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Modelling Approaches for Predictive Control of Large-Scale Sewage Systems

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Abstract

In this report, model predictive control (MPC) of large-scale sewage systems is addressed considering different modelling approaches that include several inherent continuous/discrete phenomena (overflows in sewers and tanks) and elements (weirs) in the system that result in distinct behaviour depending on the dynamic state (flow/volume) of the network. These behaviours can not be neglected nor can be modelled by a pure linear representation. In order the MPC controller takes into account these phenomena and elements, a modelling approach based on piece-wise linear functions is proposed and compared against a hybrid modelling approach previously reported by the authors. Control performance results and associated computation times of the closed-loop scheme considering both modelling approaches are compared by using a real case study based on the Barcelona sewer network.

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1 Introduction

Sewer networks are considered as complex large-scale systems since they are geographically distributed and decentralized with a hierarchical structure. Each sub-system is in itself composed of a large number of elements with time-varying behavior, exhibiting numerous operating modes and subject to changes due to external conditions (weather) and operational constraints.

Most cities around the world have sewage systems that combine sanitary and storm water flows within the same network. This is why these networks are known as *Combined Sewage Systems* (CSS). During rain storms, wastewater flows can easily overload these CSS, thereby causing operators to dump the excess water into the nearest receiver environment (rivers, streams or sea). This discharge to the environment, known as *Combined Sewage Overflow* (CSO), contains biological and chemical contaminants creating a major environmental and public health hazard. Environmental protection agencies have started forcing municipalities to find solutions in order to avoid those CSO events. A possible solution to the CSO problem would be to enhance existing sewer infrastructure by increasing the capacity of the wastewater treatment plants (WWTP) and by building new underground retention tanks. But in order to take profit of these expensive infrastructures, it is also necessary a highly sophisticated real-time control (RTC) scheme which ensures that high performance can be achieved and maintained under adverse meteorological conditions [27]. The advantage of RTC applied to sewer networks has been demonstrated by an important number of researchers during the last decades, see [10, 23, 22, 14]. Comprehensive reviews that include a discussion of some existing implementations are given by [25] and cited references therein, while practical issues are discussed by [26], among others.

The RTC scheme in sewage systems might be *local* or *global*. When local control is applied, flow regulation devices use only measurements taken at their specific locations. While this control structure is applicable in many simple cases, in a big city, with a strongly interconnected sewer network and a complex infrastructure of sensors and actuators, it may not be the most efficient alternative. Conversely, a global control strategy, which computes control actions taking into account real-time measurements all through the network, is likely the best way to use the infrastructure capacity and all the available sensor information. Global RTC deals with the problem of generating control strategies for the control elements in a sewer network, ahead of time, based on a predictive dynamic model of the system, and readings of the telemetry system, in order to avoid street flooding, prevent CSO discharges to the environment, minimize the pollution, homogenise the utilization of sewage system storage capacity and, in most of cases, minimize the operating costs [27, 9, 32, 28]. The multivariable and large-scale nature of sewer networks have lead to the use of some variants of *Model Predictive Control* (MPC), as global control strategies [10, 23, 22, 14]. The MPC strategy, also referred as *Receding Horizon Control (RHC)* or *Moving Horizon Optimal Control (MHOC)*, is one of the few advanced methodologies which has significant impact on industrial control engineering. MPC is being applied in process industry because it can handle multivariable control problems in a natural form, it can take into account actuator limitations and allows constraints consideration.

In order to use MPC within a global RTC scheme of a sewage system, a model able to predict its future states over a prediction horizon taking into account a rain forecast is needed. Sewer networks are systems with complex dynamics since water flows through sewers in open channel. These flow dynamics are described by Saint-Vennant's partial differential equations that can be used to perform simulation studies but are highly complex to be solved in real time. When developing a control-oriented model, there is always a trade-off between model description accuracy and computational complexity. As a general rule, the model used for control purposes should be descriptive enough to capture the most significant dynamics of the system but simple enough to be scalable for large-scale networks such that real-time implementation is allowed.

Several control-oriented modelling techniques have been presented in the literature that deal with the global RTC of sewage systems, see [14, 8]. In [18, 6], it is used a conceptual linear model based on assuming that a set of sewers in a catchment can be considered as a virtual tank. The main reason to use a linear model is to preserve the convexity of the optimization problems related to the MPC strategy. A similar approach can be found in an early reference on MPC applied to sewage systems [10].

However, there exist several inherent phenomena (overflows in sewers and tanks) and elements (weirs) in the system that result in distinct behaviour depending on the state (flow/volume) of the network. These discontinuous behaviours can not be neglected nor can be modelled by a pure linear model. Additionally, the presence of intense precipitation causes that new flow paths appear. Thus, some flow paths are not always present in the sewer network and depend of its state and disturbances (rain). According to this observed behaviour, a control-oriented model methodology that allows to consider and incorporate overflows and other logical dynamics in most of the sewer network elements is needed. One of the main contributions of this paper is to describe and analyze these continuous/discrete dynamic behaviours in order to propose a control-oriented modelling approach focused on designing an MPC-based RTC scheme for large-scale sewer networks.

First of all, an hybrid modelling approach based on the *Mixed Logical Dynamical* (MLD) paradigm, introduced in [2] and already used to model hybrid elements in sewer networks will be briefly presented (see [16] for further details). However, from previous works in this line done by the authors (see [17]), the inclusion of those discontinuous behaviours in the MPC problem increases the computation time of the control law. So, some relaxation should be thought in the modelling approach such that it can be considered within the RTC of large-scale sewer networks. Therefore, another contribution of this paper consists in proposing an alternative modelling approach consisting on representing the sewage system by using piece-wise linear functions (in the sequel called *PWLF-based model* or simply *PWLF model*), following the ideas proposed by Schechter [24]. The aim of this modelling approach is to reduce the complexity of the MPC problem by avoiding the logical variables introduced by the MLD system representation. The idea behind the PWLF modelling approach consists in having a description of the network using functions that, despite their discontinuous nature, are considered as quasi-convex [4], and hence the optimization problems associated to the non-linear MPC strategy used for RTC of the sewage system. In this way, the resulting optimization problems does not include integer variables what allows saving computation time.

The remainder of the paper is organized as follows. In Section 2, control-oriented modelling of sewer networks is revised and the issue of discontinuous dynamics is presented. Two modelling approaches for large-scale sewer networks are explained and discussed. RTC scheme for sewage systems based on MPC strategy is addressed in Section 3 taking into account the modelling approaches presented in previous section. Section 4 presents a real case study based on the Barcelona sewer network. This case study is used to compare the closed-loop performance when implementing a predictive controller based on the modelling approaches presented in Section 2. Section 5 shows and discusses the comparisons of performance and computation times of the closed-loop system considering the mentioned control-oriented modelling approaches. Finally, main conclusions close the paper in Section 6.

2 RTC-Oriented Modelling of Large-Scale Sewer Networks

2.1 Principles of Mathematical Modelling of Sewage Systems

One of the most important stages on design of RTC schemes for sewer networks, in the case of using a model-based control technique as MPC, lies on the modelling task. This is because performance of model-based control techniques is very dependant of model quality. So, in order to design an MPC-based RTC scheme with an acceptable performance, a system model with accuracy enough should be used but keeping complexity manageable. This section is focused on the determination of a control-oriented sewer network model taking into account the trade-off between accuracy and complexity and keeping always in mind the RTC inherent restrictions [10].

Water flow in sewer pipes is open-channel, that corresponds to the flow of a certain fluid in a channel in which the fluid shares a free surface with an empty space above. The Saint-Venant equations, based on physical principles of mass conservation and energy, allow the accurate description of the open-channel flow in sewer pipes [15] and therefore also allow to have a detailed non-linear description of the system behaviour. These equations are expressed as:

$$\frac{\partial q_{x,t}}{\partial x} + \frac{\partial A_{x,t}}{\partial t} = 0, \tag{1}$$

$$\frac{\partial q_{x,t}}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_{x,t}^2}{A_{x,t}}\right) + gA_{x,t}\frac{\partial L_{x,t}}{\partial x} - gA_{x,t}\left(I_0 - I_f\right) = 0, \tag{2}$$

where $q_{x,t}$ is the flow (m^3/s), $A_{x,t}$ is the cross-sectional area of the pipe (m^2), t is the time variable (s), x is the spatial variable measured in the direction of the sewage flow (m), g is the gravity (m/s^2), I_0 is the sewer pipe slope (dimensionless), I_f is the friction slope (dimensionless) and $L_{x,t}$ is the water level inside the sewer pipe (m). This pair of partial-differential equations constitutes a non-linear hyperbolic system. For an arbitrary geometry of the sewer pipe, these equations lack of an analytical solution. Notice that these equations describe the system behaviour in high detail. However, such a level of detail is not useful for RTC implementation due to the complexity of obtaining the solution of (1)-(2) and the associated high computational cost.

Alternatively, several modelling techniques that deal with RTC of sewer networks have been presented in the literature, see [13], [9], [8], [14], among many others. The modelling approaches presented in this paper follow closely the mathematical modelling principles proposed in [10]. Here, sewage system is divided into catchments and the set of pipes storage capacity belonging to each partition is modelled as a *virtual tank*. At any given time, the stored volumes represent the amount of water stored inside the sewer pipes associated with. The virtual tank volume is calculated through the mass balance of the stored volume, the inflows and the outflow of the catchment measured using limnimeters and the input rain intensity measured using rain-gauges.

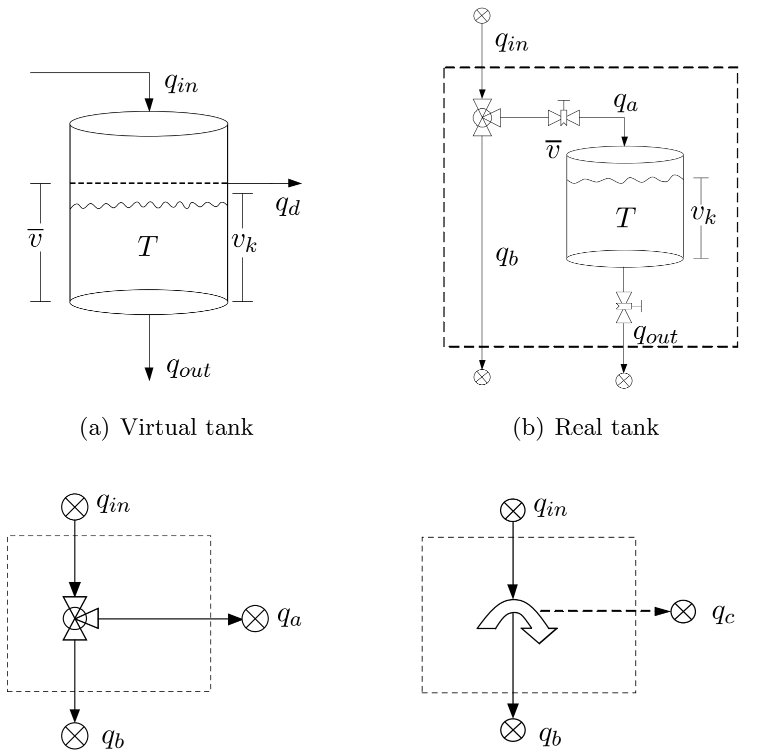
2.2 Sewer Network Constitutive Elements

Using the virtual tank modelling principle and the mass balance conservation law, a sewer network can be decomposed and described by using the elementary models explained below and shown in Figures 1 and 2, element by element and conforming a simple network, respectively. Other common sewage system elements such as pumping stations can be easily represented by using the mentioned modelling principles but will be omitted here as they are not taken into account in the case study presented in this paper. Every outlined element presented below includes a conceptual scheme which will be not only used for describing its operation but also for explaining the mathematical relations and equations derived when the modelling approaches are explained along the next sections.

2.2.1 Virtual and Real Tanks

These elements are used as storage devices. In the case of virtual tanks, the mass balance of the stored volume, the inflows and the outflow of the tank and the input rain intensity can be written as the difference equation

$$v_{ik+1} = v_{ik} + \Delta t \varphi_i S_i P_{ik} + \Delta t \left(q_{in}{}_k^i - q_{out}{}_k^i\right), \tag{3}$$



(a) Virtual tank (b) Real tank

(c) Redirection gate (d) Sewer pipe or weir with single inflow

Figure 1: Conceptual schemes for sewer networks constitutive elements.

where φ_i is the *ground absorption coefficient* of the *i-th* catchment, S is the surface area, P_k is the *rain intensity* at each sample with a sampling time Δt. Flows ${q_{in}}_k^i$ and ${q_{out}}_k^i$ are the sum of inflows and outflows, respectively. *Real retention tanks*, which correspond to the sewer network reservoirs, are modelled in the same way but without the precipitation term. Tanks are connected with flow paths or links, which represent the main sewage pipes between the tanks. Manipulated variables of the system, denoted as q_{u_i}, are related to the outflows from the control gates. Tank outflows are assumed to be proportional to the tank volume , that is,

$$ {q_{out}}_k^i = \beta_i v_{ik}, \tag{4} $$

where β_i (given in s^{-1}) is defined as the *volume/flow conversion* (VFC) coefficient as suggested in [29] by using the linear tank model approach. Notice that this relation can be made more accurate (but more complex) if (4) is considered to be non-linear (non-linear tank model approach). Limits on the volume range of real tanks are expressed as

$$ 0 \leq v_{ik} \leq \overline{v}_i, \tag{5} $$

where $\overline{v}_i$ denotes the maximum volume capacity given in cubic meters. As this constraint is physical, it is impossible to send more water to a real tank than it can store. Notice that real tanks without overflow capability have been considered. Virtual tanks do not have a physical upper limit on their capacity. When they rise above a preestablished volume, an overflow situation occurs. This fact represents the case when level in sewers has reached a limit so that an overflow situation can occur in the streets (flooding). Hence, when virtual tanks maximum volume $\overline{v}$ is reached, the excess volume above this maximum amount is redirected to another tank (catchment) within the network or to a receiver environment (pollution). This situation

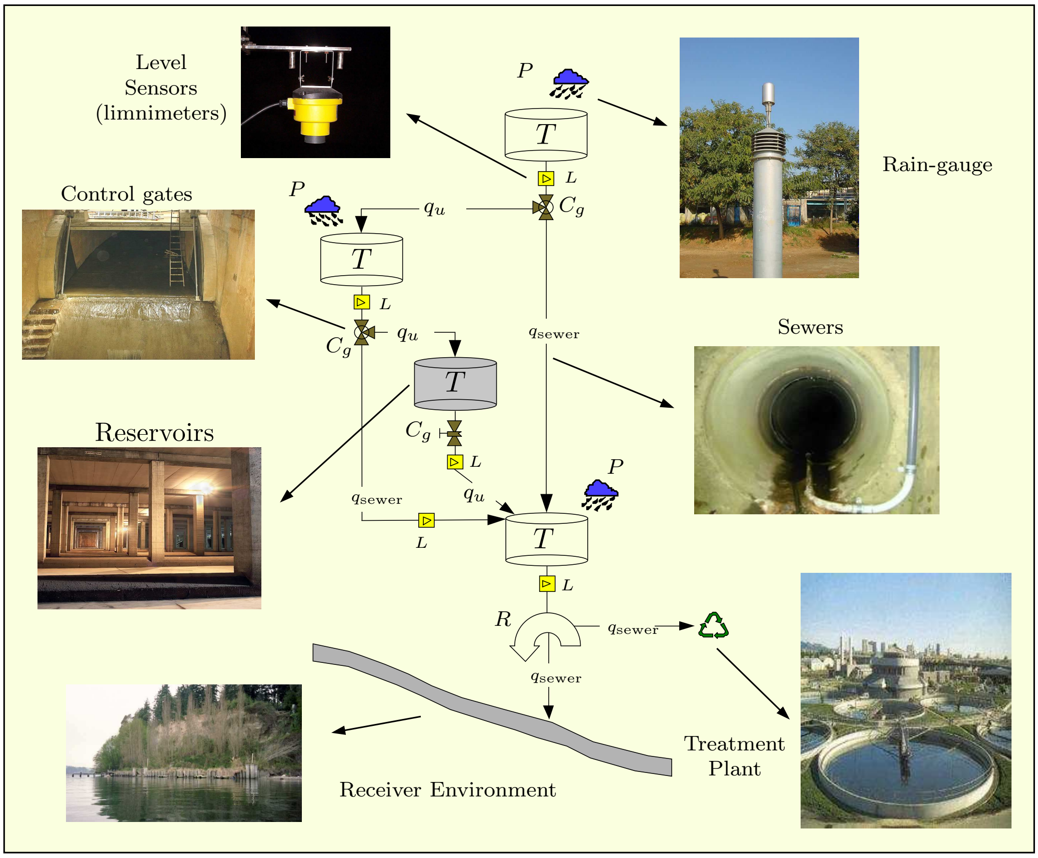


Figure 2: Simple sewer network conformed by constitutive elements described in Section 2.2.

creates a new flow path from the tank denoted as q_d (referred to as *virtual tank overflow*) that can be expressed mathematically as:

$$q_{dk} = \begin{cases} \frac{(v_k - \overline{v})}{\Delta t} & \text{if} \quad v_k \geq \overline{v} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Thus, outflow of virtual tank is then limited by its maximum volume capacity as follows:

$$q_{out\,k} = \begin{cases} \beta\overline{v} & \text{if} \quad v_k \geq \overline{v} \\ \beta v_k & \text{otherwise.} \end{cases} \tag{7}$$

Consequently, considering the tank overflow, the difference equation (3) related to virtual tanks becomes

$$v_{ik+1} = v_{ik} + \Delta t \varphi_i S_i P_{ik} + \Delta t \left(q_{in\,k}^{\,i} - q_{out\,k} - q_{dk} \right). \tag{8}$$

On the other hand and as was said before, real tanks are elements designed to retain water in case of severe weather. For this reason, both tank inflow and outflow could be controlled. In the same way, tank inflow is constrained by the actual volume within the real tank, by its maximum capacity and by tank outflow. Since real tanks are considered without overflow capabilities, inflow is pre-manipulated by using a redirection gate (explained in Section 2.2.2 below), what results in the consideration of this component within the modelling of the real tank. In order to restrict the value of the manipulated flow $q_{ak}^{\star}$ to satisfy the maximum flow condition in the input gate, flow through input link q_a should be expressed as

$$\tilde{q}_{a\,k} = \begin{cases} q^{\star}_{a\,k} & \text{if} \quad q^{\star}_{a\,k} \leq q_{in\,k} \\ q_{in\,k} & \text{otherwise.} \end{cases} \tag{9}$$

However, maximum tank capacity also restricts the inflow according to the expression

$$q_{a\,k} = \begin{cases} \tilde{q}_{a\,k} & \text{if} \quad q_{b\,k} - q_{out\,k} \leq \frac{\overline{v} - v_k}{\Delta t} \\ \frac{\overline{v} - v_k}{\Delta t} & \text{otherwise.} \end{cases} \tag{10}$$

Finally, tank outflow is given by

$$q_{out\,k} = \begin{cases} q^{\star}_{out\,k} & \text{if} \quad q^{\star}_{out\,k} \leq \beta v_k \\ \beta v_k & \text{otherwise,} \end{cases} \tag{11}$$

taking into account that $q^{\star}_{out}$ is also restricted by the maximum capacity of the outflow link, denoted by $\overline{q}_{out\,k}$. Thus, latter expressions lead to the following difference equation for real tanks in sewer networks:

$$v_{k+1} = v_k + \Delta t (q_{a\,k} - q_{out\,k}). \tag{12}$$

Notice that the flow through q_b corresponds to the mass balance

$$q_{b\,k} = q_{in\,k} - q_{a\,k}. \tag{13}$$

Figures 1(a) and 1(b) show conceptual schemes of the both virtual and real tanks considered in this paper.

2.2.2 Manipulated Gates

Within a sewer network, gates are elemets used as control devices since they can change the flow downstream. Depending on the action made, gates can be classified as *retention gates*, used to change the direction of the sewage flow, and *redirection gates*, used to retain the sewage flow in a certain point (sewer or reservoir) of the network. In the case of real tanks, a retention gate is present to control the outflow. Virtual tank outflows can not be closed but can be diverted using redirection gates. Indeed, redirection gates divert a flow from a nominal path which the flow follows if the gate is closed. This nominal flow is denoted as Q_i in the equation below, which expresses the mass conservation relation of the element:

$$q_{out}{}^{i}_{k} = Q_{i\,k} + \sum_j q^{j}_{u_i\,k}, \tag{14}$$

where j is an index over all manipulated flows coming from the gate. Figure 1(c) shows a conceptual scheme of redirection gates considered in this paper. Assuming that the flow through sewer q_a is imposed (for instance computed by means of a control law), the expressions that describe a redirection gate can be written as:

$$q_{a\,k} = \begin{cases} \overline{q}_a & \text{if} \quad q^{\star}_a > \overline{q}_a \\ q^{\star}_a & \text{otherwise,} \end{cases} \tag{15}$$

where $q^{\star}_a$ corresponds to the imposed/computed value for the flow $q_{a\,k}$. Flow $q_{b\,k}$ is directly given by the mass balance expression

$$q_{b\,k} = q_{in\,k} - q_{a\,k}. \tag{16}$$

2.2.3 Weirs and Main Sewer Pipes

These components complement the set of elements of sewer networks considered in this paper. Since the descriptions of their dynamics are relatively close, all of them are presented together in this section. *Nodes* are points of the network where the sewage can be either propagated or merged. Hence, these elements can be classified as *splitting nodes* and *merging nodes*. The first type can be treated considering a constant partition of the sewage flow in predefined portions according to the topological design characteristics. Indeed, splitting nodes exhibit a switching behaviour. In the case of a set of n inflows q_i, with $i = 1, 2, \dots, n$, the expression for the node outflow q_{out} is written as

$$q_{out} = \sum_{i=0}^{n} q_i. \tag{17}$$

Weirs can be seen as splitting nodes having a maximum capacity in the nominal outflow path related to the flow capacity of the output pipe. In the same way, *main sewer pipes* can be seen as weirs with a single inflow. They are used as connection devices between network constitutive elements. Therefore, considering the similarity between all the aforementioned elements and the notation in Figure 1(d), the set of expressions valid to represent the behaviour either a weir or a sewer pipe are the following:

$$q_{bk} = \begin{cases} \overline{q}_b & \text{if} \quad q_{in} > \overline{q}_b \\ q_{ink} & \text{otherwise,} \end{cases} \tag{18a}$$

$$q_{ck} = \begin{cases} q_{ink} - \overline{q}_b & \text{if} \quad q_{in} > \overline{q}_b \\ 0 & \text{otherwise,} \end{cases} \tag{18b}$$

where $\overline{q}_b$ is the maximum flow through q_b and q_{in} is the inflow. Notice that the outflow from virtual tanks is assumed to be unlimited in order to guarantee a feasible solution of an associated optimization problem within the design procedure of a optimization-based control strategy. The same idea applies to the outflow q_{bk} related to retention gates. But most often, sewer pipes have limited flow capacity. The description of this element given here takes into account this limited capacity. When the limit of flow capacity is exceeded, resulting overflow is possibly redirected to another element within the network or is considered as loss to the environment.

2.3 Hybrid Modelling Approach using MLD Forms

In order to obtain a control-oriented model that takes into account the switching elements and discontinuous phenomena inherent of sewage systems (as presented in previous section), the hybrid systems modelling methodology based on MLD forms proposed by [16] is briefly described.

According to [16], an entire sewer network model is constructed by connecting the system inflows (rain) and outflows (sewer treatment plants and/or outflows to the environment) with the inflows and outflows of the elements as well as connecting the elements themselves. The set of manipulated variables of the whole sewage system, denoted as q_u, is conformed by the manipulated variables of the constitutive elements of the sewer network. The logical conditions presented to describe the dynamics of the sewage system elements can be translated into linear integer inequalities as described in [2]. The whole sewer network expressed in MLD form can be written as

$$v_{k+1} = Av_k + B_1 q_{u_k} + B_2 \delta_k + B_3 z_k + B_4 d_k, \tag{19a}$$

$$y_k = Cv_k + D_1 q_{u_k} + D_2 \delta_k + D_3 z_k + D_4 d_k, \tag{19b}$$

$$E_2 \delta_k + E_3 z_k \leq E_1 q_{u_k} + E_4 v_k + E_5 + E_6 d_k, \tag{19c}$$

where $v \in \mathbb{R}_+^{n_c}$ corresponds to the vector of tank volumes (states), $q_u \in \mathbb{R}_+^{mi_c}$ is the vector of manipulated sewer flows (inputs), $d \in \mathbb{R}_+^{md}$ is the vector of rain measurements (disturbances), logic vector $\delta \in \{0,1\}^{r_\ell}$ collects the Boolean overflow conditions and vector $z \in \mathbb{R}_+^{r_c}$ is associated with variables that appear depending on system states and inputs. Variables δ and z are auxiliary variables associated with the MLD form. Equation (19c) collects the set of element constraints as well as translations from logic propositions. Notice that this model is a more general MLD than was presented in [2] due to the addition of the measured disturbances.

Next, the set of logical conditions needed to describe the dynamics of the sewage system elements presented in Section 2.2 are outlined for each considered constitutive element.

2.3.1 Virtual Tanks

The overflow existence condition in this element is considered by defining the logical variable

$$[\delta_k = 1] \longleftrightarrow [v_k \geq \overline{v}], \tag{20}$$

what implies that flows q_{dk} and q_{outk} are defined through this logical variable as:

$$\begin{aligned} z_{1k} &= q_{dk} \\ &= \delta_k \left[\frac{(v_k - \overline{v})}{\Delta t} \right], \end{aligned} \tag{21a}$$

$$\begin{aligned} z_{2k} &= q_{outk} \\ &= \delta_k \beta \overline{v} + (1 - \delta_k) \beta v_k. \end{aligned} \tag{21b}$$

Hence, the corresponding difference equation for the tank in function of the auxiliary variables is rewritten as:

$$v_{k+1} = v_k + \Delta t [q_{ink} - z_{1k} - z_{2k}], \tag{22}$$

where q_{ink} is the tank inflow, z_{1k} is related to the tank overflow and z_{2k} is related to the tank output. Notice that q_{ink} collects all inflows to the tank, which could be outflows from tanks located upstream, link flows, overflows from other tanks and/or sewers and rain inflows. Flow q_{bk} is computed as in (13).

2.3.2 Real Tanks

The MLD form for a real tank and manipulated input/output gates according to the description in Section 2.2.1 can be obtained by introducing the set of δ and z variables collected in Table 1.

Table 1: Expressions for δ and z variables for hybrid modelling of the real tank element.

Logical variable δ	Auxiliary variable z
$[\delta_{1k} = 1] \longleftrightarrow [q_a^\star \leq q_{ink}]$	$z_{1k} = q_{bk} = \delta_{1k} q_{ak}^\star + (1 - \delta_{1k}) q_{ink}$
$[\delta_{2k} = 1] \longleftrightarrow \left[z_{1k} - z_{3k} \leq \frac{\overline{v} - v_k}{\Delta t} \right]$	$z_{2k} = q_{ak} = \delta_{2k} q_{bk} + (1 - \delta_{2k}) \frac{\overline{v} - v_k}{\Delta t}$
$[\delta_{3k} = 1] \longleftrightarrow [q_{out}^\star \leq \beta v_k]$	$z_{3k} = q_{outk} = \delta_{3k} q_{outk}^\star + (1 - \delta_{3k}) \beta v_k$

Then, mass balance difference equation of real tanks can be rewritten as follows:

$$v_{k+1} = v_k + \Delta t [z_{3k} - z_{2k}]. \tag{23}$$

Again, flow q_{bk} is computed as in (13).

2.3.3 Redirection Gates

The MLD form is obtained taking into account that q_a, i.e., the manipulated flow (see Figure 1(c)), should satisfy the restriction (15). Hence, definitions

$$[\delta_k = 1] \longleftrightarrow [q_{ak} \geq q_{ink}] \tag{24}$$

and

$$z_k \quad = \quad \delta_{1k} q_{ak} + (1 - \delta_{1k}) q_{ink} \tag{25}$$

are stated in order to generate the MLD form of this element. Thus, the flow through sewer q_b is directly defined by the mass conservation relation

$$q_{bk} = q_{ink} - z_k. \tag{26}$$

2.3.4 Main Sewer Pipes (or Single Inflow Weirs)

The MLD form for either of these elements is obtained from the overflow condition

$$[\delta_k = 1] \longleftrightarrow [q_{in} \geq \overline{q}_b], \tag{27}$$

and the auxiliary continuous variables that define the flows q_{bk} and q_{ck} are, respectively:

$$\begin{aligned} z_{1k} &= q_{bk} \\ &= \delta_k \, \overline{q}_b + (1 - \delta_k) \, q_{ink}, \qquad (28a) \\ z_{2k} &= q_{ck} \\ &= \delta_k \, (q_{ink} - \overline{q}_b). \qquad (28b) \end{aligned}$$

2.4 PWL Modelling Approach

An alternative approach to the hybrid modelling consists in using continuous and monotonic functions to represent expressions that contains logical conditions, as for instance, (6) or (18), which describe the weirs behaviour and overflow capability of reservoirs, respectively. Indeed, these phenomena involve the switching and discontinuous behaviour of the sewage system.

The properties of a function being monotonic and continuous are very useful when optimization-based control strategies are designed since a quasi-convex optimization problem can be stated, what might lead in a global optimal solution [4]. The continuous and monotonic functions for the modelling approach proposed here are defined as follows:

- Saturation function, defined as

$$\mathrm{sat}(x, M) = \begin{cases} x & \text{if} \quad 0 \leq x \leq M, \\ M & \text{if} \quad x > M, \\ 0 & \text{if} \quad x < 0. \end{cases} \tag{29}$$

- Dead-zone function, defined as

$$\mathrm{dzn}(x, M) = \begin{cases} x - M & \text{if} \quad x \geq M, \\ 0 & \text{if} \quad x < M. \end{cases} \tag{30}$$

Next, sewer network constitutive elements described in Section 2.2 will be expressed using this modelling approach. Notice that the whole representation of a given sewer network modelled using this approach consists in a set of equations instead of a matricial model as the one obtained with the hybrid MLD approach presented in previous section.

2.4.1 Virtual Tanks

Using the PWLF approach, the tank outflows can be expressed as

$$q_{outk} = \beta \, \mathrm{sat}\,(v_k, \bar{v}_k)\,, \quad (31)$$

$$q_{dk} = \frac{\mathrm{dzn}(v_k, \bar{v}_k)}{\Delta t}, \quad (32)$$

what allows the difference equation for the tank volume to be written as in (12):

$$v_{k+1} = v_k + \Delta t(q_{ink} - q_{dk} - q_{outk)}, \quad (33)$$

where q_{ink} considers all possible inflows including the precipitation term (in flow units).

2.4.2 Real Tanks

The following expressions related to the tank inflow and outflow are stated for this element:

$$q_{outk} = \mathrm{sat}\,(q^{\star}_{outk}, \beta\, v_k)\,, \quad (34)$$

$$q_{ak} = \mathrm{sat}\left(q^{\star}_{ak}, \min\left(\frac{\overline{v}_k - v_k}{\Delta t}, q_{ink}\right)\right). \quad (35)$$

Thus, the difference equation for the volume of the tank is again written as in (12):

$$v_{k+1} = v_k + \Delta t(q_{ak} - q_{outk)}, \quad (36)$$

and flow q_b obeys to the mass balance $q_{bk} = q_{ink} - q_{ak}$.

2.4.3 Redirection Gates

In the case of redirection gates, the PWLF model is defined taking into account that q_a should satisfy the restriction (15) what can be rewritten in terms of the PWL functions as

$$q_{ak} = \mathrm{sat}(q_{ak}, q_i n_k). \quad (37)$$

Flow through q_b is given by the mass balance (16).

2.4.4 Main Sewer Pipes (or Single Inflow Weirs)

The PWLF model for either of these elements can be obtained from the overflow condition as follows:

$$q_{bk} = \mathrm{sat}(q_{ink}, \overline{q}_b), \quad (38)$$

$$q_{ck} = \mathrm{dzn}(q_{ink}, \overline{q}_b), \quad (39)$$

where $\overline{q}_b$ corresponds again to the maximum flow capacity of the nominal outflow pipe.

3 MPC-Based RTC on Large-scale Sewer Networks

3.1 MPC as a Tool for Implementing Global RTC

In most sewer networks, the regulated elements (pumps, gates and detention tanks) are typically controlled locally, i.e., they are controlled by a remote station according to the measurements of sensors connected to that station only. However, a global RTC system requires the use of an operational model of the network dynamics in order to compute, ahead of time, optimal control strategies for the network actuators based on the current state of the system (provided by SCADA[1] sensors), the current rain intensity measurements and appropriate rainfall predictions. The computation procedure of an optimal global control law should take into account all the physical and operational constraints of the sewage system, producing set-points which achieve minimum flooding and CSO.

As discussed in the introduction, MPC is a suitable control strategy to implement global RTC of sewer networks since it has some features to deal with complex systems such as sewer networks: big delays compensation, use of physical constraints, relatively simple for people without deep knowledge of control, multivariable systems handling, etc. Hence, according to [27], such controllers are very suitable to be used in the global control of urban drainage systems within a hierarchical control structure [21, 14].

MPC, which more than a control technique, is a set of control methodologies that use a mathematical model of a considered system to obtain a control signal minimizing a cost function related to selected performance indexes related to the system behaviour. MPC is very flexible regarding its implementation and can be used over almost all systems since it is set according to the model of the plant [5]. Notice that MPC, as the global control law, determines the references for local controllers located on different elements of the sewer network. A management level is used to provide to MPC the operational objectives, what is reflected in the controller design as the performance indexes to be minimized. In the case of urban drainage systems, these indexes are usually related to flooding, pollution, control energy, etc.

This section briefly describes the MPC strategy from the generic point of view and then describes the particularities for its use with sewage systems.

3.2 MPC Strategy Description

MPC is a wide field of control methods that share a set of basic elements in common as

- a *cost function*, that represents a performance index of the system studied,
- a *prediction model*, which should capture the representative process dynamics and allows to predict the future behaviour of the system, and
- a *control signal computation procedure* using a receding horizon strategy generally by solving an optimization problem whose objective is the cost function and the restrictions are the prediction model plus the operational constraints.

However, different tuning parameters rise to a different set of implementation algorithms [12].

3.3 General MPC Formulation

MPC strategy used in this paper follows the formulation introduced in [12]. Thus, let

$$x_{k+1} = g(x_k, u_k) \tag{40}$$

be the mapping of states $x_k \in \mathbb{X} \subseteq \mathbb{R}^n$ and control signals $u_k \in \mathbb{U} \subseteq \mathbb{R}^m$ for a given system, where $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a function that describes the system. In the case of this paper, g can be either a MLD or PWLF model obtaided by using the elementary models presented in Section 2.

Let

$$\mathbf{u}_k(x_k) \triangleq \left(u_{0|k}, u_{1|k}, \ldots, u_{H_p-1|k}\right) \in \mathbb{U}^{H_p} \tag{41}$$

[1]Supervisory Control And Data Adquisition (SCADA)

be an input control sequence over a fixed time horizon H_p. Then, an *admissible input sequence* with respect to the state $x_k \in \mathbb{X}$ is defined by

$$\mathcal{U}_{H_p}(x_k) \triangleq \left\{ \mathbf{u}_k \in \mathbb{U}^{H_p} | \mathbf{x}_k \in \mathbb{X}^{H_p} \right\}, \tag{42}$$

where

$$\mathbf{x}_k(x_k, \mathbf{u}_k) \triangleq \left(x_{1|k}, x_{2|k}, \ldots, x_{H_p|k} \right) \in \mathbb{X}^{H_p} \tag{43}$$

corresponds to the state sequence generated by applying the input sequence (41) to the system (40) from initial state $x_{0|k} \triangleq x_k$, where x_k is the measurement (or the estimation) of the current state. Hence, the receding horizon approach is based on the solution of the open-loop optimization problem (OOP) [3]

$$\min_{\{\mathbf{u}_k \in \mathcal{U}_{H_p}\}} J\left(\mathbf{u}_k, x_k, H_p\right), \tag{44a}$$

subject to

$$H_1 \mathbf{u}_k \leq b_1, \tag{44b}$$

$$G_2 \mathbf{x}_k + H_2 \mathbf{u}_k \leq b_2, \tag{44c}$$

where $J(\cdot) : \mathbb{X}_f(H_p) \mapsto \mathbb{R}_+$ is the cost function with domain in the *set of feasible states* $\mathbb{X}_f(H_p) \subseteq \mathbb{X}$ [11], H_p denotes the *prediction horizon* or *output horizon* and G_2, H_i and b_i are matrices of suitable dimensions. In sequence (43), $x_{k+i|k}$ denotes the prediction of the state at time $k+i$ done in k, starting from $x_{0|k} = x_k$. When $H_p = \infty$, the OOP is called *infinite horizon problem*, while with $H_p \neq \infty$, the OOP is called *finite horizon problem*. Constrains stated to guarantee system stability in closed loop would be added in (44b)-(44c).

Assuming that the OOP (44) is feasible for $x \in \mathbb{X}$, i.e., $\mathcal{U}_{H_p}(x) \neq \emptyset$, there exists an optimal solution given by the sequence

$$\mathbf{u}_k^* \triangleq \left(u_{0|k}^*, u_{1|k}^*, \ldots, u_{H_p-1|k}^* \right) \in \mathcal{U}_{H_p}, \tag{45}$$

and then the receding horizon philosophy sets [12], [5]

$$u_{\text{MPC}}(x_k) \triangleq u_{0|k}^*, \tag{46}$$

and disregards the computed inputs from $k = 1$ to $k = H_p - 1$, repeating the whole process at the following time step. Equation (46) is known in the MPC literature as *the MPC law*. Summarizing, Algorithm 1 briefly describes the basic MPC law computing process.

Algorithm 1 Basic MPC law computation.

$k = 0$
loop
    $x_{k+0|k} = x_k$
    $\mathbf{u}_k^*(x_k) \Leftarrow$ solve OOP (44)
    Apply only $u_k = u_{k+0|k}^*$
    $k = k + 1$
end loop

3.4 MPC on Sewer Networks

3.4.1 Control Objectives

The sewage system control problem has multiple objectives with varying priority, see [14]. The type, number and priority of those objectives can also be different depending on the particular sewage system design. However, in general, the most common objectives are related to the manipulation of the sewage in order to avoid undesired sewage flows outside of the main sewers (flooding). Another type of control objectives are related for instance to the control energy, i.e., the energy cost of the regulation gates movements. The main considered objectives for the case study presented in this paper are listed below in order of decreasing priority:

- *Objective 1:* minimize flooding in streets (virtual tank overflow).
- *Objective 2:* minimize flooding in links between virtual tanks.
- *Objective 3:* maximize sewage treatment.

A secondary purpose of the third objective is to reduce the volume in the tanks to anticipate future rainstorms. This objective also indirectly reduces pollution to the environment. This is because if the treatment plants are used optimally with the storage capacity of the network, pollution should be strongly minimized. Moreover, this objective can be complemented by conditioning minimum volume in real tanks at the end of the prediction horizon. It could be seen as a fourth objective. It should be noted that in practice the difference between the first two objectives is small.

3.4.2 Problem Constraints

When using the modelling approach based on virtual tanks either in MLD or PWLF form, only flow rates are manipulated in such way that some the inherent nonlinearities (e.g., nonlinear relation between gate opening and discharge flow) of the sewer network are simplified as discussed in [10]. But, in turn, some physical restrictions need to be included as constraints on system variables. For instance, variables q_{ui}^{j} that redirect outflow from a virtual tank should never be larger than the outflow from the tank. This is expressed with the following inequality

$$\sum_{j} q_{uik}^{j} \leq q_{out}{}_{k}^{i} = \beta\, v_{ik} \tag{47}$$

Additionally, operational constraints associated to the range of gates actuation leads to the manipulated flows has to fulfill $q_{uik}^{j} \leq \overline{q}_{ui}^{j}$, where $\overline{q}_{ui}^{j}$ denotes its upper limit. Similarly, operational limits on the range of real tank volumes should be included (see (5)) to limit the amount of sewage that can be stored.

3.4.3 MPC disturbances

Rain plays the role of measured disturbance in the MPC problem on sewer networks. The type of disturbance model to be used depends on the rain prediction procedure available [30]. Existing methods include from the use of time series [30] to the sophisticated utilization of meteorological radars [33]. According to [14], different assumptions can be done for the rain prediction when an optimal control law is used in the RTC of sewer networks. Results show that the assumption of constant rain over a short prediction horizon gives results that can be compared with the assumption of known rain over the considered horizon, confirming similar results are reported in [10] and [19].

4 Case Study Description

4.1 The Barcelona Sewer Network

The city of Barcelona has a CSS of approximately 1697 km length in the municipal area plus 335 Km in the metropolitan area, but only 514.43 km are considered as the main sewer network. Its storage capacity is of 3038622 m^3, which implies a dimension three times greater than other cities comparable to Barcelona. It is worth to notice that Barcelona has a population which is around 1.59 million inhabitants on a surface of 98km^2, approximately. This fact results in a very high density of population. Additionally, the yearly rainfall is not very high (600mm/year), but it includes heavy storms (up to 90mm/h) typical of the Mediterranean climate that can cause a lot of flooding problems and CSO to the receiving environments.

Clavegueram de Barcelona, S.A. (CLABSA) is the company in charge of the sewage system management in Barcelona. There is a remote control system in operation since 1994 which includes, sensors, regulators, remote stations, communications and a Control Center in CLABSA. Nowadays, as regulators, the urban drainage system contains 21 pumping stations, 36 gates, 10 valves and 8 detention tanks which are regulated in order to prevent flooding and CSO. The remote control system is equipped with 56 remote stations including 23 rain-gauges and 136 water-level sensors which provide real-time information about rainfall and water levels into the sewage system. All this information is centralized at the CLABSA Control Center through a SCADA system. The regulated elements (pumps, gates and detention tanks) are currently controlled locally, i.e., they are handled from the remote control center according to the measurements of sensors connected only to local stations.

4.2 Barcelona Test Catchment

From the whole sewer network of Barcelona, which was described beforehand, this paper considers a portion that represents the main phenomena and the most common characteristics appeared in the entire network. This representative portion is selected to be the case study of this paper because a calibrated and validated model of the network obtained using the virtual modelling methodology (see Section 2) is available as well as rain gauge data for an interval of several years. The considered Barcelona Test Catchment (BTC) has a surface of 22,6 km^2 and includes typical elements of the larger network.

The BTC has one retention gate associated with one real tank, three redirection gates and one retention gate, 11 sub-catchments defining equal number virtual tanks, several level gauges (limnimeters) and a two WWTPs. Also, there are five rain-gauges used to measure the rain entering in each sub-catchment. Notice that some sub-catchments (virtual tanks) share the same rain sensor. These sensors count the amount of tipping events in five minutes (sampling time) and such values is multiplied by 1.2 mm/h in order to obtain the rain intensity P in m/s at each sampling time, after the appropriate units conversion. The difference between the rain inflows for virtual tanks that share sensor lies in the surface area S_i and the ground absorbtion coefficient φ_i of the i-th sub-catchment (see (3)), what yields in different amount of the rain inflows.

Using the virtual tanks representation principle, resulting BTC model has 12 state variables corresponding to the volumes in the 12 tanks (one real, 11 virtual), four control inputs corresponding to the manipulated links and five measured disturbances corresponding to the measurements of rain precipitation at the sub-catchments. Two WWTPs are used to treat the sewage before it is released to the environment. It is supposed that all states (virtual tank volumes) are estimated by using the limnimeters shown with capital letter L in Figure 3. The free flows to the environment as pollution ($q_{10\text{M}}$, $q_{7\text{M}}$, $q_{8\text{M}}$ and $q_{11\text{M}}$ to the Mediterranean sea and q_{12s} to other catchment) and the flows to the WWTPs ($q_{7\text{L}}$ and $q_{11\text{B}}$) are also shown in

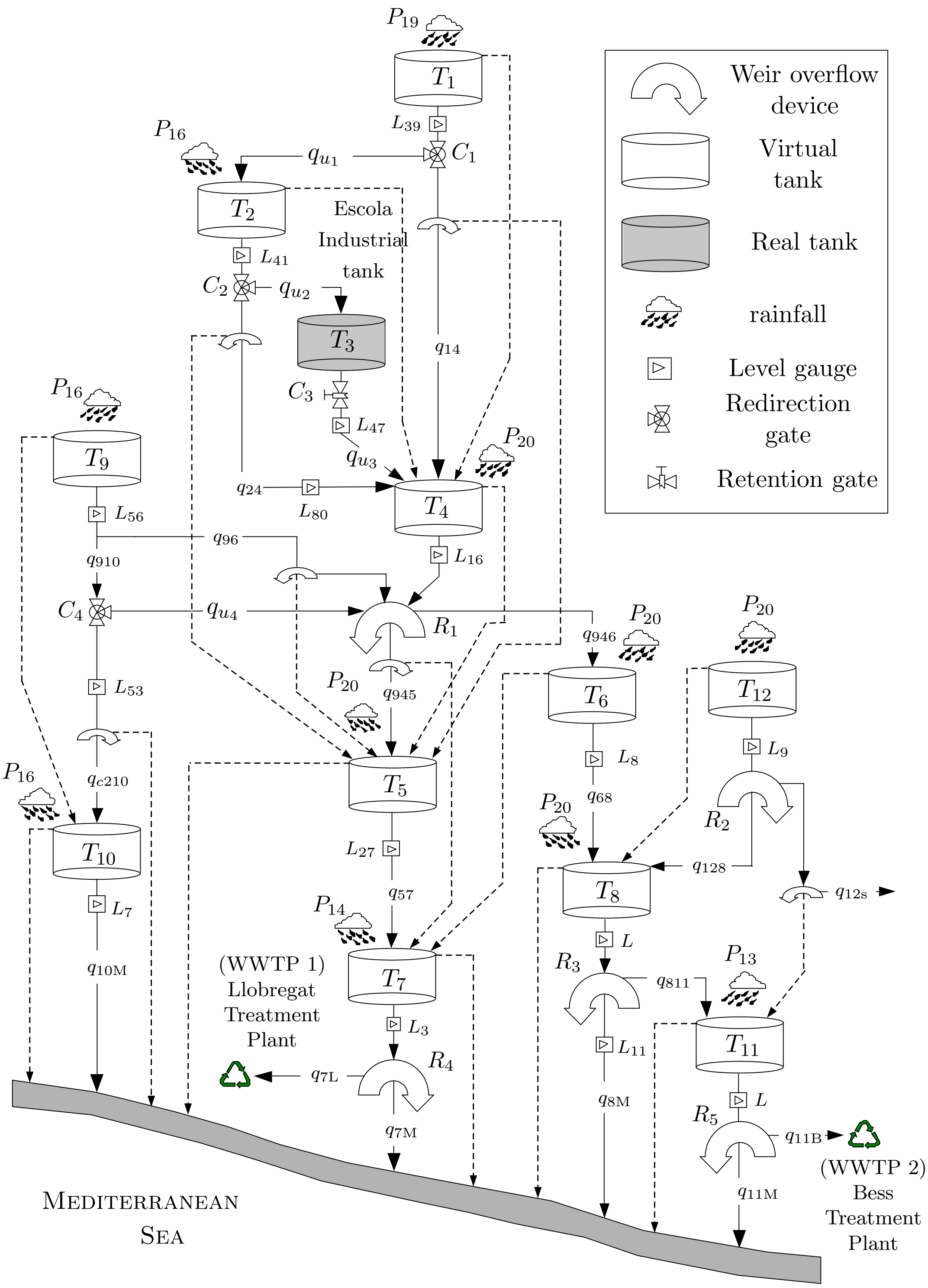


Figure 3: Barcelona test catchment scheme.

the figure as well as rain intensities P_{13}, P_{14}, P_{16}, P_{19} and P_{20} according to the case. The four manipulated links, denoted as q_{u_i} have a maximum flow capacity of 9.14, 25, 7 and 29.3 m^3/s, respectively, and these amounts can not be relaxed, being physical restrictions of the system (hard constraints).

Table 2: Rain episodes used for comparing modelling approaches

Rain Episode	Maximum Return Rate (years)	Return Rate average (years)
1999-09-14	16.3	4.3
2002-07-31	8.3	1.0
2002-10-09	2.8	0.6
1999-10-17	1.2	0.7
2000-09-28	1.1	0.4

4.3 Rain Episodes

Rain episodes used for the simulation of the BTC and for the design of MPC strategies are based on real rain gauge data obtained within the city of Barcelona on the given dates (yyyy-mm-dd) as presented in Table 2. These episodes were selected to represent the meteorological behaviour of Barcelona, i.e., they contain representative meteorologic phenomena in the city. Table 2 also shows the maximum *return rate*[2] among all five rain gauges for each episode. In the third column of the table, the return rate for the whole Barcelona network is shown. The number is lower because it includes in total 20 rain gauges. Notice that one of the rain storms had a return rate of 4.3 years in the case of the whole network while for one of the rain gauges the return rate was 16.3 years.

5 Simulation and Results

5.1 Preliminaries

This section is focused on comparing the performance of a MPC-based sewer network RTC using a set of real rain episodes in Table 2 when the hybrid and PWLF modelling approaches, proposed in Section 2, are applied to the case study in Section 4. Computation time, when every modelling approach is used, is also compared. Results of such comparison would be a key issue to decide which of the two modelling approaches should be used for a RTC implementation in the real network. The assumptions made for all the implementations will be presented and their validity discussed before the results are given.

The detailed description of BTC case study including operating ranges of the control signals and state variables as well as the description of all variables and parameters can be found in [16] and [17].

The computation times presented in this paper has been obtained using Matlab® 7.2 implementations running on an Intel® Core™2, 2.4 GHz machine with 4Gb RAM. Notice that computation time results reported here related with hybrid models are different with respect to [16] due to the machine characteristics and solver versions.

5.2 Simulation and Prediction Models

Results presented in this paper are obtained in simulation by using two different models: one used as the plant (sewer network), which in the sequel will be called as *open-loop model*, and the other used by the MPC controller or *prediction model*. The open-loop model is implemented considering a non-linear representation of the sewer network based on mass balances where ranges

[2]The return rate or return period is defined as the average interval of time within which a hydrological event of given magnitude is expected to be equaled or exceeded exactly once. In general, this amount is given in years.

and bounds for every variable (control signals, volumes, rain disturbances) are strictly considered and all passible logical or discontinuous dynamics are included (as the case of weirs and overflows). On the other hand, prediction model is obtained by using the modelling approaches presented in Section 2.

To obtain a model of the BTC using the hybrid modelling approach, the elementary models and transformations presented in Section 2 should be used. However, in order to avoid the tedious procedure of deriving the MLD form of the BTC by hand, the higher level language and associated compiler HYSDEL (see [31]) are used here. The resulting model in MLD form has 22 logical variables and 44 auxiliary variables. Once the hybrid model based on MLD form has been obtained, a hybrid MPC controller has been designed and the set of considered rain scenarios were simulated using the *Hybrid Toolbox* for Matlab® (see [1]) and ILOG CPLEX 11.2. This latter solver allows to solve efficiently the MIP problems associated to the hybrid MPC controller. Using the hybrid model, to determine the control actions using hybrid MPC implies that for each time instant, considering a prediction horizon $H_p = 6$, $2^{22\times 6} = 5.4 \times 10^{39}$ LP problems (for a linear norm in the cost function) or QP problems (for a quadratic norm in the cost function) should be solved in the worst case.

On the other hand, the model of BTC using the PWLF-based modelling approach was obtained by joining the different compositional elements described in Section and following the network diagram of Figure 3, resulting in a non-linear representation as a set of expressions for the whole network. The implementation of an MPC using the PWLF modelling approach leads to a non-linear optimization problem. The selection of the algorithm to solve such problem was done after the evaluation of several solvers available on Tomlab® (e.g., *conSolve*, *nlpSolve*, among others). The *Structured Trust Region* algorithm (see [7]) was finally chosen because it provides an *acceptable* trade-off between system performance and computation time.

5.3 MPC Controller Set-up

Different parameters of the MPC controller should be defined and tuned according to the control objectives and their prioritization. Following the discussion in Section 3.4.1 regarding the control objectives of a sewer network, the following system outputs have been included in both the hybrid and PWLF modelling approaches:

$$y_{1k} = \sum_i q_{\mathrm{str}_v k} + \sum_j q_{\mathrm{str}_q k}, \tag{48a}$$

$$y_{2k} = \sum_l q_{\mathrm{sea} k}, \tag{48b}$$

$$y_{3k} = q_{\mathrm{trp1} k}, \tag{48c}$$

$$y_{4k} = q_{\mathrm{trp2} k}, \tag{48d}$$

where y_{1k} represents the sum of the i overflows to street from virtual tanks at time k, denoted by $q_{\mathrm{str}_v k}$, plus the sum of the j overflows to street from links (main pipes) at time k, denoted by $q_{\mathrm{str}_q k}$. Output y_{2k} represents the sum of the l overflows which go to sea (as receiver environment) at time k, denoted as $q_{\mathrm{sea}_v k}$, and finally y_{3k} and y_{3k} represent the flows towards the WWTPs at time k, denoted by $q_{\mathrm{trp1} k}$ and $q_{\mathrm{trp2} k}$. Note that for the case study of this paper, $q_{\mathrm{trp1} k} = q_{7\mathrm{L} k}$ and $q_{\mathrm{trp1} k} = q_{11\mathrm{B} k}$.

Using the outputs (48), the cost function for the BTC can be written as follows

$$J(u_k, x_k) = \sum_{i=0}^{H_p-1} \left\| y_{k+i|k} - y_r \right\|_Q^2, \tag{49}$$

where $y_{k+i|k}$ is the output vector (defined by (48)) at the instant $k + i$ with respect to time instant k, H_p denotes the prediction horizon and y_r is a known vector containing the system references and defined for this case as

$$y_r = [0 \quad 0 \quad \mathbb{1}^T \overline{q}_{7\text{L}} \quad \mathbb{1}^T \overline{q}_{11\text{B}}]^T, \tag{50}$$

where $\mathbb{1}$ is a vector of ones with suitable dimensions and $\overline{q}_{7\text{L}}$ and $\overline{q}_{11\text{B}}$ correspond to the maximum flow capacity through sewers $q_{7\text{L}}$ and $q_{11\text{B}}$, respectively.

In order to tune the MPC controller, the weighted approach technique has been used. Hence, in (49), Q corresponds to the weight matrix containing the weights w_i, each one related to a control objective. Notice that the desired prioritization of the control objectives is given by the values w_i that, for this case, determine a Q matrix of the following form:

$$Q = diag\{w_{\text{str}}\, I \quad w_{\text{sea}}\, I \quad w_{\text{trp1}}\, I \quad w_{\text{trp2}}\, I\}, \tag{51}$$

where I corresponds to a identity matrix of suitable dimensions. Here, $w_{\text{str}} = 1$, $w_{\text{sea}} = 10^{-1}$, $w_{\text{trp1}} = 10^{-3}$, and $w_{\text{trp2}} = 10^{-3}$.

The prediction horizon H_p has been set to 6, which is equivalent to 30 minutes with a sampling time $\Delta t = 300s$. This selection was based on the reaction time of the system to disturbances. Another reason for this selection is that the constant rain prediction assumed in this paper becomes less reliable for larger horizons. The length of the simulation scenarios is 100 samples, what allows to see the influence of the peak of the rain (disturbance) from the selected rain episode over the dynamics of the network and also over the dynamic of the closed loop.

5.4 Control performance and computation time comparisons

In this section, the comparison between results obtained using the MPC controllers based on the hybrid and PWLF modelling approaches is presented and discussed. Moreover, results for the performance indexes when the open-loop scheme is simulated are also outlined. This latter case consists in the sewage system without control so the manipulated links are used as passive elements, i.e., the amount of flows q_{u1}, q_{u2} and q_{u4} only depend on the inflow to the corresponding gate and they are not manipulated while q_{u3} is the outflow of the real tank given by gravity (tank discharge). Results related to the control performance are summarized in Tables 3, 4, and 5 for five of the more representative rain episodes in Barcelona between 1998 and 2002 (yyyy-mm-dd in tables). Table 3 shows the comparison of the volumes of sewage that go to street (flooding) during a simulation scenario while Table 4 shows the same comparison but regarding the volumes to receiver environments (pollution). Finally, Table 5 shows the comparison of volumes regarding the treated sewage at the WWTPs.

Table 3: Performance results. Index: Flooding [$\times 10^3$ m^3].

Rain Episodes	Open Loop	Hybrid Model	PWLF Model
1999-09-14	108	92.9	88.2
2002-10-09	116.1	97	113.3
2002-07-31	160.3	139.7	132.8
1999-10-17	0	0	0
2000-09-28	1	1	1

Notice from Tables 3, 4, and 5 that the performance of the system is better when a MPC control law is considered no matter the modelling approach utilized with respect to the performance in open-loop. This justifies the use of closed-loop control. Moreover, notice also that

Table 4: Performance results. Index: Pollution ($\times 10^3$ m^3].

Rain Episodes	Open Loop	Hybrid Model	PWLF Model
1999-09-14	225.8	223.5	226.1 (1.16%)
2002-10-09	409.8	398.7	407.7 (2.25%)
2002-07-31	378	374.6	380 (1.44%)
1999-10-17	65	58.1	59.9 (3.09%)
2000-09-28	104.5	98	102 (4.08%)

Table 5: Performance results. Index: Treated sewage at WWTPs ($\times 10^3$ m^3].

Rain Episodes	Open Loop	Hybrid Model	PWLF Model
1999-09-14	278.3	280.7	276.7 (1.43%)
2002-10-09	533.8	545	534.2 (1.98%)
2002-07-31	324.3	327.8	321.9 (1.80%)
1999-10-17	288.4	295.3	293.5 (0.61%)
2000-09-28	285.3	291.9	287.5 (1.51%)

the use of the hybrid modelling approach implies in average an better system performance (always respecting the prioritization of the control objectives) with respect to the performance improvement obtained by using the PWLF modelling approach. Notice also that the performance improvement is basically related to the improvement of the main control objective and then, following in a hierarchical order, to the second objective and so on.

These results, in general, were expected since the MPC controller based on the hybrid modelling approach achieves its optimum by solving a set of convex linear programs using a branch and bound scheme. However, the MPC based on PWLF modelling approach leads to a non-linear network model representation what results in a non-convex optimization. Therefore, the global optimum can not be assured leading possibly to a sequence of suboptimal controls when the computation of the receding horizon control law is done. This explains why the performance obtained using the PWLF model is in general worse than the one obtained using the hybrid model. However, it is very difficult to ensure that the optima reached for a complex problem that involves multi-objectives optimization and trial and error tuning procedures is the suitable for the particular case study. Suboptimality levels of the results obtained using the PWLF model were never greater that 4.1 % for the cases of the second and third objective (as shown in Tables 4, and 5 in parenthesis at the last column for some rain episodes). For the case of the first control objective (related to flooding), results were not so homogeneous since for some scenarios one of the modelling approaches leads in better system performance while for other scenarios occurred just the opposite.

On the other hand, the main difference of using the hybrid or the PWLF modelling approaches is in the computation time required to compute the control actions at each iteration. As mentioned in Section 5.2, the model in MLD form contains an important number of Boolean and auxiliary variables. The complexity of the MIP associated to the MPC law becomes bigger by increasing the number of Bolean variables since the underlying optimization problem is combinatorial and $\mathcal{NP}$-hard [20]. Thus, the worst-case computation time is exponential in the amount of integer variables. In large-scale systems such as a sewer network, the amount of elements with logical/discontinuous dynamics can augment according to the topology of the particular case study. Therefore, computation times increase towards a point where the use of this modelling for obtaining a MPC-based RTC law becomes almost impossible. On the other

Table 6: Computation time results [s].

Rain Episodes	Hybrid Model		PWLF Model	
	Total CPU time	max. CPU time in a sample	Total CPU time	max. CPU time in a sample
1999-09-14	1109.29	787.17	695.33	91.32
2002-10-09	561.73	85.31	293.23	66.01
2002-07-31	1050.54	381.49	830.20	83.04
1999-10-17	79.14	10.39	180.22	16.15
2000-09-28	84.76	13.27	120.88	12.13

hand, the use of alternative modelling approach based on the PWL functions proposed on this paper allows to have control sequences computed in lower times at the price of some degree of suboptimality due to the possible local optimum. Table 6 summarizes the computation times for both the modelling approaches proposed on this paper and for the six rain episodes previously considered.

Summarizing, despite the suboptimal nature of the solutions as a consequence the minor improvement of the control performance, the MPC controller based on the PWLF modelling approach not only leads to a faster control sequences computation but also to feasible ones respecting the real-time restriction imposed by the sampling time. In average, all the maximum computation times to compute the MPC control action when the PWLF modelling approach is used are less than the third part of the sampling time. This is not the case when using the hybrid modelling approach.

6 Conclusions

In this paper, model predictive control (MPC) of large-scale sewage systems has been addressed considering different modelling approacdhes that include several inherent continuous/discrete phenomena (overflows in sewers and tanks) and elements (weirs) in the system that result in distinct behaviour depending on the state (flow/volume) of the network. These behaviours can not be neglected nor can be represented by a pure linear model. After describing and analyzing these continuous/discrete dynamic behaviours, a modelling approach based on piece-wise linear functions is proposed and compared against the hybrid modelling approach previously reported by the authors. Control performance results and associated computation times of both approaches are compared by using a real case study based on the Barcelona sewer network. It was seen that, although the hybrid approach provides better performance since with the resulting MIP formulation and existing branch and bound solvers, the global optimum can be reach, the required computational time could be prohibitive for large-scale networks. On the other hand, with the PWLF modelling formulation, although a small amount of suboptimally is introduced since the resulting non-linear constraints are non-convex what leads to the optimization algorithms could be stack at some local optimum, the reduction of computation time allows to face the control of large-scale sewer networks.

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