Fault Prognosis Approach using Data-driven Structurally Generated Residuals

Xin Fang[1], Joaquim Blesa[1,2] and Vicenç Puig[1]

Abstract— **This paper presents a fault prognosis approach using data-driven structurally generated residuals. It assumes that a set of residuals generated using structural analysis (SA) and identified using data-driven approach are available. Residuals are used for fault detection purposes activating fault signals when residual values reach anomalous values. In addition, it is possible to predict future faults by means of the detection of anomalous residual deviations. Once an anomalous change in the residual trend has been detected, it is proceed to estimate when this residual deviation will result in a fault detection and therefore which will be the Remaining Useful Life (RUL) time of the system. For this purpose, the future residual evolution is estimated by means of a regressor function. Nominal and interval parameters of regressor function are estimated with available residual data providing nominal and interval values of the RUL of the system. A brushless direct current (BLDC) motor is used as the application case study to illustrate the performance of proposed approach.**

I. INTRODUCTION

Fault diagnosis (FD) is of vital importance in nearly all different industrial fields already known. It consists in detecting faults by means of analysing current available information provided by the available sensors. In last years, FD has been the subject of investigation among diverse communities, particularly those in the fields of automatic control and artificial intelligence.[1]. Both communities have devised their distinct diagnosis approaches: FDI (Fault Detection and Isolation) methods rooted in engineering disciplines like control theory and statistical decision-making, and DX approaches drawing from fields such as logic, combinatorial optimization, search algorithms, and complexity analysis. In recent years, there has been a trend towards the development of fault diagnosis methodologies that leverage aspects of both FDI and DX approaches [2]. Both quantitative and qualitative models of the system are required for both approaches to conduct fault diagnosis effectively. The fundamental model-based fault diagnosis approach involves comparing the observed behavior of the system with its expected behavior predicted by the model. This comparison typically involves computing a residual, which represents the difference between a measured process variable and its estimated value provided by the model. Ideally, these residuals should be zero to ensure the system functions correctly. However, factors such as external disturbances and modeling inaccuracies can cause residuals to deviate from zero even in the absence of faults. Therefore, model uncertainty must be considered during the fault detection stage. This can be achieved, for instance, by computing a threshold that accounts for the maximum possible value of model uncertainties.

While obtaining an accurate mathematical model to describe the system under monitoring is crucial, it can be particularly challenging in complex industrial systems. Addressing this challenge, some innovative methods have been explored. Instead of relying on exact physical equations of the system, these methods focus on utilizing structural Analytical Redundancy Relations (ARRs) obtained through Structural Analysis (SA) and data, as discussed in [3]. SA plays a pivotal role in assessing the detectability and isolability of the system. It transforms the mathematical model of the system into a structural model, facilitating the diagnostic process by analyzing the system analytical redundancy. Moreover, in [4], a hybrid method has been proposed. This method employs model decomposition techniques to analyze the internal relationships between system variables, leading to the design of reduced grey-box models using data-driven techniques. Various tools such as Possible Conflicts (PCs) [5] and state space Neural Networks (ssNN) [6] are utilized in this approach. Recurrent Neural Networks (RNNs) are particularly effective in simulating the performance of complex dynamic systems.

Furthermore, the maintenance is also an essential phase in all complex industrial systems. The evolution of maintenance has been developed from post-failure repair (corrective maintenance) to preventive maintenance to Condition-Based Maintenance (CBM). In recent years, the preventive maintenance and CBM have been increased their influence among many industrial companies. The preventive maintenance is an expensive and time-consuming process that consists in performing maintenance process regularly regardless of its current health condition of system. In large-scale factories, with its high reliability requirements, the preventive maintenance will be an extremely high-cost process. Therefore, it leads to the incorporation of CBM because of its cost-effective strategy of maintenance. As the name implies, maintenance is applied when it is needed, and it is closely related to the concept so-called Prognostics and Health Management (PHM). The basic theory of PHM is to establish a real-time assessment of health condition of monitored system, as well as the prediction of future state with its up-to-date information. While PHM has been originated from aerospace industry, now it is applied in many fields

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including manufacturing, automotive, railway, energy and heavy industry [7].

Prognosis is the key technology to accomplish CBM in industrial manufacturing factories, in which the timely forecasting of potential faults leads to considerable reduction of economical losses. Additionally, the prediction of future behaviours of system allow the supervision system to make important decisions in advance. Beyond the FD mentioned before, system prognosis tends to predict the future evolution of behaviours of system. Unlike the FD that has been well-studied by numerous investigators, it has already established its own matured benchmark of investigation, the field of prognosis is remaining many undiscovered ways of study since it is still a relatively new approach.

This paper introduces a method for prognosis using residuals, merging SA and data-driven algorithms. It determines ARRs from system structural information given by graphic (or textual) system description. Thereby, instead of finding ARRs from physical mathematical model, combining SA and data-driven modelling approach, a set of data-driven ARRs can be obtained and used to implement the prognosis of system. The prognostic approach relies on detecting anomaly or unusual tendency of deviation that reflexes on the evolution of residuals by using some sequential analysis techniques for change detection, in particular the Cumulative Sum (CUSUM) approach [8]. Once the tendency is detected, the future evolution of tendency of residuals deviation will be modeled using least-squares (LS) regression by assuming some degradation function, such as e.g. a polynomial function. This function is able to forecast the fault evolution, as well as the Remaining Useful Life (RUL) of system. According to [9], parametric and additive unknown but bounded errors are considered in the forecast function providing an interval in the residual prediction, so that instead of finding a single RUL, an interval of RUL will be estimated.

The paper has the following structure: Section II presents the generation of data-driven residuals based on ARR obtained using SA, and it is followed by the Section III where change detection process is developed. In Section IV, an explanation of RUL estimation is presented. In Section V, a case study of a Brushless Direct Current (BLDC) motor is presented to show the practical application of the proposed method. Finally, Section VI draws the conclusions of the present paper and present future research paths.

II. DATA-DRIVEN RESIDUALS

Given a system graph and a set of sensors, using SA analysis a set of n_r ARRs relating a subset of measured variables can be derived. For each ARR, one of the variables can be explained from the remaining variables in the ARR as follows

$$\begin{aligned}\hat{y}_i(k) =& f_i(y_i(k-1),...,y_i(k-n_a),\boldsymbol{y}_{\neg i}(k-1),\\ &...,\boldsymbol{y}_{\neg i}(k-n_a),\boldsymbol{u}_i(k-1),...,\boldsymbol{u}_i(k-n_a))\\ &i=1,...,n_r\end{aligned} \quad (1)$$

where $\hat{y}_i(k) \in \Re$ represents the estimation of the i-th component of $\boldsymbol{y}$ at instant k, $f_i()$ is an unknown complex function of order n_a with

$$\boldsymbol{y}_{\neg i}(k-j) = \boldsymbol{y}(k-j) \backslash y_i(k-j) \quad j=1,...,n_a \quad (2)$$

$$\boldsymbol{u}_i(k-j) = \boldsymbol{u}(k-j) \quad j=1,...,n_a \quad (3)$$

and $f_i()$ is a function that considers the n_a previous values of the measured variables to provide an estimation of the i-th component of $\boldsymbol{y}$ at instant k denoted by $\hat{y}_i(k) \in \Re$.

Defining

$$\boldsymbol{v}_i(k-j) = (\boldsymbol{y}_{\neg i}(k-j) \quad \boldsymbol{u}_i(k-j)) \quad j=1,...,n_a$$

equation (1) can be rewritten as

$$\begin{aligned}\hat{y}_i(k) =& f_i(y_i(k-1),...,y_i(k-n_a),\boldsymbol{v}_i(k-1),...,\boldsymbol{v}_i(k-n_a))\\ &i=1,...,n_y\end{aligned} \quad (4)$$

Model (4) can be obtained by means of the the physical knowledge of the system or using system identification tools as the ones proposed in [10] considering linearity or some kind of non-linearity in function $f_i()$ or using other data-driven estimation techniques as proposed in [11] where an adaptive network-based fuzzy inference system (ANFIS) and data is proposed to obtain model (4).

The consistency of model (4) and the actual behaviour of the system can be assessed by evaluatng the difference (residual) of the actual output $y_i(k)$ and its estimation

$$r_i(k) = y_i(k) - \hat{y}_i(k) \quad i=1,...,n_r \quad (5)$$

Model (4) can be fit considering non-faulty historical data $r_i(j)\ j=1,...,N_{nf}$ and assuming linearity or some kind of non-linearity in function $f_i()$ using computational tools [10] or using other parameter estimation techniques.

This non-faulty historical data can also be used to compute thresholds $\bar{\sigma}_i$ and $\underline{\sigma}_i$ as the maximum and minimum observed errors as

$$\bar{\sigma}_i = \max_{j=1,...,N_{nf}} \beta r_i(j) \quad (6)$$

with security factor $\beta \geq 1$. The computation of $\underline{\sigma}_i$ will be as in (6) but substituting 'max' by 'min'. Then, a fault detection test considering these thresholds could be defined as

$$\begin{cases} r_i(k) \in [\underline{\sigma}_i, \bar{\sigma}_i] \Rightarrow \text{ No Fault} \\ \text{Otherwise} \Rightarrow \text{ Fault}\end{cases} \quad (7)$$

III. CHANGE DETECTION

In order to anticipate the appearance of potential faults before fault detection test (7) is activated, the trend of residuals $r_i(k) \quad i=1,...,n_r$ should be analyzed.

In particular, the Cumulative Sum (CUSUM) approach [8] is a statistical method that tracks cumulative deviations from a reference value. In a first step, CUSUM algorithm can be used to detect maximum cumulative residual deviations in available fault free data as

$$
\begin{array}{l}
s_i^+(j) = r_i(j) \\
s_i^-(j) = -r_i(j) \\
g_i^+(j) = max(g_i^+(j-1) + s_i^+(j) - v, 0) \\
g_i^-(j) = max(g_i^-(j-1) + s_i^-(j) - v, 0) \\
j = 1, ..., N_{nf}
\end{array} \tag{8}
$$

where v is the drift parameter and $g_i^+(0)$ and $g_i^-(0)$ are initialized to zero.

Once, $g_i^+(j)$ and $g_i^-(j)$ are computed for $j = 1, ..., N_{nf}$, CUSUM fault free thresholds $\bar{\delta}_i$ and $\underline{\delta}_i$ are computed with maximum cumulative residual deviations as follows

$$
\begin{aligned}
\bar{\delta}_i &= \max_{j=1,\ldots,N_{nf}} \alpha g_i^+(j) \\
\underline{\delta}_i &= \max_{j=1,\ldots,N_{nf}} \alpha g_i^-(j)
\end{aligned} \tag{9}
$$

with security factor $\alpha \geq 1$. Then, on-line CUSUM algorithm is applied to the residuals $r_i(k)$ to compute $g_i^+(k)$ and $g_i^-(k)$ values. If $g_i^+(k)$ or $g_i^-(k)$ violate fault free thresholds $\bar{\delta}_i$ or $\underline{\delta}_i$ defined in (9) an anomalous change in the residual trend is detected.

IV. Remaining useful life (RUL)

Once an anomalous change in the residual trend has been detected, we can proceed to estimate when this residual deviation will result in a fault detection. To do this, it will be necessary to estimate the evolution of the residual in the future considering the observed deviation. In this section, for the sake of notation simplicity, it will be considered a generic residual $r(k)$ with the residual bounds $\underline{\sigma}$ and $\bar{\sigma}$. In addition, it will be considered that the results could be applied to the all particular residuals $r_i(k)$ $i = 1, ..., n_r$.

It will be considered that the residual deviation can be expressed in the following regressor form

$$
r(k) = \varphi(k)\theta^t \tag{10}
$$

where $\varphi(k) \in \Re^{n_\theta}$ is the regressor vector and $\theta \in \Re^{n_\theta}$ is the parameter vector. Residual model (10) covers a long range of possible drifts modelled using an n order polynomial function

$$
r(k) = a_0 + a_1 k + a_2 k^2 + ... + a_n k^n \tag{11}
$$

that can be expressed as in (10) defining

$$
\begin{array}{l}
\varphi(k) = \begin{pmatrix} 1 & k & k^2 & \cdots & k^n \end{pmatrix} \\
\theta = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_n \end{pmatrix}
\end{array} \tag{12}
$$

or considering a exponential function

$$
r(k) = a_0 + a_1 e^k \tag{13}
$$

that can be expressed as in (10) with

$$
\begin{array}{l}
\varphi(k) = \begin{pmatrix} 1 & e^k \end{pmatrix} \\
\theta = \begin{pmatrix} a_0 & a_1 \end{pmatrix}
\end{array} \tag{14}
$$

Then, parameter vector θ can be estimated from residual and regressor data as

$$
\hat{\theta}(k) = (\Phi^t(k)\Phi(k))^{-1}\Phi^t(k)R(k) \tag{15}
$$

with

$$
R(k) = \begin{pmatrix} r(0) \\ \vdots \\ r(k) \end{pmatrix} \Phi(k) = \begin{pmatrix} \varphi(0) \\ \vdots \\ \varphi(k) \end{pmatrix} \tag{16}
$$

Remark 1: It is supposed that time instant k is big enough to have enough data to obtain an accurate identification.

In addition, at the same time instant k residual evolution can be predicted for future time steps. In particular, for a horizon prediction H this prediction will be computed as

$$
\hat{r}(k+H|k) = \varphi(k+H)\hat{\theta}^t(k) \tag{17}
$$

Finally, we can predict the time instant H_f when residual $r(k)$ will lead to a fault scenario as

$$
\begin{array}{l}
\min_{H_f} \quad H_f \\
\text{subject to} \\
\qquad \hat{r}(k+H_f|k) \notin [\underline{\sigma}, \bar{\sigma}]
\end{array} \tag{18}
$$

Remark 2: H_f can be seen as the Remaining Useful Life (RUL) time. i.e. the time duration that the system is expected to remain operational and functional before it reaches a fault scenario.

In case of a linear drift residual deviation $r(k) = a_0 + a_1 k$ leads to the regressor model (10)

$$
\varphi(k) = (1 \quad k) \quad \text{and} \quad \theta = (a_0 \quad a_1) \tag{19}
$$

and the forecast of the residual evolution is given by

$$
\hat{r}(k+H|k) = \varphi(k+H)\hat{\theta}^t(k) = a_0 + (k+H)a_1 \tag{20}
$$

Hence, considering (18) predicted fault time instant H_f should fulfill

$$
\begin{cases}
a_0 + (k+H_f)a_1 > \bar{\sigma}, & \text{if } a_1 > 0 \\
a_0 + (k+H_f)a_1 < \underline{\sigma}, & \text{if } a_1 < 0
\end{cases} \tag{21}
$$

For example, considering $a_1 > 0$

$$
H_f > \frac{\bar{\sigma} - a_0}{a_1} - k \tag{22}
$$

A. Parameter uncertainty

Considering unknown but bounded parametric and additive uncertainties in model (10)

$$
r(k) = \varphi(k)\hat{\theta}^t(k) + e(k) \tag{23}
$$

where

$$
\begin{array}{l}
|e(k)| \leq \epsilon \\
\hat{\theta}(k) \in \mathbf{\Theta}
\end{array} \tag{24}
$$

with ϵ a suitable constant that considers the additive noise in measurements and $\mathbf{\Theta}$ is the set that bounds parameter values.

We will assume that the parameter set is a zonotope

$$\mathbf{\Theta} = \theta^0 \oplus \mathbf{P}\mathbb{B}^n = \left\{\theta^0 + \mathbf{P}\mathbf{z} : \mathbf{z} \in \mathbb{B}^n\right\} \tag{25}$$

where θ^0 is the nominal parameter vector, $\mathbf{P} \in \Re^{n_\theta \times n}$ is a suitable matrix that takes into account the dependence between parameters if any, $\mathbb{B}^n$ is a unitary box composed by n unitary $\mathbb{B}^n \in [-1, 1]$ interval vectors and $\oplus$ denotes the Minkowski sum.

Residual prediction model (23) considering additive and parametric uncertainties (24)(25) leads to an interval

$$\hat{r}(k) \in [\underline{\hat{r}}(k), \bar{\hat{r}}(k)] \tag{26}$$

that according to [9]

$$\begin{aligned} \bar{\hat{r}}(k) &= \hat{r}^0 + \|\varphi(k)\mathbf{P}\|_1 + \epsilon \\ \underline{\hat{r}}(k) &= \hat{r}^0 - \|\varphi(k)\mathbf{P}\|_1 - \epsilon \end{aligned} \tag{27}$$

where $\hat{r}^0(k) = \varphi(k)\theta^0(k)$

On the other hand, the computation of parameter set $\mathbf{\Theta}$ consistent with available data can be formulated as the following optimization problem

$$\begin{aligned} &\min_{\mathbf{P}} \mathcal{J}(\mathbf{\Theta}) \\ &\text{subject to} \\ &\qquad r(j) \le \hat{r}^0 + \|\varphi(j)\mathbf{P}\|_1 + \epsilon \\ &\qquad r(j) \ge \hat{r}^0 - \|\varphi(j)\mathbf{P}\|_1 - \epsilon \\ &\qquad j = 1, ..., N \end{aligned} \tag{28}$$

where $\mathcal{J}(\mathbf{\Theta})$ is the volume of $\mathbf{\Theta}$ and N is the data horizon considered. The resolution of optimization problem (28) is characterized by significant difficulty in general [12]. However if a predefined shape of matrix $\mathbf{P}$ is considered as proposed in [9]

$$\mathbf{P} = \lambda \mathbf{P}_0 \tag{29}$$

Optimization problem (28) leads to the optimal solution (29) with λ computed as

$$\lambda = \max_{k \in \{1,...,N\}} \left(\frac{|r(k) - \hat{r}(k)| - \epsilon}{\|\varphi(k)\mathbf{P}_0\|_1}, 0 \right) \tag{30}$$

Once parameter set has been computed, predicted residual bounds can be obtained:

$$\begin{aligned} \bar{\hat{r}}(k + H|k) &= \hat{r}^0(k + H) + \|\varphi(k)\mathbf{P}\|_1 + \epsilon \\ \underline{\hat{r}}(k + H|k) &= \hat{r}^0(k + H) - \|\varphi(k)\mathbf{P}\|_1 - \epsilon \end{aligned} \tag{31}$$

Therefore, an interval for H_f can be found as follows

$$\begin{aligned} &H_f \in [\underline{H}_f, \bar{H}_f] \\ &\text{such that} \\ &\hat{r}^0(k + H_f) \in [\sigma - \|\varphi(k)\mathbf{P}\| - \epsilon, \sigma + \|\varphi(k)\mathbf{P}\| + \epsilon] \end{aligned} \tag{32}$$

where σ is $\bar{\sigma}$ if the residual trend is increasing or $\underline{\sigma}$ if the residual trend is decreasing.

Considering a linear drift residual deviation as in (20), the intervals of H_f when an increasing drift in residual trend is detected will be given by

$$\begin{aligned} \bar{H}_f &= \frac{\sigma + \epsilon - a_0 + \Delta a_0}{a_1 - \Delta a_1} - k \\ \underline{H}_f &= \frac{\sigma - \epsilon - a_0 - \Delta a_0}{a_1 + \Delta a_1} - k \end{aligned} \tag{33}$$

In the case of having different residuals r_i $i = 1, ..., n_r$, nominal and interval RULs could be computed for every residual and the minimum one will determine the RUL of the system.

V. Application example

The Brushless Direct Current (BLDC) motor presented in [13] is used as the application example to illustrate the performance of proposed method. The operation condition of motor is on 2-phase conduction mode, the dynamic model of system consists in two parts: electrical equation and mechanical equation. The model equations can be rewritten in state-space form as follows

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} i \\ \omega_r \end{bmatrix} &= \begin{bmatrix} -\frac{R_{eq}}{L_{eq}} & -\frac{k_e}{L_{eq}} \\ \frac{k_T}{J} & -\frac{B_r}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega_r \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{L_{eq}} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} V_{dc} \\ T_L \end{bmatrix} \end{aligned} \tag{34}$$

where R_{eq} is the equivalent resistance, V_{dc} is the DC voltage, L_{eq} is the equivalent inductance, T_L is the resisting (or load) torque, J is the moment of inertia of the rotational system (BLDC motor and load), B_r is the damping (or viscous friction) coefficient, k_T is the torque coefficient and k_e is the coefficient of the back electromotive force (emf), the two state variables are current i and rotor angular velocity ω_r .

As discussed in [14], considering the absence of physical analytical model for the BLDC motor, the following structural model of system can be obtained just considering the input and output variables of each subsystem:

$$\begin{aligned} &e_1 : i' = f_1(i, \omega_r, v_{dc}) \\ &e_2 : \omega_r' = f_2(i, \omega_r, T_L) \\ &e_3 : y_1 = i \\ &e_4 : y_2 = \omega_r \\ &e_5 : i' = \frac{di}{dt} \\ &e_6 : \omega_r' = \frac{d\omega_r}{dt} \end{aligned} \tag{35}$$

Considering the structural model of system obtained above, the following residuals [14] can be generated using Minimal Structurally Over-determined (MSO) set approach:

$$\begin{aligned} r_1 &= f_1(\omega_r, \omega_r', i, T_L) \\ r_2 &= f_2(i, i', \omega_r, V_{dc}) \\ r_3 &= f_3(\omega_r, \omega_r', T_L, V_{dc}) \\ r_4 &= f_4(i, i', T_L, V_{dc}) \end{aligned} \tag{36}$$

From this structural information of residuals and considering the faults proposed in [13]

- Sensor faults: position (f_θ) and current (f_i)

- Parametric faults: resistance (f_R), inductance (f_L), friction (f_B) and inertia (f_J)
- Systems faults: voltage supply (f_{Vdc}), load (f_{Load})

the following binary Fault Signature Matrix (FSM) is obtained

TABLE I
THEORETICAL FSM

	f_R	f_L	$f_{V_{dc}}$	f_B	f_J	f_θ	f_i	f_{Load}
r_1	0	0	0	1	1	1	1	1
r_2	1	1	1	0	0	1	1	0
r_3	1	1	1	1	1	1	0	1
r_4	1	1	1	1	1	0	1	1

The analytical expression of residuals (36) considering a linear behaviour, first order functions and calibrated by means of the System Identification Toolbox™ of MATLAB® [10] as follows

$$\begin{aligned} r_1(k) =& \omega_r(k) - 0.9952 \cdot \hat{\omega_r}(k-1) - 0.1107 \cdot i(k-1) \\ &- 0.09952 \cdot (\omega_r(k-1) - \hat{\omega_r}(k-1)) \\ r_2(k) =& i(k) - 0.5215 \cdot \hat{i}(k-1) - 0.01045 \cdot \omega_r(k-1) \\ &- 0.06813 \cdot v_{dc}(k-1) \\ r_3(k) =& \omega_r(k) - 0.9949 \cdot \hat{\omega_r}(k-1) - 0.03421 \cdot v_{dc}(k-1) \\ &- 0.09949 \cdot (\omega_r(k-1) - \hat{\omega_r}(k-1)) \\ r_4(k) =& i(k) - 0.7894 \cdot \hat{i}(k-1) - 0.06094 \cdot v_{dc}(k-1) \end{aligned} \tag{37}$$

where ω_r and $\hat{\omega_r}$ are measured and estimated value of rotor angular velocity. Similarly, i and $\hat{i}$ are measured and estimated value of electric current. Residuals r_1 and r_2 have been generated using an observer.

The evolution of residuals $r_i(k)$ $i = 1, ..., 4$ in a fault free scenario is presented in Figure 1. The thresholds are calculated using (6) considering security factor $\beta = 1.3$ and are also shown in Figure 1 in green color. In addition, maximum cumulative residual deviations are computing by means of the CUSUM approach (9) considering security factor $\alpha = 1$ and drift parameter v equal to the residual mean of the first time instants of the residual.

On the other hand, Figure 2 shows the evolution of the four residuals when a linear drift fault is introduced to the current sensor ($f_i = -2mA/s$) at instant $t = 10s$. As it can be seen in Figure 2, the sensor fault affects to residuals r_1,r_2 and r_4 that is consistent with the FSM of Table I.

The CUSUM approach (8) detects anomalous decreasing cumulative deviations in residuals r_2,r_4 and increasing deviation in residual r_1. In particular, this anomalous deviation is detected at $t = 14.2s$ in residual r_2 whose detailed evolution is depicted in Figure 3. Once the anomalous deviations are detected, nominal and interval residual predictions are computed as can be seen in Figure 3 for residual r_2. In case of residual r_2, nominal and interval predictions are computed at time $t = 19.2s$, that is 5 seconds later that anomalous deviation in this residual has been detected. Finally, nominal and interval RULs are calculated by means of (18) and (32), respectively. The minimum, nominal and maximum computed RULs are $\underline{H}_f = 0.9s$ $H_f = 33,2s$ and $\overline{H}_f = 65.3s$. Considering the time when the prediction is done ($t = 19.2s$), the fault prediction time is stated from $t = 21.1s$ to $t = 84,5s$ as can be seen in Figure 3. This prediction reasonably conforms to the actual future evolution of the residual: the residual violates threshold first at instant $23.7s$, then fault signal start oscillating and at the end the faulty signal remains completely activated at instant $75.9s$. In conclusion, the potential fault can be anticipated $3.6s$. This procedure is also carried out for residuals r_1 and r_4 obtaining similar results.

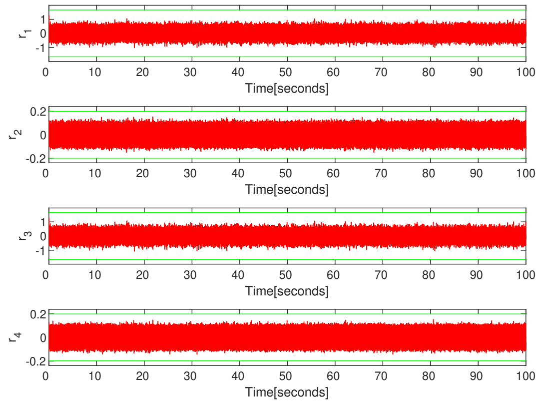

Fig. 1. Evolution of four residuals in fault free scenario

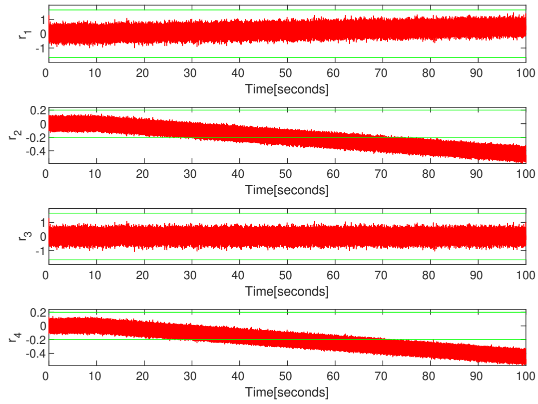

Fig. 2. Evolution of four residuals with linear fault

In addition, the proposed methodology is assessed in the case of parabolic drifts as the one depicted in Figure 4 with satisfactory results.

VI. CONCLUSIONS

This paper introduces a fault prognosis methodology employing data-driven structurally generated residuals. The proposed approach assumes the availability of a set of residuals derived through structural analysis (SA) and identified via a data-driven methodology. These residuals serve the purpose of fault detection, activating fault signals upon reaching

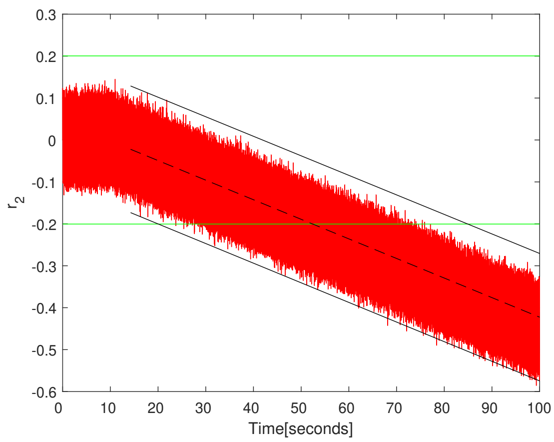

Fig. 3. Interval prediction of residual r_2 with linear fault

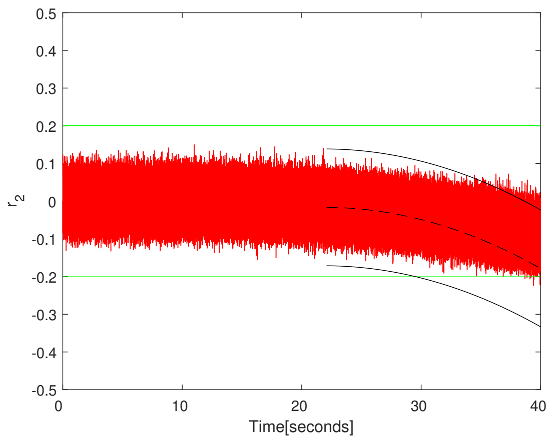

Fig. 4. Interval prediction of residual r_2 with parabolic fault

anomalous values. Furthermore, the prediction of future faults is facilitated through the identification of anomalous residual deviations detected by the CUSUM approach. Upon detection of an anomalous change in the residual trend, the estimation of when this deviation will culminate in fault detection is performed, thereby determining the Remaining Useful Life (RUL) time of the system. To achieve this, a regressor function is employed to estimate the future residual evolution. Nominal and interval parameters of the regressor function are deduced utilizing available residual data, providing nominal and interval values for the RUL of the system. The performance of the proposed method is exemplified through the utilization of a brushless direct current (BLDC) motor as a case study. As future research, the extension of the proposed method to deal with prognosing the fault isolation will be explored.

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