

Modelling and Adaptive Parameter Estimation for a Piezoelectric Cantilever Beam

Bin Wang, Ramon Costa-Castelló, *Senior Member, IEEE*, Jing Na, *Member, IEEE*, Oscar de la Torre, Xavier Escaler

Abstract—This paper proposes a new adaptive estimation approach to online estimate the model parameters of a piezoelectric cantilever beam. The beam behavior is firstly modeled using partial differential equations (PDE) considering the Kelvin-Voigt damping. To facilitate the estimation of unknown model parameters, the Galerkin’s method is introduced to extract desired vibration modes by separating the time and space variables of the PDE. Then, considering two major vibration modes, the corresponding system model can be represented by a fourth-order ordinary differential equation (ODE). Finally, by using measured input and output information, a novel adaptive parameter estimation strategy is introduced to estimate the unknown parameters of the derived ODE model in real time. For the purpose of driving the parameter updating law, the estimation error is extracted by using an auxiliary variables and a time-varying gain. Consequently, the convergence of the parameter estimation error is rigorously proved based on the Lyapunov theory. Simulations and experimental results show the validity and practicability of the proposed estimation method.

Index Terms—Piezoelectric cantilever beam, beam modelling, partial differential equation (PDE), adaptive parameter estimation.

I. INTRODUCTION

During the past decades, many researchers have been interested in modelling of beam to explore modal dynamics [1] [2] [3] due to its application in many engineering practice. The well known Euler–Bernoulli beam [1], Timoshenko beam [4] and Rayleigh beam [5] were investigated, leading to the developments of a broad class of beam models [6], [7]. In [8], a piezoelectric laminate beam model was developed using piezoelectric actuator and sensor. In [9], the Rayleigh damping was included into the Euler–Bernoulli beam to represent a flexible-link robot. However, most of these beam models are described by using partial differential equations (PDE). Although the control-oriented adaptive estimation can be theoretically achieved [10], a critical issue lies in the PDE models is that estimating the unknown parameters of PDE is time

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consuming and sometimes impossible to complete in practice [11]. To simplify the PDE models, the Galerkin’s summation method was proposed and its convergence properties were studied under different boundary conditions in [12]. In [13], a specific procedure for implementing the Galerkin’s method was provided and illustrated with numerical examples. In [14], a reduced-order model was introduced by using the truncated set of linear mode shapes, where different vibration modes of the elastic beams were studied. Another approach named differential quadrature method that can simplify the PDE model in an approximate form was proposed in [15]. The effect of different varying parameters was discussed based on the simplified form. In [16], the balanced model reduction method was proposed to simplify the high-order model. However, the states information is replaced by the new states that have no physical meanings. Although the modelling of the beam and the simplification of several PDE models have been studied in the above-mentioned literatures, a model that is suitable for online estimating the parameters of the beam by using the piezoelectric actuator and strain gauge was not developed yet.

For the vibrating systems like the beam, system parameters may be unknown and varying under different operating conditions [17], [18]. For example, the frequency and damping of the beam in different liquids are not the same [19]. It is highly desired to design parameter estimation strategies to online estimate the unknown varying model parameters, which in turn contributes to the precise modeling and control synthesis. For parameter estimation techniques of the PDE models, many challenging mathematical problems (e.g. the uniqueness and stability) remain to be addressed. In [20], a high-gain adaptive regulator was introduced to stabilize and estimate the unknown parameters of a PDE based beam model. Although the uniqueness of the classical solution is justified, the computational load is high for solving the PDE numerically. Hence, in practice, it is preferable to construct the ordinary differential equation (ODE) based models, which can be solved analytically. In [16], an offline identification approach was adopted to estimate the parameters of the discrete-time beam model. In [21], an online estimator was proposed to estimate the frequencies of the two main vibration modes of a cantilever beam. The work was done in the frequency domain based on the algebraic derivative approach.

For parameter estimation of ODE-based systems, many researchers have been working on adaptive parameter estimation techniques [22]. In [12], a two-stage gradient based method was proposed by means of iterative algorithms for autoregressive systems. In [23], an online optimization algorithm based on the gradient method together with an extended

Kalman filter was investigated. Another well-known parameter estimation technique is the least-squares method or recursive least-squares method [24], [25]. Although the convergence of these algorithms can be proved under the persistent excitation condition, there are still challenging issues accounting for measurement noises for the beam systems and the verification of the required excitation condition.

Moreover, for parameter estimation, it is more convenient to parameterize the system model to separate the lumped parameters from signals [26]. In this procedure, the use of differentiation (or system states) is not desirable because of the amplification of the noises, which would even be worse for the plants that contain high-frequency modes [24]. To address this problem, in our previous work [27], a stable filter is applied to derive a new parameterized formula without using the derivatives. Then the unknown parameters are estimated by using the input and output signals only. In addition, the convergence of parameter estimation error is proved under the persistent excitation (PE) condition. However, with respect to the vibration problem in the beam system, there are multiple oscillation modes with higher frequencies. In such cases, the estimation algorithm in [27] may not retain its performance since only a constant gain set by the designer is adopted.

In this paper, a novel adaptive estimation approach is introduced to online estimate the unknown parameters of a cantilever beam modeled with the Euler-Bernoulli theory. We first derive the motion equation of the cantilever beam with the help of Hamilton's principle. The piezoelectric actuator model is included in the derived PDE model and the deflection-strain relationship is also presented for the purpose of experimental validation. To make it more convenient for parameter estimation, the Galerkin's method is adopted to convert the PDE model to an ODE model. The system behavior is further described as a state-space model. To derive an adaptive algorithm for estimating the system parameters with multiple oscillation modes, we select the first and second modes to seek for a tradeoff between model complexity and accuracy. The model with two oscillation modes is then written as a compact parameterized form. Based on the parameterized model, a new adaptive parameter updating law with a time-varying gain is proposed which is different to the adaptive law with a constant gain provided in [27]. The convergence of the parameter estimation error is proved by using the Lyapunov theory. Compared to the conventional gradient method and least-squares method, the proposed estimation approach can achieve better performance. Simulations and experiments are carried out to verify the validity of the proposed method.

Main contributions of the paper are summarized as follows:

- 1) A piezoelectric cantilever beam model is developed considering the Kelvin-Voigt damping and piezoelectric actuator. Based on the PDE model, a simplified ODE model is derived and the parameters can be online estimated.
- 2) A novel parameter estimation scheme by using the parameter estimation error is proposed to online estimate the unknown beam model parameters to retain convergence. Specifically, a time-varying gain is designed to handle the effects of regressor to enhance convergence performance and simplify the tuning of learning gains.

This paper is organized as follows: Section II describes the beam dynamics and the problem formulation. The adaptive parameter estimation method is introduced in Section III. Comparative simulations and experiments are given in Section IV. Conclusions are outlined in Section V.

II. BEAM MODELLING AND PROBLEM FORMULATION

In this section, we will derive a PDE based beam model via the Euler-Bernoulli beam theory. The piezoelectric actuator model will be included in the beam model. In order to collect the output information with strain gauge in the experiments, we will derive the relationship between deflection and strain. The derived beam model will be then simplified into an ODE based model and then a parameterized model will be constructed for parameter estimation.

A. Cantilever beam model

The sketch of the cantilever beam considering the flapwise deflection is depicted in Fig. 1.

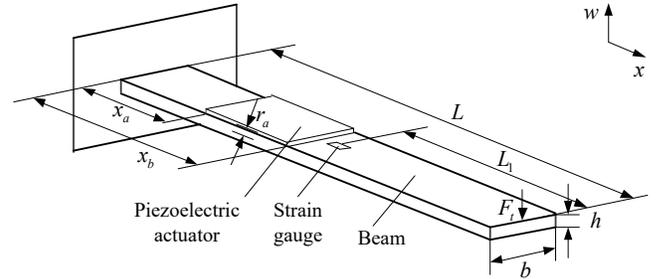


Fig. 1: Sketch of a cantilever beam

For an inextensible beam (neglecting the stretch deformation), the kinetic energy T can be given as [28], [29]

$$T = \int_0^L \frac{1}{2} m V_y^2 dx = \frac{1}{2} \rho A \int_0^L \left(\frac{\partial w}{\partial t} \right)^2 dx \quad (1)$$

where m is the mass of the cantilever beam, V_y is the flapwise velocity, ρ is the density, A is the cross-sectional area of the beam ($A = b \cdot h$, b is the beam width and h is the beam thickness), w is the flapwise displacement and L is the length of the beam.

To describe the dynamics associated with the damping, the Kelvin-Voigt damping model is considered. Based on the Kelvin-Voigt hypothesis, the conservative moment M_c and the non-conservative moment M_{nc} can be given as [30], [31]

$$M_c = EI \frac{\partial^2 w}{\partial x^2} \quad (2)$$

$$M_{nc} = CI \frac{d}{dt} \left(\frac{\partial^2 w}{\partial x^2} \right) \quad (3)$$

where E is the Young's modulus, I is the Bending moment of inertia and C is the damping factor of the beam.

Considering the flapwise deformation [32], the potential energy (or strain energy) U of an inextensible beam may be obtained by using the conservative moment in (2) [31], [33]

$$U = \frac{1}{2} \int_0^L EI \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx. \quad (4)$$

Using the non-conservative moment (3), the non-conservative force F_{nc} can be written as [31]

$$F_{nc} = \int_0^L \frac{\partial^2}{\partial x^2} M_{nc} dx = \int_0^L CI \frac{\partial^2}{\partial x^2} \left[\frac{d}{dt} \left(\frac{\partial^2 w}{\partial x^2} \right) \right] dx. \quad (5)$$

By using Hamilton's principle [29], the governing equation of motion can be derived along with (1), (4) and (5) as

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + CI \frac{\partial^5 w}{\partial t \partial x^4} = 0. \quad (6)$$

The boundary conditions correspond to

$$w|_{x=0} = 0; \quad \frac{\partial w}{\partial x} \Big|_{x=0} = 0; \quad \frac{\partial^2 w}{\partial x^2} \Big|_{x=L} = 0; \quad \frac{\partial^3 w}{\partial x^3} \Big|_{x=L} = 0. \quad (7)$$

This means that at the fixed end both the deflection and the slope are zero. At the free end of the cantilever beam, there are no bending moments or shear forces.

B. Cantilever beam with piezoelectric actuator

It is common to assume that the piezoelectric actuator's layer is much thinner than the cantilever beam. The effect of the bonding material and the layer are negligible. In addition, the twisting (shear) effect is not considered [34].

The voltage applied to the actuator introduces a strain in the actuator. Since there is a distance between the actuator and the neutral surface, the strain introduces counteracting control moment on the mechanical structure. The control moment can be described as [34], [35]

$$M_a = r_a d_{31} E_p U_a(x, t) \quad (8)$$

where r_a is the effective moment arm (distance between the surface of the beam and the mid-plane of the piezoelectric actuator), d_{31} is the transverse piezoelectric coefficient [36], E_p is the Young's modulus of the piezoelectric patch and $U_a(x, t)$ is the distributed voltage on the patch.

The distributed voltage on the piezoelectric actuator can be given as

$$U_a(x, t) = U_a(t) [H(x - x_a) - H(x - x_b)] \quad (9)$$

where $U_a(t)$ is the voltage imposed on the actuator, x_a, x_b represent the two ends of the piezoelectric actuator as shown in Fig. 1. $H(\cdot)$ is the Heaviside step function which is defined as

$$H(x - x_i) = \begin{cases} 0 & x < x_i \\ 1 & x \geq x_i. \end{cases} \quad (10)$$

The piezoelectric force can be written as

$$F_a = \int_0^L \frac{\partial^2}{\partial x^2} M_a dx. \quad (11)$$

Then, the governing equation (6) can be derived as

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + CI \frac{\partial^5 w}{\partial t \partial x^4} + \frac{\partial^2 M_a}{\partial x^2} = 0. \quad (12)$$

The derived PDE beam model (12) describes the dynamics of a piezoelectric cantilever beam considering Kelvin-Voigt damping and piezoelectric actuator. The model can better describe the characteristics of a real beam compared to the beam model in [8] due to the effect of the damping in practice. Compared to the proportional (Rayleigh) damping considered in [9], the Kelvin-Voigt damping included in (12) has its advantage in representing the behaviour of stress and strain state [37].

C. Deflection calculation from strain

From the stress-strain relationship in the Kelvin-Voigt model, the strain can be written as

$$\varepsilon(t) = \frac{\sigma}{E} (1 - e^{-\lambda t}) \quad (13)$$

where σ is the bending stress, $\varepsilon(t)$ is the strain and $\lambda = E/C$.

When we calculate the strain $\varepsilon(t)$ from the known bending stress σ , the strain can be approximately given as

$$\varepsilon(t) = \frac{\sigma}{E}. \quad (14)$$

The theoretical force F_t for a cantilever beam can be calculated as [38]

$$F_t = \frac{3EIw}{L^3}. \quad (15)$$

Then the bending stress can be given by

$$\sigma = \frac{Mh}{2I} = \frac{F_t L_1 h}{2I} = \frac{3EwL_1 h}{2L^3} \quad (16)$$

where L_1 is the distance from the strain gauge to the theoretical force location.

Consider the stress-strain relationship from (14), we have the deflection-strain relationship as

$$w = \frac{2L^3 \varepsilon}{3L_1 h}. \quad (17)$$

Note that the deflection-strain relationship will only be used in experiments to transform the strain to displacement and will not be included in the model. This is because the displacement and velocity are more straightforward for analysis in practice compared to the strain.

Remark 1. Compared to the classical formulation of Euler-Bernoulli beam, herein we include the Kelvin-Voigt damping and the piezoelectric force in this study to derive a more precise model. Besides, the deflection-strain relationship is derived. The derived model (12) together with the deflection-strain relationship (17) make it possible to represent the practical beam behavior bonding with the piezoelectric actuator and the strain gauge. Compared to the beam model introduced in [8], the proposed model (12) included the damping term, which can better describe the dynamics of a real beam. The included Kelvin-Voigt damping in (12) has its advantage in representing the behaviour of stress and strain state [37] in contrast to the Rayleigh damping in [9].

Remark 2. To estimate the unknown parameters of the PDE model (12), an intuitive way is to use the finite dimensional approximation approaches (e.g. finite difference method) to approximate the PDE and then apply the parameter estimation methods. However, these PDE-based methods are time-consuming and have high computational costs. To make it more feasible for practical implementation, in the following, we will transform the PDE model to an ODE model which is more suitable for online estimation. Hence, the following developments are essentially different to the existing results [16], [21].

D. Model transformation with the Galerkin's method

The Galerkin's method is adopted to convert the PDE to an ODE. The Galerkin method is valid for such a transformation [39]. The displacement of the beam can be written as a summation of different oscillation modes as

$$w(x, t) = \sum_{i=1}^n W_i(x) \eta_i(t) \quad (18)$$

where $W_i(x)$ are the mode shape functions and $\eta_i(t)$ are the corresponding modal coefficients.

Based on the separation of variables approach, the mode shape functions are defined as [30]

$$W_i(x) = \frac{\cos \beta_i x - \cosh \beta_i x}{(\cos \beta_i L + \cosh \beta_i L) (\sin \beta_i x - \sinh \beta_i x)} - \frac{\sin \beta_i L + \sinh \beta_i L}{\sin \beta_i L + \sinh \beta_i L} \quad (19)$$

where the values of β_i satisfy the frequency equation

$$\cos \beta L \cosh \beta L + 1 = 0. \quad (20)$$

By using Galerkin's method (see Appendix), we derive the motion equation of the beam (12) as a matrix-vector form

$$\mathbf{M}\ddot{\boldsymbol{\eta}} + \mathbf{C}\dot{\boldsymbol{\eta}} + \mathbf{K}\boldsymbol{\eta} = \mathbf{z}u \quad (21)$$

where

$$\mathbf{M} = \begin{bmatrix} m_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & m_{nn} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & c_{nn} \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} k_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & k_{nn} \end{bmatrix}, \mathbf{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}.$$

Based on the definition of natural frequency and damping ratio [30], equation (21) together with (18) can be further written as the following state-space model

$$\begin{cases} \dot{X} = AX + Bu \\ y = CX \end{cases} \quad (22)$$

where the system state vector is chosen as $X = [\eta_1, \eta_2, \dots, \eta_{n-1}, \eta_n, \dot{\eta}_1, \dot{\eta}_2, \dots, \dot{\eta}_{n-1}, \dot{\eta}_n]^T$ and

$$A = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{I}_{n \times n} \\ -\boldsymbol{\Omega}^2 & -2\boldsymbol{\zeta}\boldsymbol{\Omega} \end{bmatrix}, B = \begin{bmatrix} \mathbf{0}_{n \times 1} \\ \mathbf{B}_w \end{bmatrix},$$

$$C = \begin{bmatrix} \mathbf{C}_w & \mathbf{0}_{1 \times n} \end{bmatrix}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \omega_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \omega_n \end{bmatrix}, \boldsymbol{\zeta} = \begin{bmatrix} \zeta_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \zeta_n \end{bmatrix},$$

$$\mathbf{B}_w = \begin{bmatrix} \frac{z_1}{m_{11}} & \cdots & \frac{z_n}{m_{nn}} \end{bmatrix},$$

$$\mathbf{C}_w = \begin{bmatrix} W_1 & \cdots & W_n \end{bmatrix}$$

where ω_i and ζ_i are the natural frequency and damping ratio of i_{th} ($i = 1, 2, \dots, n$) oscillation mode.

In the system model (22), the unknown parameters ω_i and ζ_i are essential for modeling and control synthesis. Hence, the aim of this study is to introduce a constructive estimation scheme to derive the values of ω_i and ζ_i by using the input and output measurement.

Remark 3. System (22) describes the piezoelectric cantilever beam behavior with multiple oscillation modes. Theoretically, the system model will be more accurate with more oscillation modes. However, this will lead to a high order system model, which is more complicated to implement in practice. As a result, a trade-off should be made between the accuracy and the practicability. In this paper, we take two major oscillation modes into consideration to demonstrate the effectiveness of the proposed adaptive parameter estimation method.

III. ADAPTIVE PARAMETER ESTIMATION

This section will introduce a real-time parameter estimation scheme for the above ODE based model (22) with two main vibration modes. After parameterizing the system model, an adaptive law with a time-varying gain will be proposed. Without loss of generality, we assume the input u and output y are bounded. For the unbounded cases, the normalization operation [40] can be applied to fulfill this condition.

A. Model Parameterization

Considering the first and second oscillation modes, the model (22) can be rewritten as

$$\begin{cases} \dot{X} = \bar{A}X + \bar{B}u \\ y = \bar{C}X \end{cases} \quad (23)$$

where the system states are chosen as $\bar{X} = [\eta_1, \eta_2, \dot{\eta}_1, \dot{\eta}_2]^T$ and

$$\bar{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \bar{k}_1 & 0 & \bar{c}_1 & 0 \\ 0 & \bar{k}_2 & 0 & \bar{c}_2 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ 0 \\ \bar{b}_1 \\ \bar{b}_2 \end{bmatrix}, \quad (24)$$

$$\bar{C} = \begin{bmatrix} -2 & 2 & 0 & 0 \end{bmatrix}$$

with $\bar{k}_1 = -\omega_1^2$, $\bar{k}_2 = -\omega_2^2$, $\bar{c}_1 = -2\zeta_1\omega_1$, $\bar{c}_2 = -2\zeta_2\omega_2$, $\bar{b}_1 = z_1/m_{11}$ and $\bar{b}_2 = z_2/m_{22}$.

As shown in the above model (23), the parameters $\bar{k}_1, \bar{k}_2, \bar{c}_1, \bar{c}_2, \bar{b}_1, \bar{b}_2$ involving ω_i and ζ_i are essential for modeling the beam system, which will be estimated in this study.

However, one difficulty for using model (23) is that the states \bar{X} cannot be directly measured. In practice, only the input voltage $u = U_a(t)$ and the output displacement $y = 2L^3\varepsilon/(3L_1h)$ can be measured. Hence, an alternative model that is described by the input and output is desirable for parameter estimation. To address this issue, we can obtain the transfer function of model (23) as

$$G(s) = \bar{C}(sI - \bar{A})^{-1}\bar{B} = \frac{b_2s^2 + b_1s + b_0}{s^4 + a_3s^3 + a_2s^2 + a_1s + a_0} \quad (25)$$

where

$$\begin{cases} b_2 = 2\bar{b}_2 - 2\bar{b}_1 \\ b_1 = 2\bar{b}_1\bar{c}_2 - 2\bar{b}_2\bar{c}_1 \\ b_0 = 2\bar{b}_1\bar{k}_2 - 2\bar{b}_2\bar{k}_1 \\ a_3 = -\bar{c}_1 - \bar{c}_2 \\ a_2 = -\bar{k}_1 - \bar{k}_2 + \bar{c}_1\bar{c}_2 \\ a_1 = \bar{c}_1\bar{k}_2 + \bar{k}_1\bar{c}_2 \\ a_0 = \bar{k}_1\bar{k}_2 \end{cases} \quad (26)$$

are unknown parameters including ω_i and ζ_i as shown in (24).

To accomplish the parameter estimation in the time-domain, the model (25) can be further expressed as an ODE as

$$y^{(4)} + a_3y^{(3)} + a_2y^{(2)} + a_1y^{(1)} + a_0y = b_2u^{(2)} + b_1u^{(1)} + b_0u \quad (27)$$

To avoid using derivatives of output y as [27], i.e. $y^{(4)}, y^{(3)}, y^{(2)}, y^{(1)}$, we filter each side of (27) with a fourth-order filter $1/\Lambda(s)$, where $\Lambda(s) = (s + \lambda)^4 = s^4 + k_3s^3 + k_2s^2 + k_1s + k_0$, and then have

$$\begin{aligned} \frac{s^4}{\Lambda(s)}y + a_3\frac{s^3}{\Lambda(s)}y + a_2\frac{s^2}{\Lambda(s)}y + a_1\frac{s}{\Lambda(s)}y + a_0\frac{1}{\Lambda(s)}y \\ = b_2\frac{s^2}{\Lambda(s)}u + b_1\frac{s}{\Lambda(s)}u + b_0\frac{1}{\Lambda(s)}u. \end{aligned} \quad (28)$$

Then, equation (28) can be rewritten as a parameterized form

$$\begin{aligned} y = b_2\frac{s^2}{\Lambda(s)}u + b_1\frac{s}{\Lambda(s)}u + b_0\frac{1}{\Lambda(s)}u + (k_3 - a_3)\frac{s^3}{\Lambda(s)}y \\ + (k_2 - a_2)\frac{s^2}{\Lambda(s)}y + (k_1 - a_1)\frac{s}{\Lambda(s)}y + (k_0 - a_0)\frac{1}{\Lambda(s)}y \\ = \Theta^T \Psi \end{aligned} \quad (29)$$

where

$$\begin{aligned} \Theta &= [b_2, b_1, b_0, k_3 - a_3, k_2 - a_2, k_1 - a_1, k_0 - a_0]^T, \\ \Psi &= \left[\frac{s^2}{\Lambda(s)}u, \frac{s}{\Lambda(s)}u, \frac{1}{\Lambda(s)}u, \frac{s^3}{\Lambda(s)}y, \frac{s^2}{\Lambda(s)}y, \frac{s}{\Lambda(s)}y, \frac{1}{\Lambda(s)}y \right]^T. \end{aligned}$$

The values of frequencies ω_1, ω_2 and damping ratios ζ_1, ζ_2 to be online estimated are determined by $\bar{k}_1, \bar{k}_2, \bar{c}_1, \bar{c}_2$, which can be obtained by using (24) and (25). One can obtain the eigenvalues of \bar{A} in (24) as

$$\begin{cases} p_1 = \frac{\bar{c}_2}{2} + \frac{\sqrt{\bar{c}_2^2 + 4\bar{k}_2}}{2} \\ p_2 = \frac{\bar{c}_2}{2} - \frac{\sqrt{\bar{c}_2^2 + 4\bar{k}_2}}{2} \\ p_3 = \frac{\bar{c}_1}{2} + \frac{\sqrt{\bar{c}_1^2 + 4\bar{k}_1}}{2} \\ p_4 = \frac{\bar{c}_1}{2} - \frac{\sqrt{\bar{c}_1^2 + 4\bar{k}_1}}{2} \end{cases} \quad (30)$$

From (25), one can obtain the eigenvalues p_1, p_2, p_3, p_4 by computing the roots of the characteristic polynomial $s^4 + a_3s^3 + a_2s^2 + a_1s + a_0$. Substituting p_1, p_2, p_3, p_4 into (30), we can solve (30) and obtain $\bar{k}_1, \bar{k}_2, \bar{c}_1, \bar{c}_2$. Then the frequencies ω_1, ω_2 and damping ratios ζ_1, ζ_2 can be computed.

For the other two parameters \bar{b}_1 and \bar{b}_2 in (24), one can obtain the values with the help of (26). We can rewrite the first three equations of (26) as

$$\begin{bmatrix} -2 & 2 \\ 2\bar{c}_2 & -2\bar{c}_1 \\ 2\bar{k}_2 & -2\bar{k}_1 \end{bmatrix} \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \\ b_0 \end{bmatrix}. \quad (31)$$

Since (31) is over-constrained, we can obtain the approximate solution of (31) by introducing Moore–Penrose inverse. Then \bar{b}_1 and \bar{b}_2 can be obtained as

$$\begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2\bar{c}_2 & -2\bar{c}_1 \\ 2\bar{k}_2 & -2\bar{k}_1 \end{bmatrix}^+ \begin{bmatrix} b_2 \\ b_1 \\ b_0 \end{bmatrix} \quad (32)$$

where $[\cdot]^+$ is the Moore–Penrose inverse. Another way to compute \bar{b}_1 and \bar{b}_2 is to use only two equations of (31). The computational burden will be less and the differences will be only in transient.

Clearly, the system parameters $\bar{k}_1, \bar{k}_2, \bar{c}_1, \bar{c}_2, \bar{b}_1, \bar{b}_2$ in (24) can be obtained as long as the parameters $a_i, i = 0, 1, 2, 3$ and $b_j, j = 0, 1, 2$ in (26) are estimated online. Hence, the problem now is to online estimate the unknown model parameters Θ based on the measured input u and output y .

To ensure the convergence of parameter estimation, the following definition of regressor should be used:

Definition 1. [22] A vector Ψ is persistently excited if there exist $T_1 > 0, \varsigma > 0$, such that $\int_t^{t+T_1} \Psi(r)\Psi^T(r)dr \geq \varsigma I, \forall t \geq 0$ is true.

The above condition implies that sufficiently rich information should be involved in the regressor, which has been recognized as a mandatory condition for parameter estimation [22]. This condition can usually be fulfilled in the beam system. However, the verification of this condition is not a trivial task. This will be studied in the following subsection.

B. Adaptive Parameter Estimation

Although the classical gradient algorithm has been applied for system (29), the tuning of learning gain that affects the performance is generally difficult, since it is also sensitive to measurement noise. In this subsection, we introduce an alternative adaptive parameter estimation algorithm to estimate the unknown parameter Θ of parameterized model (29), and allow verifying the required excitation condition.

To eliminate the influence of noise, we first define the auxiliary regressor matrix P and vector Q as

$$\begin{cases} \dot{P} = -lP + \Psi\Psi^T, P(0) = 0 \\ \dot{Q} = -lQ + \Psi y, Q(0) = 0 \end{cases} \quad (33)$$

where $l > 0$ is a forgetting factor set as a small constant.

One can obtain the solution of (33) as

$$\begin{cases} P = \int_0^t e^{-l(t-r)}\Psi(r)\Psi^T(r)dr \\ Q = \int_0^t e^{-l(t-r)}\Psi(r)y(r)dr \end{cases} \quad (34)$$

which are bounded for any bounded Ψ and y .

To obtain the estimation error $\tilde{\Theta} = \Theta - \hat{\Theta}$ to design an adaptive law, we define another auxiliary vector W as

$$W = P\hat{\Theta} - Q \quad (35)$$

where $\hat{\Theta}$ is the estimated parameter.

Combining (34) and (35), the vector W can be further reformulated as

$$W = -P\tilde{\Theta}. \quad (36)$$

Hence, the parameter estimation error $\tilde{\Theta}$ can be explicitly calculated online based on the measured input u and output y , which can be used to design the adaptive laws [26] to obtain better performance than the classical gradient algorithm. Moreover, the purpose of introducing the above filtered matrix P is to provide a feasible approach to verify the required excitation condition, which can be summarized as the following lemma:

Lemma 1. *If the regressor Ψ defined in (29) is persistently excited, the matrix P is positively definite (i.e., $\lambda_{\min}(P(t)) > \sigma > 0$ for a positive constant σ), and vice versa.*

We refer to [27] for a similar proof of the above lemma. The value of this lemma lies in that one can calculate the minimum eigenvalue of matrix P to evaluate if the required excitation condition of regressor Ψ is fulfilled.

Since W is a function of parameter estimation error $\tilde{\Theta}$, one can use it to design the adaptive laws as shown in [27], where the adopted constant learning gain should be carefully set by the designers. However, from (33) and (36), we can see that the regressor Ψ is involved in the vector W , which may influence the convergence response of the estimated parameters [41]. Hence, we will introduce a new adaptive law with a time-varying gain to handle effects of regressor so as to simplify the tuning of learning gains. Taking the effect of the regressor Ψ into consideration, we define a time-varying gain H as

$$\dot{H} = \gamma H - H\Psi\Psi^T H \quad (37)$$

where $\gamma > 0$ is a design constant and $H(0) = H_0 > 0$ is the initial condition.

Using the matrix identity

$$\frac{d}{dt} H H^{-1} = \dot{H} H^{-1} + H \frac{d}{dt} H^{-1} = 0 \quad (38)$$

we have its solution

$$H(t) = \left[e^{-\gamma t} H_0^{-1} + \int_0^t e^{-\gamma(t-r)} \Psi(r) \Psi^T(r) dr \right]^{-1}. \quad (39)$$

Now, the boundedness of matrix H is shown as [41]:

Lemma 2. *If the regressor Ψ is persistently excited, the matrix H^{-1} defined in (39) is bounded by*

$$\beta I \leq H^{-1}(t) \leq \alpha I \quad (40)$$

where $\beta = e^{-\gamma T_1} \varsigma$ and $\alpha = \lambda_{\min}(H_0^{-1}) + \varpi^2$, $\varpi \geq \|\Psi\|$.

The proof of the above claims can be inspired by [41], which is not presented here due to the page limit.

As shown in (34) and (39), the matrix H converges to the inverse of auxiliary matrix P . Hence, it can be used to

compensate the effect of regressor in the variable W . We can use W and H together to design the following adaptive parameter updating law

$$\dot{\hat{\Theta}} = -\Gamma H W \quad (41)$$

where $\Gamma > 0$ is a positive constant.

The main results of this paper can be given as the following Theorem:

Theorem 1. *Consider system (27) with the parameter updating law (41), if the regressor Ψ is persistently excited, the estimation error $\tilde{\Theta}$ exponentially converges to zero.*

Proof. We choose the Lyapunov candidate function as

$$V = \frac{1}{2\Gamma} \tilde{\Theta}^T H^{-1} \tilde{\Theta}. \quad (42)$$

One can obtain the time derivative of V along with (41) as

$$\begin{aligned} \dot{V} &= \frac{1}{2\Gamma} \left(\dot{\tilde{\Theta}}^T H^{-1} \tilde{\Theta} + \tilde{\Theta}^T \dot{H}^{-1} \tilde{\Theta} + \tilde{\Theta}^T H^{-1} \dot{\tilde{\Theta}} \right) \\ &= \frac{1}{\Gamma} \tilde{\Theta}^T H^{-1} \dot{\tilde{\Theta}} + \frac{1}{2\Gamma} \tilde{\Theta}^T \dot{H}^{-1} \tilde{\Theta} \\ &= -\tilde{\Theta}^T P \tilde{\Theta} + \frac{1}{2\Gamma} \tilde{\Theta}^T \dot{H}^{-1} \tilde{\Theta}. \end{aligned} \quad (43)$$

From (39), we have

$$\dot{H}^{-1} = -\gamma H^{-1} + \Psi\Psi^T. \quad (44)$$

Then, equation (43) can be further written as

$$\begin{aligned} \dot{V} &= -\tilde{\Theta}^T P \tilde{\Theta} + \frac{1}{2\Gamma} \tilde{\Theta}^T (-\gamma H^{-1} + \Psi\Psi^T) \tilde{\Theta} \\ &\leq -\frac{\sigma}{\alpha} \tilde{\Theta}^T H^{-1} \tilde{\Theta} - \frac{\gamma}{2\Gamma} \tilde{\Theta}^T H^{-1} \tilde{\Theta} + \frac{\varpi^2}{2\Gamma\beta} \tilde{\Theta}^T H^{-1} \tilde{\Theta} \\ &\leq -\left(\frac{2\Gamma\sigma}{\alpha} + \gamma - \frac{\varpi^2}{\beta} \right) \frac{1}{2\Gamma} \tilde{\Theta}^T H^{-1} \tilde{\Theta} \\ &\leq -\mu V \end{aligned} \quad (45)$$

where $\mu = 2\Gamma\sigma/\alpha + \gamma - \varpi^2/\beta$ is a positive constant for large gains Γ, γ . Based on the Lyapunov's Theorem, the estimation error $\tilde{\Theta}$ exponentially converges to zero. \square

Remark 4. *One can find that the adaptive law (41) is constructed based on the derived variable W , which is driven by the parameter estimation error $\tilde{\Theta}$ as shown in (36). Therefore, the convergence of this algorithm can be rigorously proved. Clearly, the proposed adaptive estimation method avoids using any predictor/observer, which is different from the gradient algorithms and least-squares methods driven by the observer error (as shown in [24]). Besides, a time-varying gain H is adopted to compensate the effect of regressor Ψ in the adaptive law to enhance the convergence over that given in [27]. In this line, as shown in the proof of Theorem 1, the learning gain Γ can be trivially selected compared with the gradient counterparts. Moreover, online validation of the persistent excitation can be achieved by testing the positive definiteness of P according to Lemma 1.*

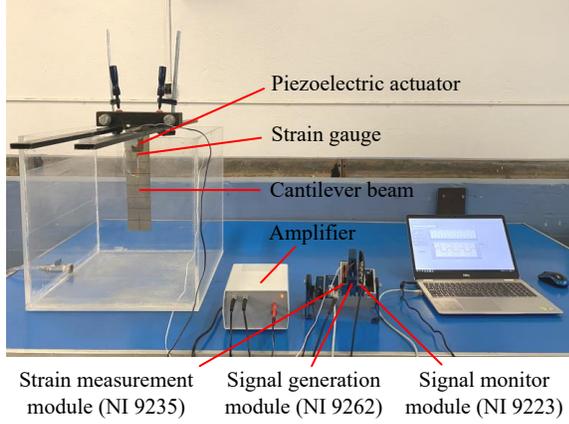


Fig. 2: Configuration of the experimental setup.

IV. SIMULATIONS AND EXPERIMENTS

To illustrate the effectiveness and practicality of the proposed estimation algorithm, simulations and experiments are conducted based on a steel cantilever beam as shown in Fig. 2. In the simulation, the nominal model values were computed from the empirical material data of the steel beam and piezoelectric actuator. The geometric and material properties of the steel beam and the piezoelectric actuator [36] are described in Table I and Table II, respectively.

TABLE I: Material data of the steel beam

Variables	Values
Density of the beam (ρ)	7723.13 kg/m ³
Cross sectional area (A)	0.00008 m ²
Young's modulus of elasticity (E)	1.93 · 10 ¹¹ N/m ²
Bending moment of inertia (I)	6.67 · 10 ⁻¹² m ⁴
Damping factor (C)	9.8 · 10 ⁷ N/(m/s)
Length of the beam (L)	0.36 m

TABLE II: Material data of the piezoelectric ceramics

Variables	Values
Young's modulus of elasticity (E_p)	7.06 · 10 ¹⁰ N/m ²
Piezoelectric coefficient (d_{31})	-1.8 · 10 ⁻¹⁰ C/N
Effective moment arm (r_a)	0.001 m

A. Simulations

In the simulations, the beam system model (27) can be written as

$$y^{(4)} + 65.42y^{(3)} + 6.18 \cdot 10^4 y^{(2)} + 1.95 \cdot 10^5 y^{(1)} + 9.23 \cdot 10^7 y = 13.02u^{(2)} - 1.18 \cdot 10^3 u^{(1)} - 1.11 \cdot 10^6 u \quad (46)$$

where $a_3 = 65.42$, $a_2 = 6.18 \cdot 10^4$, $a_1 = 1.95 \cdot 10^5$, $a_0 = 9.23 \cdot 10^7$, $b_2 = 13.02$, $b_1 = -1.18 \cdot 10^3$, $b_0 = -1.11 \cdot 10^6$ are the model parameters to be estimated. These nominal values are computed from the material properties given in Table I. The input signal is a square wave with amplitude 0.1 and frequency 7 Hz to ensure the required excitation condition given in Lemma 1. The polynomial used to derive the parameterized form is

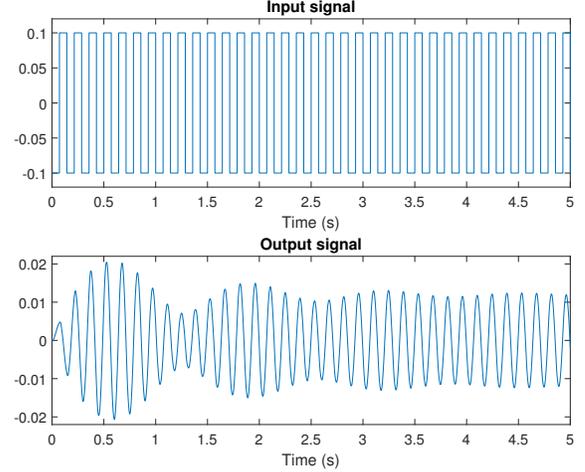


Fig. 3: Input and output signals (simulation results)

chosen as $\Lambda(s) = (s+100)^4$. The other parameters of the proposed adaptive law are $\Gamma = 80 \cdot \text{diag}(1, 1, 1, 1, 1, 1)$, $H(0) = 1 \cdot 10^{22} \cdot \text{diag}(1, 1, 10, 10, 10, 10, 10)$ and $l = 0.01$, $\gamma = 0.1$.

For comparison, the gradient-based method and least-squares method [24] are used to estimate the unknown system parameters under the same operation condition. The gradient-based adaptive law is designed as [24]

$$\dot{\hat{\Theta}} = \Gamma_1 e \Psi, \quad e = y - \hat{\Theta}^T \Psi \quad (47)$$

and the least-squares algorithm as [24]

$$\dot{\hat{\Theta}} = \Gamma_2 e \Psi, \quad e = y - \hat{\Theta}^T \Psi, \quad \dot{\Gamma}_2 = -\Gamma_2 \Psi \Psi^T \Gamma_2. \quad (48)$$

The learning gain of the gradient-based method in (47) is $\Gamma_1 = 1 \cdot \text{diag}(2 \cdot 10^{11}, 2 \cdot 10^{15}, 6 \cdot 10^{19}, 3 \cdot 10^{12}, 2 \cdot 10^{17}, 7 \cdot 10^{18}, 4 \cdot 10^{23})$. The initial value of the covariance matrix Z in the least-squares method (48) is $\Gamma_2(0) = 1 \cdot 10^{23} \cdot \text{diag}(1, 1, 1, 10, 10, 10, 10)$.

Clearly, the tuning of these gains for the gradient-based method and least-squares method should consider the effect of regressor Ψ (e.g. the amplitude of each component), and thus it is a time-consuming phase, requiring preliminary information of the plant. In contrast, for the proposed estimation algorithm, since the effect of regressor Ψ on the convergence is handled by introducing the time-varying gain H , such that the tuning of the learning constant Γ is more straightforward, i.e. all components in the gain Γ can be set as a constant.

Simulation results are depicted in Fig. 3-Fig. 5. Fig. 3 shows the profile of the input and output signal of the system (46). Fig. 4 provides the estimation of the parameters a_i, b_j . We can see that the proposed adaptive estimation method has a smoother and faster response compared with the gradient method and least-squares method. Hence, the proposed adaptive method can accurately estimate the unknown model parameters. This is because the adaptive law (41) is designed by using the extracted parameter estimation error instead of the predictor/observer error in the conventional methods. The gradient method achieves larger overshoots and slower convergence performance. Moreover, the least-squares method suffers from a windup problem in the gain Γ_2 given in

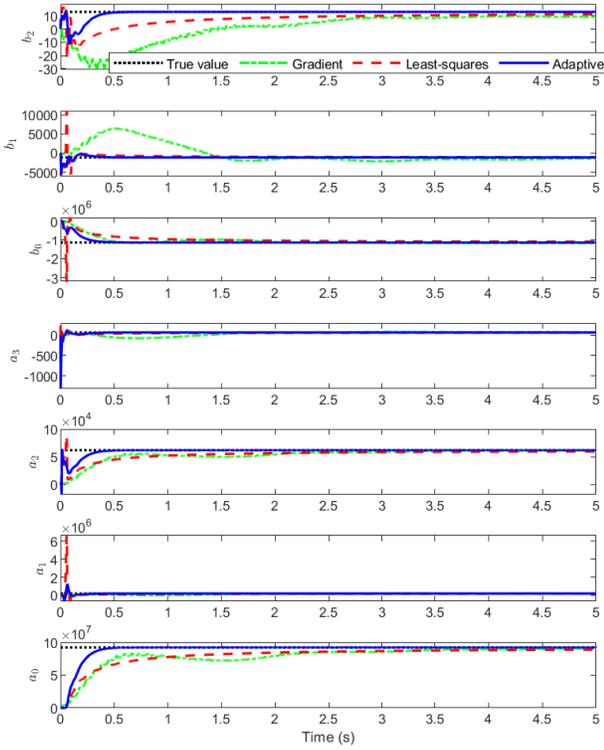


Fig. 4: Comparative convergence performance of gradient method, least-squares method and the proposed adaptive method (simulation results)

(48). To implement the parameter estimation algorithms in real-time, the algorithms are discretized using a 4th order Runge-Kutta method. To estimate the required computation burden, the different algorithms have been implemented using MATLAB/Simulink and executed in a computer with Intel(R) Core(TM) i7-8700K CPU @ 3.70 GHz processor and 16.0 GB of memory running a 64-bit windows OS. The computational time of each time step with the proposed adaptive method is 0.039ms while the gradient method is 0.025ms and Least-squares method is 0.032ms. Although the computational cost of the proposed method is slightly larger than the gradient method and Least-squares method, the computational time of each time step is still much less than the sampling time (0.1ms), which indicates the feasibility for real-time estimation of the proposed adaptive method. To illustrate the accuracy of the proposed approach, we further reconstruct and compare the outputs of the model using the estimated parameters with these three different methods. The system output and model output with the estimated parameters are shown in Fig. 5. We can see that the output profile can be accurately reconstructed by using the proposed adaptive method while the gradient method and least-squares method can not achieve such a good performance.

According to (23) and (25), we can obtain the first mode frequency ω_1 , the second mode frequency ω_2 , the first mode damping ratio ζ_1 and the second mode damping ratio ζ_2 .

TABLE III: Frequency and damping ratio

	$\omega_1(\text{rad/s})$	$\omega_2(\text{rad/s})$	ζ_1	ζ_2
True value	39.1533	245.3711	0.0207	0.13
Gradient	38.8304	245.0479	0.0216	0.1374
least-squares	39.1339	231.6241	0.02	0.1205
Adaptive	39.1682	245.3184	0.0207	0.13

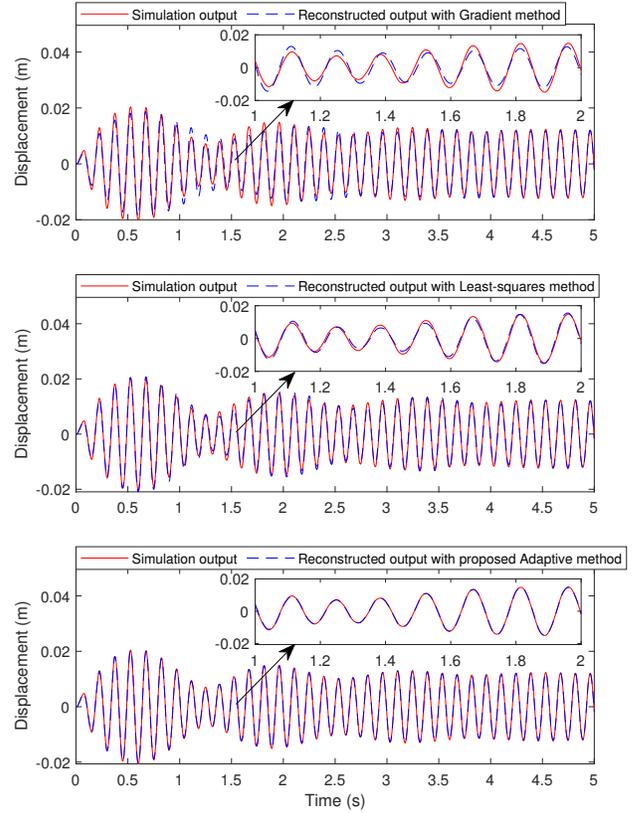


Fig. 5: Reconstructed model output with gradient method, least-squares method and the proposed adaptive method (simulation results)

Comparative values are shown in Table III. It is shown that the proposed adaptive method has better estimation results than the other two methods.

B. Practical Experiments

In the experiments, we use a piezoelectric actuator (type number: P-876.A12) from PI Ceramic GmbH to excite the beam. The voltage applied on the actuator is an amplified signal (ranges: $\pm 100\text{V}$) from the voltage amplifier and the original voltage signal (ranges: $\pm 10\text{V}$) is generated from NI 9262 module which is a simultaneously updating analog output module from National Instruments. The NI 9223 module is used to monitor the amplified signal. To configure the sensor subsystem, we use a strain gauge (type number: WFLA-6-11-3LDBTB) from Tokyo Measuring Instruments Laboratory to measure the strain of the beam. The NI 9235 quarter-bridge strain measurement module is used to collect the signal of the strain gauge. A bridge offset nulling calibration is introduced

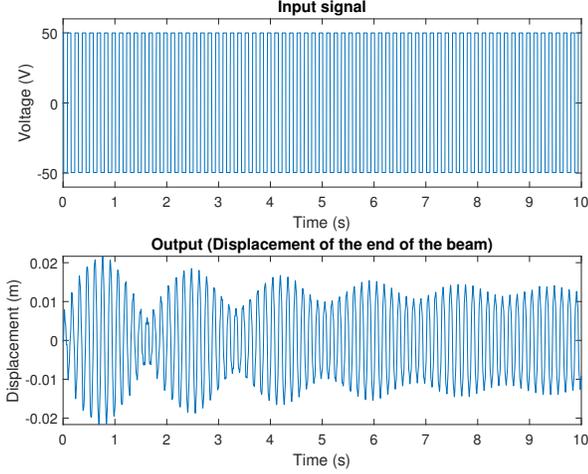


Fig. 6: Input and output signals (experiment results)

to eliminate the offset of the strain gauge. The programs are run on a PC with Intel(R) Core(TM) i7-8700K CPU @ 3.70 GHz, 16.0 GB memory, 64-bit OS. Please refer to Fig.2 for the detailed configuration of the experimental setup.

Following the analysis given in the simulations, the parameters of the adaptive law (41) used in the experiments are $\Gamma = 10 \cdot \text{diag}(1, 1, 1, 1, 1, 1)$, $H(0) = 1 \cdot \text{diag}(1 \cdot 10^{23}, 1 \cdot 10^{15}, 1 \cdot 10^{24}, 1 \cdot 10^{21}, 1 \cdot 10^{22}, 1 \cdot 10^{24}, 1 \cdot 10^{25})$, $l = 0.01$, $\gamma = 0.1$. The learning gain of the gradient-based method in (47) is $\Gamma_1 = 1 \cdot \text{diag}(1 \cdot 10^{11}, 1 \cdot 10^{10}, 1 \cdot 10^{19}, 1 \cdot 10^{16}, 1 \cdot 10^{17}, 1 \cdot 10^{19}, 1 \cdot 10^{21})$. The initial value of the covariance matrix Γ_2 in the least-squares method (48) is $\Gamma_2(0) = 1 \cdot \text{diag}(1 \cdot 10^{10}, 1 \cdot 10^{16}, 1 \cdot 10^{17}, 1 \cdot 10^{21}, 1 \cdot 10^{24}, 1 \cdot 10^{23}, 1 \cdot 10^{27})$. Clearly, since a time-varying gain H is used to handle the effect of regressor, the tuning of learning gain for the proposed algorithm is easier than the gradient algorithm.

Experimental results are given in Fig.6-Fig.8. Fig.6 describes the profile of the input and output signal collected from the test bench. The input signal is a square wave with amplitude 50 V and frequency 7 Hz. To alleviate the effect of the high-frequency electrical noises, a low-pass filter is applied before feeding into the estimation algorithm. The parameter estimation results of gradient method, least-squares method and the proposed adaptive method are shown in Fig.7. We can see that the proposed adaptive method (41) has an overall better performance compared to the gradient-based method (47) and the least-squares method (48). In particular, smaller oscillations of the parameters can be achieved with the proposed adaptive method. However, the gradient algorithm has a slow convergence rate and more oscillations or even steady-state errors in the estimated parameters. To illustrate the accuracy of the proposed approach, we further reconstruct and compare the output of the model using the estimated parameters with these three different methods. The measured output and model output with the estimated parameters are shown in Fig. 8, from which we can see that the proposed method can accurately estimate the unknown model parameters and thus approximate the output dynamics of the real beam system. In contrast, the gradient method and the least-

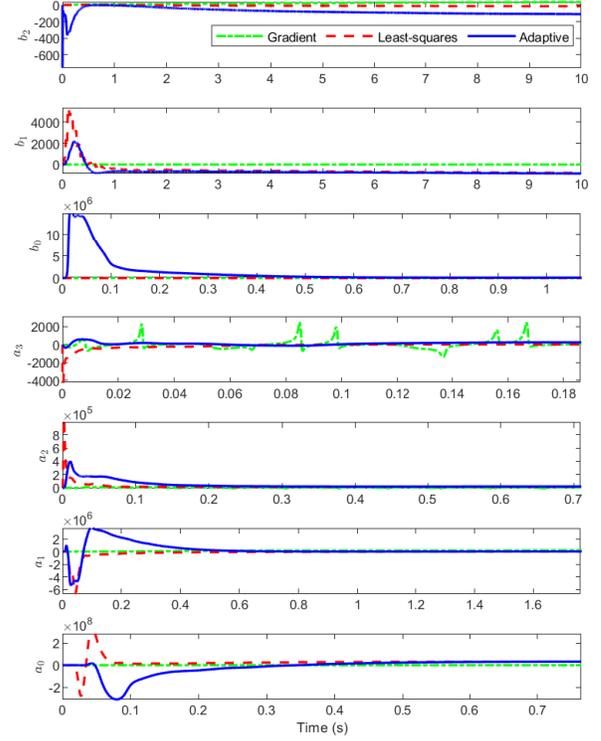


Fig. 7: Comparative convergence performance of gradient method, least-squares method and the proposed adaptive method (experiment results)

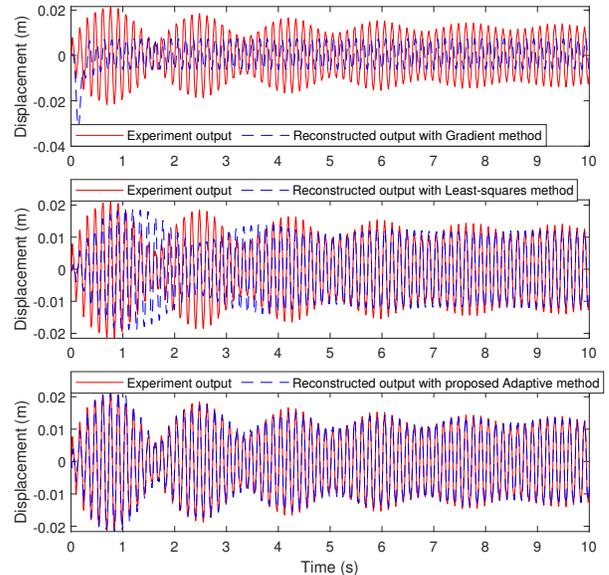


Fig. 8: Reconstructed model output with gradient method, least-squares method and the proposed adaptive method (experiment results)

squares method can not achieve such a good performance. The gradient algorithm leads to steady-state errors in the model output, implying unprecise parameter estimation, while the least-squares method has a transient error during the first 3 sec, indicating a sluggish convergence for parameter estimation. These experimental results showcase the advantages of the proposed learning algorithm with the introduced time-varying gains.

V. CONCLUSION

This paper developed a constructive model for a cantilever beam established by using the Euler-Bernoulli theory and then presented an alternative adaptive parameter estimation scheme to determine the model parameters. A PDE based model is first provided including the beam motion equation and the piezoelectric actuator dynamics. The Kelvin-Voigt damping is also considered in the modelling procedure. By using the Galerkin's method, the system model is reconstructed to a new form described by an ODE. Taking into consideration of the first mode and second mode dynamics, the beam model is finally reduced to a fourth-order ODE based model, which is suitable for online parameter estimation. Then, an adaptive parameter estimation method is proposed to estimate the unknown model parameters, where the online estimation can be achieved using the input and output only, and the system states are not required. Specifically, a time-varying gain is suggested in the learning law to eliminate the influence of regressor and retain the convergence of estimation error. Simulation and experiment results validate the effectiveness and improved performance of the proposed method.

APPENDIX

In order to obtain the matrix-vector form of the beam equation, we use Galerkin's method to rewrite the PDE model (12).

By substituting (18) into (12), the residual can be obtained as

$$R_u = \rho A \sum_{i=1}^n W_i(x) \frac{d^2 \eta_i(t)}{dt^2} + EI \sum_{i=1}^n \frac{d^4 W_i(x)}{dx^4} \eta_i(t) + CI \sum_{i=1}^n \frac{d^4 W_i(x)}{dx^4} \frac{d\eta_i(t)}{dt} + \frac{d^2 M_a}{dx^2}. \quad (49)$$

The Galerkin's method gives

$$\int_0^L R_u W_j(x) dx = 0. \quad (50)$$

Then, we have

$$\begin{aligned} & \int_0^L \rho A \sum_{i=1}^n W_i(x) \frac{d^2 \eta_i(t)}{dt^2} W_j(x) dx \\ & + \int_0^L EI \sum_{i=1}^n \frac{d^4 W_i(x)}{dx^4} \eta_i(t) W_j(x) dx \\ & + \int_0^L CI \sum_{i=1}^n \frac{d^4 W_i(x)}{dx^4} \frac{d\eta_i(t)}{dt} W_j(x) dx \\ & + \int_0^L \frac{d^2 M_a}{dx^2} W_j(x) dx = 0. \end{aligned} \quad (51)$$

Using the orthogonality principle of the modes shapes [30]

$$\int_0^L W_i(x) W_j(x) dx = 0, \quad i \neq j, \quad (52)$$

we can rewrite equation (51) as

$$\sum_{i=1}^n \left\{ m_{ij} \frac{d^2 \eta_i(t)}{dt^2} + c_{ij} \frac{d\eta_i(t)}{dt} + k_{ij} \eta_i(t) + z_i u(t) \right\} = 0, \quad j = 1, 2, \dots, n \quad (53)$$

where

$$m_{ij} = \int_0^L \rho A W_i(x) W_j(x) dx, \quad (54)$$

$$c_{ij} = \int_0^L CI \frac{d^4 W_i(x)}{dx^4} W_j(x) dx, \quad (55)$$

$$k_{ij} = \int_0^L EI \frac{d^4 W_i(x)}{dx^4} W_j(x) dx, \quad (56)$$

$$\begin{aligned} z_i &= r_a d_{31} E_p \left[\frac{d}{dx} W_i(x_1) - \frac{d}{dx} W_i(x_2) \right] \\ &= r_a d_{31} E_p \int_0^L W_j(x) \frac{d^2}{dx^2} [H(x - x_1) - H(x - x_2)] dx, \\ &u(t) = U_a(t). \end{aligned} \quad (57)$$

Then, we can obtain the matrix-vector form of the beam as given in (21).

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