the latter considers a nonlinear model predictive control problem. The proposed approach allows us to decouple the mixed-integer and nonlinear parts of the whole problem, and thus provides more flexibility on the optimization solvers that can be employed. In addition, the first stage also evaluates the attainability of the demand agreements, and provides a mechanism to minimally adjust such constraints so that their satisfaction can be guaranteed at the second stage. The overall model predictive control approach is based on experimental data collected at VAXA Technologies Ltd., and the effectiveness of the proposed method is validated through numerical simulations including multiple manufacturing units and uncertainties.

Keywords: Model predictive control; microalgae manufacturing plant control; production planning and control; maintenance scheduling; mixed-integer optimization.

1. INTRODUCTION

Due to the increasing environmental concerns and motivated by the united nations (UN) seventeen sustainable development goals (UN General Assembly, 2015), the modern industry has shown a significant interest in microalgae as a promising alternative for sustainable food production. For instance, VAXA Technologies Ltd. cultivates microalgae rich in protein and Omega-3 as a method to convert clean energy into sustainable nutrition, while maintaining a carbon negative profile and addressing the UN sustainable development challenges $\#2, 3, 9, 12-14$. Furthermore, recent researches have explored the potential of microalgae not only for human and animal nutrition (Amorim et al., 2021; Mahata et al., 2022), but also for the production of pigments (Pagels et al., 2020), biofuels (Eldiehy et al., 2022), and as wastewater treatment and CO_2 biofixation mechanisms (Al-Jabri et al., 2021; Molazadeh et al., 2019), among many other applications. As such, the automation and optimization of microalgae industrial production is a relevant research topic for control engineers.

The control and optimization of microalgae production imposes three main challenges. First, control-oriented models for microalgae production optimization are difficult to develop as complex bioprocesses are involved, e.g., the photosynthesis in photoautotrophic microalgae species (Ifrim et al., 2022). Second, at the industrial level, microalgae production is often subject to demand agreements that must be satisfied. Hence, the underlying control algorithm should determine the amount of microalgae to be harvested every day, so that the production demands can be met over a given horizon. Third, as with other manufacturing systems, microalgae production units must undergo recurrent maintenance and cleaning. This fact signifies an additional layer of planning because maintenance/cleaning operations often interfere with microalgae production in the units that undergo maintenance, and thus affect the satisfaction of the overall demand constraints. Based on the previous discussion, in this paper we focus on the

⋆ Juan Martinez-Piazuelo gratefully acknowledges the Universitat Politècnica de Catalunya and Banco Santander for the financial support of his predoctoral grant FPI-UPC. In addition, the authors would like to thank VAXA Technologies Ltd. as well as the project PID2020-115905RB-C21 (L-BEST) funded by MCIN/ AEI /10.13039/501100011033 for supporting this research.

second and third challenges, and we design a mixed-integer model predictive controller to tackle them. Namely, we design a model predictive control approach for the joint production and maintenance management of microalgae harvesting systems. Regarding the first challenge, on the other hand, we use a high-level modeling approach based on experimental data collected at VAXA Technologies Ltd. Hence, we avoid the difficulties of complex low-level modeling by employing high-level descriptive experimental data.

Related to our research, the problem of joint production and maintenance scheduling has been recently studied from the general perspective of manufacturing systems. For instance, Kang and Subramaniam (2018) propose an integrated control model for the dynamic maintenance and production of a two-machine manufacturing system subject to usage deterioration. Similarly, Polotski et al. (2019) study the problem of joint production and maintenance policy optimization in a single-machine hybrid manufacturing-remanufacturing system subject to age-dependent deterioration. Both of the previous works exploit ideas of dynamic programming and optimal control, and the proposed control policies seek to minimize the total production cost, which includes inventory, backlog, and maintenance costs. In contrast, Rokhforoz and Fink (2021) design a distributed model predictive controller for the joint production and maintenance scheduling in multi-machine manufacturing systems subject to usage deterioration. Motivated by this later approach, in this paper we also design a model predictive controller for the joint production and maintenance optimization of multi-machine microalgae harvesting systems. In contrast to Rokhforoz and Fink (2021), however, we consider a centralized control approach but we include coupling constraints over the maintenance of the multiple machines. Namely, we impose that the maintenance of only a limited number of machines can be done simultaneously over the same day, which is often the case in actual industrial manufacturing systems. This fact renders the underlying optimization problem as a mixed-integer programming problem with coupled constraints both on the production demands and the maintenance schedule.

Based on the above, the main contributions of this paper are as follows.

- First, we formulate the microalgae production and maintenance control task as a mixed-integer nonlinear programming problem within a model predictive control scheme. Such a problem considers coupled demand and maintenance constraints over a given time horizon, and the underlying nonlinear model for the microalgae growth process is based on experimental data collected at VAXA Technologies Ltd.
- Second, we split the mixed-integer and nonlinear parts of the aforementioned programming problem into a modular two-stage optimization approach comprised of a mixed-integer quadratic programming problem and a nonlinear programming problem. Since general mixed-integer nonlinear programming problems are quite challenging to solve, the proposed splitting allows us to have more flexibility on the optimization solvers to be employed. In addition, the first stage of the proposed approach allows us to estimate whether the considered demand constraints are attainable, and provides a mechanism to adjust the demand agreements to guarantee their attainability under a worst-case scenario.
- Third, we illustrate the performance of the proposed two-stage optimization approach via numerical simulations considering multiple microalgae manufacturing units. Moreover, model uncertainties are included in the simulations to further validate the proposed approach.

The remainder of this paper is organized as follows. First, in Section 2 we formally state the problem that is studied in this paper. Second, in Section 3 we formulate our proposed approach to solve the considered problem. Then, in Section 4 we provide some numerical simulations to illustrate the effectiveness of our proposed solution. Finally, in Section 5 we conclude the paper and mention some future directions of research.

2. PROBLEM STATEMENT

In this paper, we consider the problem of joint production and maintenance optimization of a set of microalgae manufacturing units, also referred to as miroalgae cultures (as illustration, Fig. 1 shows an actual microalgae production unit from VAXA Technologies Ltd). Each culture is readily equipped with low-level control loops to keep the temperature, pH, light, and nutrients at adequate levels for microalgae growth. Hence, our focus is on a high-level optimization task to decide the amount of biomass to be harvested from each culture every day (so that certain production demands are satisfied), and to schedule the day for maintenance/cleaning of each culture (so that the operational constraints of the system are met).

To formally model the problem, we let $\mathcal{C} = \{1, 2, \dots, N\}$ denote the set of microalgae cultures, where $N \in \mathbb{Z}_{\geq 1}$ is the total number of cultures, and we let $x_i \in \mathbb{R}_{\geq 0}$ be the biomass of microalgae (in kg) contained in culture $i \in \mathcal{C}$. Additionally, we let $v_i \in \mathbb{Z}_{\geq 0}$ be an auxiliary variable denoting the operational running-time (in days) of culture $i \in \mathcal{C}$ since its last maintenance. Based on the above, the day-to-day dynamics of every culture $i \in \mathcal{C}$ are given by

$$v_i[k+1] = (1 - z_i[k])\,(v_i[k] + 1) \tag{1a}$$

$$x_i[k+1] = (1 - z_i[k])\,(x_i[k] + \alpha\,(x_i[k]) - y_i[k]) + \underline{x} z_i[k]. \tag{1b}$$

Here, $y_i[k] \in \mathbb{R}_{\geq 0}$ is the amount of biomass (in kg) to be harvested from the culture i at day $k \in \mathbb{Z}_{\geq 0}$; $z_i[k] \in \{0, 1\}$ is a binary variable that takes the value 1 if the maintenance of culture i is to be done during day k, and takes the value 0 otherwise; $\underline{x} \in \mathbb{R}_{>0}$ denotes the initial biomass (in kg) introduced at each culture after its maintenance; and $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a scalar-valued map characterizing the daily growth rate of each culture as a function of the biomass of microalgae available in the culture. Namely, based on methodological suggestions and experimental data collected at VAXA Technologies Ltd. (see Fig. 2), we model $\alpha(\cdot)$ as the quadratic polynomial given by

$$\alpha\,(x_i) = -0.5305 x_i^2 + 0.4435 x_i - 0.0655. \tag{2}$$

According to the model in (1), each culture $i \in \mathcal{C}$ is characterized by two state variables, v_i and x_i, and is subject to two control inputs, y_i and z_i. Furthermore, at every day k, the system's states and control inputs must satisfy the constraints given by

$$\sum_{i \in \mathcal{C}} \left(y_i[k] + x_i[k] z_i[k] - \underline{x} z_i[k] \right) \geq d[k] \tag{3a}$$

$$\sum_{i \in \mathcal{C}} z_i[k] \leq N_z \tag{3b}$$

$$z_i[k] \in \{0, 1\}, \quad \forall i \in \mathcal{C} \tag{3c}$$

$$y_i[k] \geq 0, \quad \forall i \in \mathcal{C} \tag{3d}$$

$$y_i[k] z_i[k] = 0, \quad \forall i \in \mathcal{C} \tag{3e}$$

$$x_i[k] - y_i[k] \geq \underline{x}, \quad \forall i \in \mathcal{C} \tag{3f}$$

$$x_i[k] \leq \overline{x}, \quad \forall i \in \mathcal{C} \tag{3g}$$

$$v_i[k] \geq \underline{v} z_i[k], \quad \forall i \in \mathcal{C} \tag{3h}$$

$$v_i[k] \leq \overline{v}, \quad \forall i \in \mathcal{C}. \tag{3i}$$

Namely, the constraints in (3a) reflect the fact that a microalgae production demand $d[k] \in \mathbb{R}_{\geq 0}$ must be met for every day k (note that such a constraint presumes that $x_i[k] - \underline{x}$ biomass of culture $i \in \mathcal{C}$ is harvested when the corresponding maintenance/cleaning is performed, given that $\underline{x}$ biomass is used to restart culture i after maintenance); the constraints in (3b) establish that the maintenance of at most $N_z \in \mathbb{Z}_{\geq 1}$ cultures can be done at the same day, while the constraints in (3c) force the maintenance decision variables to be binary; the constraints in (3d) state that the harvested biomass must be non-negative; the constraints in (3e) establish that no harvesting and maintenance can be performed simultaneously; the constraints in (3f) ensure that the harvesting action keeps the biomass in each culture always above the lower limit $\underline{x}$, while the constraints in (3g) seek to maintain the biomass in each culture below the upper limit $\overline{x} \in \mathbb{R}_{> \underline{x}}$; finally, the constraints in (3h) impose a minimum number of $\underline{v} \in \mathbb{Z}_{\geq 0}$ days between successive maintenance operations for each culture, whilst the constraints in (3i) ensure that no more than $\overline{v} \in \mathbb{Z}_{\geq \max\{\underline{v}, 1\}}$ days can go by without performing the maintenance of each culture.

Based on the model in (1) and the constraints in (3), the goal is to compute the control inputs for each culture $i \in \mathcal{C}$ following a model predictive control (MPC) approach over a prediction horizon of $H_p \in \mathbb{Z}_{\geq 1}$ days. To formulate the MPC problem, let us first introduce the following notations. Namely, we let $v = \mathrm{col}\,(v_1, v_2, \ldots, v_N)$, $x = \mathrm{col}\,(x_1, x_2, \ldots, x_N)$, $y = \mathrm{col}\,(y_1, y_2, \ldots, y_N)$, and $z = \mathrm{col}\,(z_1, z_2, \ldots, z_N)$, be the corresponding column vector concatenations of the overall state and control variables. Moreover, we let

$$\begin{aligned}
\mathbf{v}[k] &= \mathrm{col}\,(v[k], v[k+1], \ldots, v[k+H_p]) \\
\mathbf{x}[k] &= \mathrm{col}\,(x[k], x[k+1], \ldots, x[k+H_p]) \\
\mathbf{y}[k] &= \mathrm{col}\,(y[k], y[k+1], \ldots, y[k+H_p-1]) \\
\mathbf{z}[k] &= \mathrm{col}\,(z[k], z[k+1], \ldots, z[k+H_p-1]),
\end{aligned}$$

be the corresponding sequences of state and control variables over H_p. Observe that while the state sequences $\mathbf{v}[k]$ and $\mathbf{x}[k]$ belong to an $N(H_p+1)$-dimensional space, the control sequences $\mathbf{y}[k]$ and $\mathbf{z}[k]$ belong to an NH_p-dimensional space. Following the previous notation, in this paper we consider the MPC problem defined next.

Definition 1. The MPC-related optimization problem to be solved at day k is given by

$$\max_{\mathbf{v}[k], \mathbf{x}[k], \mathbf{y}[k], \mathbf{z}[k]} \sum_{\tau=1}^{H_p} \sum_{i \in \mathcal{C}} y_i \left[k + \tau \right], \tag{4}$$

subject to the constraints in (1) and (3), the initial conditions $v[k] = v_0$ and $x[k] = x_0$, and the constraints given by

$$\sum_{\tau=0}^{H_p-1} z_i[k+\tau] \leq 1 + \lfloor H_p / \overline{v} \rfloor, \quad \forall i \in \mathcal{C}, \tag{5}$$

where $\lfloor H_p / \overline{v} \rfloor$ denotes the (floor) integer division of H_p over $\overline{v}$. Namely, the constraints in (5) restrict the maximum number of maintenance operations that can be scheduled for each culture over the prediction horizon. □

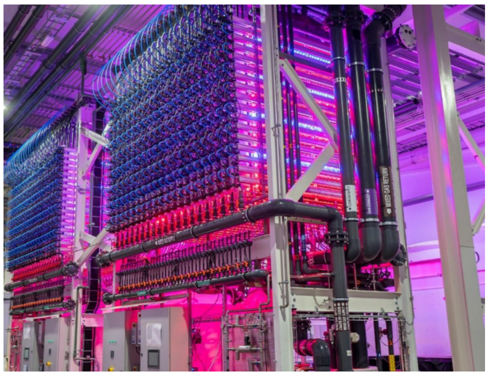

Fig. 1. Photo of a microalgae production unit (culture) from VAXA Technologies Ltd.

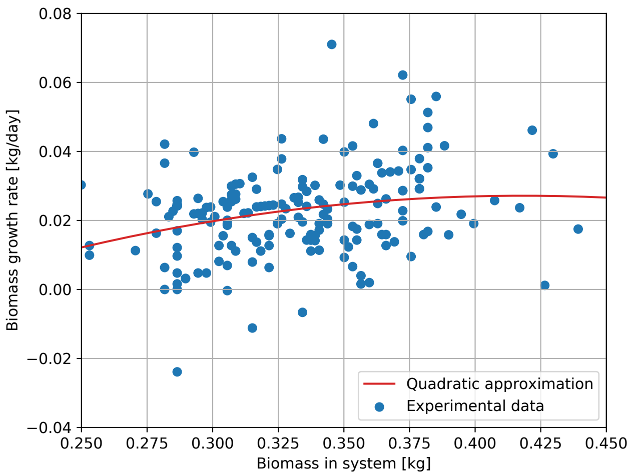

Fig. 2. Experimental data collected at VAXA Technologies Ltd. characterizing the daily biomass growth rate as a function of the biomass available in the culture (the data has been scaled for privacy preserving reasons). The quadratic approximation fit is given in (2).

The optimization problem in Definition 1 comprises a mixed-integer nonlinear programming (MINLP) problem to be solved at each day k. Such a problem regards the maximization of microalgae production over H_p and its solution provides the control sequences regarding the harvesting $\mathbf{y}[k]$ and maintenance $\mathbf{z}[k]$ decision variables. Clearly, the parameters of such an MPC-related optimization problem must satisfy the following condition.

Standing Assumption 1: The parameters N, N_z, $\underline{v}$, $\overline{v}$, and v_0, are such that there exists a feasible maintenance schedule satisfying the constraints in (3b), (3h), and (3i), for every day k. □

3. PROPOSED APPROACH

The MINLP problem in Definition 1 is a challenging optimization task for three main reasons: i) the dynamics in (1) and the constraints in (3a) and (3e) involve nonlinear terms over the optimization variables, which render the problem as a nonlinear programming problem; ii) the constraints in (3c) force the problem to be of mixed-integer nature, which increases its difficulty for larger numbers of cultures and prediction horizons; and iii) given an arbitrary demand profile $\mathbf{d}[k] = \mathrm{col}\left(d[k], d[k+1], \ldots, d[k+H_p-1]\right)$, there are no guarantees that a feasible solution exists and it is not immediately obvious how to check whether the given demands are attainable. Hence, to handle these difficulties, we propose a two-stage optimization approach as follows.

- First, we formulate a mixed-integer quadratic programming (MIQP) problem to determine the maintenance schedule and to adjust the production demands to guarantee their attainability. Such an MIQP problem regards a (worst-case) linear approximation of the quadratic growth rate in (2).
- Second, we formulate a nonlinear programming (NLP) problem to compensate for the linear approximation of the first stage and to further maximize the microalgae production while satisfying the adjusted demands. Notice, however, that the NLP problem of the second stage does not involve mixed-integer optimization variables as the maintenance schedule is determined in the first stage.

The main motivation behind the proposed two-stage approach is that many commercial optimization solvers are better suited to solve MIQP problems than general MINLP problems [1]. Hence, by splitting the mixed-integer and nonlinear parts of the problem into the two stages, we decouple the main challenges of the MPC-related MINLP problem of Definition 1. We now proceed to explain both stages.

3.1 First Optimization Stage: MIQP Problem

First, to handle the nonlinear constraints involving products of optimization variables, we introduce the change of variables given by

$$V_i = v_i z_i, \quad \forall i \in \mathcal{C} \tag{6a}$$
$$X_i = x_i z_i, \quad \forall i \in \mathcal{C} \tag{6b}$$
$$Y_i = y_i z_i, \quad \forall i \in \mathcal{C}. \tag{6c}$$

Based on (6), the constraints in (3a) can be rewritten as

$$\sum_{i \in \mathcal{C}} \left(y_i[k] + X_i[k] - \underline{x} z_i[k]\right) \geq d[k], \tag{7}$$

while the constraints in (3e) can be rewritten as

$$Y_i[k] = 0, \quad \forall i \in \mathcal{C}. \tag{8}$$

Here, we highlight that the variables Y_i can be eliminated from the problem as they are always zero. Now, to effectively contemplate the change of variables in (6) within the optimization problem, we employ the big-M method (Bemporad and Morari, 1999, Section 2) and include the additional constraints given by

$$V_i[k] \geq 0, \qquad \forall i \in \mathcal{C} \tag{9a}$$
$$V_i[k] \leq z_i[k] M_V, \qquad \forall i \in \mathcal{C} \tag{9b}$$
$$V_i[k] \geq v_i[k] - (1 - z_i[k])\, M_V, \quad \forall i \in \mathcal{C} \tag{9c}$$
$$V_i[k] \leq v_i[k] + (1 - z_i[k])\, M_V, \quad \forall i \in \mathcal{C} \tag{9d}$$
$$X_i[k] \geq 0, \qquad \forall i \in \mathcal{C} \tag{9e}$$
$$X_i[k] \leq z_i[k] M_X, \qquad \forall i \in \mathcal{C} \tag{9f}$$
$$X_i[k] \geq x_i[k] - (1 - z_i[k])\, M_X, \quad \forall i \in \mathcal{C} \tag{9g}$$
$$X_i[k] \leq x_i[k] + (1 - z_i[k])\, M_X, \quad \forall i \in \mathcal{C} \tag{9h}$$
$$0 \geq y_i[k] - (1 - z_i[k])\, M_Y, \quad \forall i \in \mathcal{C} \tag{9i}$$
$$0 \leq y_i[k] + (1 - z_i[k])\, M_Y, \quad \forall i \in \mathcal{C}, \tag{9j}$$

where (9i) and (9j) contemplate (8), and the big-M constants are set to $M_V = \overline{v}$ [due to (3i)], $M_X = \overline{x}$ [due to (3g)], and $M_Y = \overline{x} - \underline{x}$ [due to (3f) and (3g)].

Second, to handle the nonlinearity of the model in (1) caused by the quadratic growth rate in (2), we consider the worst-case linear approximation of $\alpha(x_i)$ over the interval $[\underline{x}, \overline{x}]$. More precisely, we consider the map $\beta : \mathbb{R}_{\geq 0} \to \mathbb{R}$ given by

$$\beta\left(x_i\right) = \frac{\alpha(\overline{x}) - \alpha(\underline{x})}{\overline{x} - \underline{x}} \left(x_i - \underline{x}\right) + \alpha\left(\underline{x}\right). \tag{10}$$

Namely, $\beta(x_i)$ is the line that lies under $\alpha(x_i)$ whenever $x_i \in [\underline{x}, \overline{x}]$, and takes the same value as $\alpha(x_i)$ at the boundaries of the interval. Hence, we approximate $\alpha(x_i) \approx \beta(x_i)$. Consequently, based on (6), (8), and (10), the model in (1) can be rewritten as

$$v_i[k+1] = v_i[k] + 1 - V_i[k] - z_i[k] \tag{11a}$$
$$\begin{aligned} x_i[k+1] = {} & (1+m) x_i[k] - m\underline{x} + \alpha(\underline{x}) - y_i[k] \\ & - (1+m) X_i[k] + \left(m\underline{x} - \alpha(\underline{x}) + \underline{x}\right) z_i[k], \end{aligned} \tag{11b}$$

for all $i \in \mathcal{C}$, where $m = \left(\alpha(\overline{x}) - \alpha(\underline{x})\right) / \left(\overline{x} - \underline{x}\right)$.

Under the above formulations, the MINLP problem in Definition 1 can be transformed into a mixed-integer linear programming (MILP) problem where all the constraints are now affine. Nevertheless, such an MILP problem might still not have a solution under arbitrary production demands (due to the infeasibility of (7)) and it is not immediately obvious how to check whether the given demands are attainable. To handle such an issue, we formulate an MIQP problem to determine the maximum attainable demands as well as the maintenance schedule. To do so, we introduce some auxiliary optimization variables $\epsilon[\tau] \in \mathbb{R}_{\geq 0}$, for all $\tau = k, k+1, \ldots, k+H_p-1$, and we set the notations

$$V = \mathrm{col}\left(V_1, V_2, \ldots, V_N\right)$$
$$X = \mathrm{col}\left(X_1, X_2, \ldots, X_N\right)$$
$$\mathbf{V}[k] = \mathrm{col}\left(V[k], V[k+1], \ldots, V[k+H_p-1]\right)$$
$$\mathbf{X}[k] = \mathrm{col}\left(X[k], X[k+1], \ldots, X[k+H_p-1]\right)$$
$$\boldsymbol{\epsilon}[k] = \mathrm{col}\left(\epsilon[k], \epsilon[k+1], \ldots, \epsilon[k+H_p-1]\right).$$

Based on the above, the resultant MIQP problem to be solved is defined as follows.

Definition 2. The first-stage MPC-related MIQP problem to be solved at day k is given by

$$\min_{\mathbf{v}[k], \mathbf{x}[k], \mathbf{y}[k], \mathbf{z}[k], \mathbf{V}[k], \mathbf{X}[k], \boldsymbol{\epsilon}[k]} \quad \boldsymbol{\epsilon}[k]^\top \mathbf{W} \boldsymbol{\epsilon}[k],$$

subject to the constraints in (3b)-(3d), (3f)-(3i), (5), (9), and (11), the initial conditions $v[k] = v_0$ and $x[k] = x_0$, and the constraints given by

[1] Some popular solvers well-suited for MIQP problems are CPLEX and Gurobi (CPLEX, 2009; Gurobi Optimization, LLC, 2022).

$$\sum_{i \in \mathcal{C}} (y_i[k] + X_i[k] - \underline{x} z_i[k]) \geq d[k] - \epsilon[k] \tag{12a}$$

$$\epsilon[k] \geq 0. \tag{12b}$$

Here, $\mathbf{W} \in \mathbb{R}_{\geq 0}^{H_p \times H_p}$ is a non-negative diagonal weighting matrix that weights the importance of the production demands over the prediction horizon. □

By solving the MIQP problem in Definition 2, one can determine i) the optimal maintenance schedule $\mathbf{z}^*$, and ii) the minimal production demand adjustment $\boldsymbol{\epsilon}^*$ to guarantee the feasibility of (12a) under arbitrary production demands. Moreover, given that such an MIQP problem considers the worst-case linear approximation of the microalgae growth rate, the obtained adjusted demands are also attainable when considering the quadratic growth rate model in (2). This fact is important to guarantee the feasibility of the second-stage optimization problem that we introduce next.

3.2 Second Optimization Stage: NLP Problem

At the second optimization stage, we take the solutions $\mathbf{z}^*$ and $\boldsymbol{\epsilon}^*$ determined by solving the MIQP problem of the first stage, and we solve an MPC-related NLP problem to compensate for the worst-case linear approximation of the microalgae growth rate, and to further maximize the production of microalgae. More formally, at the second stage we consider the NLP problem defined next.

Definition 3. The second-stage MPC-related NLP problem to be solved at day k is given by

$$\max_{\mathbf{x}[k], \mathbf{y}[k]} \quad \sum_{\tau=1}^{H_p} \sum_{i \in \mathcal{C}} y_i [k+\tau],$$

subject to the constraints in (1b), (3d)-(3g), and (12a), the initial condition $x[k] = x_0$, and the solutions given by $\mathbf{z}[k] = \mathbf{z}^*$ and $\boldsymbol{\epsilon}[k] = \boldsymbol{\epsilon}^*$, where $\mathbf{z}^*$ and $\boldsymbol{\epsilon}^*$ are those obtained in the first stage. Therefore, $\mathbf{z}[k]$ and $\boldsymbol{\epsilon}[k]$ are not taken as optimization variables in the second stage, and thus it is not necessary to consider the states $\mathbf{v}[k]$ nor the dynamics in (1a) within the optimization (since with $\mathbf{z}[k] = \mathbf{z}^*$ fixed, the dynamics in (1a) and (1b) are decoupled). □

For the sake of clarity, in Algorithm 1 we summarize the proposed two-stage approach.

Algorithm 1 Proposed two-stage optimization approach

Require: $H_p \in \mathbb{Z}_{\geq 1}$
1: Compute $\mathbf{z}^*$ and $\boldsymbol{\epsilon}^*$ by solving the MPC-related MIQP problem of Definition 2.
2: Compute $\mathbf{y}^*$ by solving the MPC-related NLP problem of Definition 3.
3: Apply $y^*[k]$ and $z^*[k]$ to the system.
4: Repeat steps 1-3 for every day k.

Remark 1. The proposed two-stage approach in Algorithm 1 offers various practical advantages. First, many commercial optimization solvers are better suited to solve MIQP problems than MINLP problems. Hence, by splitting the mixed-integer and nonlinear parts of the problem over the two stages, we allow for more flexibility on the optimization solvers to be employed. Second, the $\boldsymbol{\epsilon}^*$ computed in the first stage provides an estimate of the attainability of the production demands. Thus, microalgae production companies as VAXA Technologies Ltd. can use such information to better size their microalgae cultures based on the minimum production demands that they seek to achieve. Finally, given that the proposed approach is modular, microalgae growth rates more complex (non-necessarily quadratic) than (2) can be immediately considered in the NLP problem of the second stage without increasing the complexity of the first stage. □

4. NUMERICAL SIMULATIONS

In this section, we provide some numerical simulations to illustrate the effectiveness of the proposed approach. Namely, we consider two scenarios, one with $N = 4$ and $N_z = 1$ and one with $N = 26$ and $N_z = 2$. For both cases we let $H_p = 40$ days, $\overline{v} = 28$ days, $\underline{v} = 14$ days, $\overline{x} = 0.450$ kg, and $\underline{x} = 0.250$ kg. Moreover, without loss of generality, in both of the corresponding MIQP problems we let $\mathbf{W}$ be the $H_p \times H_p$ identity matrix. Based on these parameters, we simulate 41 days of operation following the receding horizon optimization approach depicted in Algorithm 1. To add uncertainty to the simulation, we take random initial conditions v_0 and x_0 for the first day, and we add a random noise to the nominal growth rate given by $\alpha(\cdot)$. For each culture, such a noise is taken according to the normal distribution $\mathcal{N}(0, 0.05^2)$. On the other hand, regarding the production demand agreements we consider the profile presented in Figs. 3 and 4. Such a profile is taken according to some of the weekly production demands at VAXA Technologies Ltd., yet the actual values are randomized due to privacy preserving reasons. All simulations are executed using Casadi (Andersson et al., 2019) on an Intel Core i7-9750H CPU running at 2.60 GHz and with 16 GB of RAM. For the MIQP problem we employ the ILOG CPLEX solver 12.80 (CPLEX, 2009), whilst for the NLP problem we use the IPOPT solver distributed with Casadi.

Figures 3 and 4 depict the performance of the proposed approach under the considered scenarios. We highlight that with $N = 4$ the production demand agreements are unattainable. Yet, the first stage adjusts the demands to guarantee the feasibility of the NLP problem of the second stage. In contrast, with $N = 26$ the original production demands are indeed satisfied. As comparison, in the $N = 4$ scenario the average controller's computing time for each day is 12.47 ± 3.53 s, whereas in the $N = 26$ scenario it is 3.24 ± 0.60 s. This fact shows that having attainable demands speeds up the computing time even for a higher number of cultures. Hence the importance of the proposed first stage as it provides information on how to adjust the demands to be attainable. On the other hand, Fig. 5 depicts the executed maintenance operations in the case with $N = 26$. Clearly, it is verified that all the maintenance-related constraints are satisfied. Finally, it is worth to highlight that directly solving the full MINLP problem of Definition 1 is quite challenging even for a reduced number of cultures. In fact, for a random instance of the simple scenario with $N = 1$ (for larger N the employed MINLP solver often fails to find a solution and a good guess for the initial point is required) and attainable production demand agreements, our proposed two-stage approach takes 0.09 ± 0.01 s to

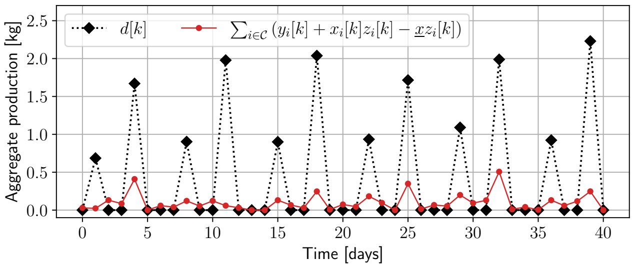

Fig. 3. Total biomass production and demand agreements over the simulation horizon for the case with $N = 4$.

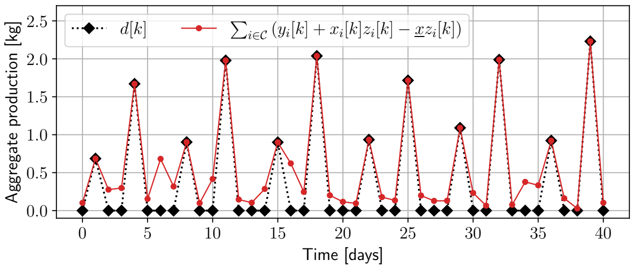

Fig. 4. Total biomass production and demand agreements over the simulation horizon for the case with $N = 26$.

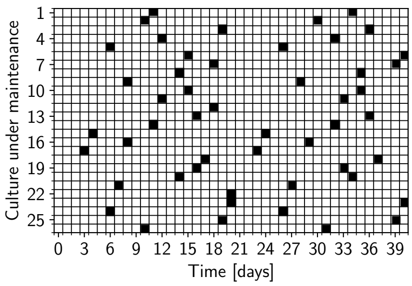

Fig. 5. Maintenance operations (black squares) performed over the simulation horizon for the case with $N = 26$.

achieve a solution, while solving the full MINLP problem with the BONMIN solver distributed with Casadi takes 84.89 ± 4.86 s (roughly 800 times slower than the proposed approach). Besides, our proposed approach achieves a total biomass production of 0.87 kg, whilst the MINLP one produces 0.88 kg of microalgae. Thus, we only have a 1.14% drop on performance for the considered instance.

5. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we have studied the problem of joint production and maintenance scheduling for microalgae harvesting systems. We have formulated the considered optimization task as a mixed-integer nonlinear programming (MINLP) problem within a model predictive control scheme, and we have proposed a two-stage approach to split the MINLP problem into an mixed-integer quadratic programming problem and an nonlinear programming problem, so that the mixed-integer and nonlinear parts of the whole optimization problem are decoupled. Future work should explore distributed implementations of the proposed approach to further scale for larger systems.

REFERENCES

Al-Jabri, H., Das, P., Khan, S., Thaher, M., and AbdulQuadir, M. (2021). Treatment of wastewaters by microalgae and the potential applications of the produced biomass — A review. *Water*, 13(1), 27.

Amorim, M.L., Soares, J., dos Reis Coimbra, J.S., de Oliveira Leite, M., Albino, L.F.T., and Martins, M.A. (2021). Microalgae proteins: production, separation, isolation, quantification, and application in food and feed. *Critical Reviews in Food Science and Nutrition*, 61(12), 1976–2002.

Andersson, J.A.E., Gillis, J., Horn, G., Rawlings, J.B., and Diehl, M. (2019). CasADi – A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, 11(1), 1–36.

Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3), 407–427.

CPLEX, I.I. (2009). V12. 1: User's manual for IBM ILOG CPLEX. *International Business Machines Corporation*, 46(53), 157.

Eldiehy, K.S., Bardhan, P., Borah, D., Gohain, M., Ahmad Rather, M., Deka, D., and Mandal, M. (2022). A comprehensive review on microalgal biomass production and processing for biodiesel production. *Fuel*, 324, 124773.

Gurobi Optimization, LLC (2022). Gurobi Optimizer Reference Manual. URL `https://www.gurobi.com`.

Ifrim, G.A., Titica, M., Horincar, G., Antache, A., Baicu, L., Barbu, M., and Guzmán, J.L. (2022). Model based optimal control of the photosynthetic growth of microalgae in a batch photobioreactor. *Energies*, 15(18), 6535.

Kang, K. and Subramaniam, V. (2018). Joint control of dynamic maintenance and production in a failure-prone manufacturing system subjected to deterioration. *Computers & Industrial Engineering*, 119, 309–320.

Mahata, C., Das, P., Khan, S., Thaher, M.I.A., Abdul Quadir, M., Annamalai, S.N., and Al Jabri, H. (2022). The potential of marine microalgae for the production of food, feed, and fuel (3F). *Fermentation*, 8(7), 316.

Molazadeh, M., Ahmadzadeh, H., Pourianfar, H.R., Lyon, S., and Rampelotto, P.H. (2019). The use of microalgae for coupling wastewater treatment with CO_2 biofixation. *Frontiers in Bioengineering and Biotechnology*, 7, 42.

Pagels, F., Salvaterra, D., Amaro, H.M., and Guedes, A.C. (2020). Pigments from microalgae. In E. Jacob-Lopes, M.M. Maroneze, M.I. Queiroz, and L.Q. Zepka (eds.), *Handbook of Microalgae-Based Processes and Products*, 465–492. Academic Press.

Polotski, V., Kenne, J.P., and Gharbi, A. (2019). Joint production and maintenance optimization in flexible hybrid manufacturing–remanufacturing systems under age-dependent deterioration. *International Journal of Production Economics*, 216, 239–254.

Rokhforoz, P. and Fink, O. (2021). Distributed joint dynamic maintenance and production scheduling in manufacturing systems: Framework based on model predictive control and benders decomposition. *Journal of Manufacturing Systems*, 59, 596–606.

UN General Assembly (2015). Transforming our world: the 2030 agenda for sustainable development. URL `https://www.refworld.org/docid/57b6e3e44.html`.