## 4. SENSOR PLACEMENT

An optimal sensor placement is defined as a sensor configuration that achieves the minimum economical cost (number of sensors) while observing pre-specified performance criteria (groups of nodes that are not isolable with a minimum number of elements).

A model of water network can be represented as a graph $G=(V,E)$, where $E$ is the set of edges that represent the pipes and $V$ is the set of vertices (nodes) where pipes meet. Vertices can represent sources, such as reservoirs or tanks, where water is introduced or sinks (demand points) where water is consumed. Each pipe connects two vertices $v_i$ and $v_j$ and usually is denoted as $(v_i, v_j)$.

Using the graph representation, the problem of optimal sensor placement can be formulated as an integer programming problem, where each decision variable $x_i$ associated to a node $v_i$ of the network can be 1 or 0, meaning respectively that the sensor will be or not installed in this node (Bagajewicz, 2000). The starting point of the algorithm is the leakage sensitivity matrix obtained by simulation. Each element of this matrix contains the sensitivity that presents a node (in rows) to the considered possible leakages (in columns), assuming a single leakage at time.

The objective function is the number of elements for the largest set of leaks with the same signature. In order to increase isolability this cost should be minimized but at the same time keeping the economical cost reasonable, that is installing the less number of sensor that is possible. The problem is solved for a number of sensors; this number is increased till the cost does not decrease. A constraint is included such that all leaks should be detected. It is introduced by forcing that signature with all *0's* is not accepted. Thus the cost of non-detection has not to be into account.

This optimization problem can be solved using either deterministic method based for example in Branch and Bound or heuristic methods based for example in Genetic Algorithms. The first type of methods guarantee the optimal solution but the computation time tends to be exponential with the number of nodes/faults (Sarrate *et al.*, 2007). On the other hand, the second type of methods just guarantees a suboptimal solution that tends to the optimal one when the size of considered population tends to infinity. Besides the formulation of solutions in series of 1's and 0's are most convenient for a GA.

In Figure 5, the evolution of cost function is presented. The cost has been taken as the number of nodes in the biggest group of possible leakage isolated with a number of sensors and a threshold between 0.8 and 0.9. It can be seen that the best choice is threshold 0.8 and that 8 sensors are as good as 11. Therefore only 8 sensors are used.

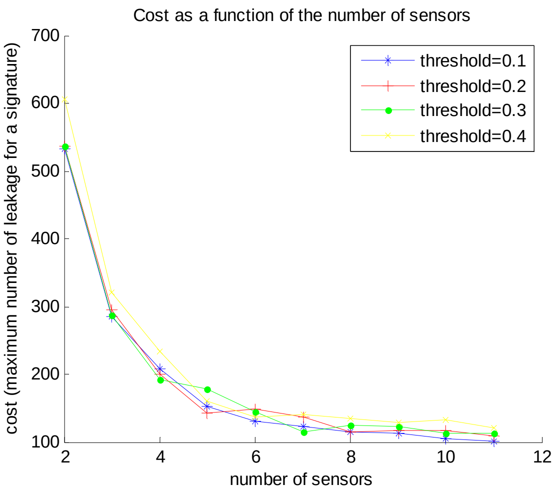

Fig. 5: Evolution of the cost function depending on number of sensors and threshold

In Figure 6, the different groups of nodes with the same leakage signature are shown. There are 39 groups and the hugest contains 190 nodes. The localization of the sensors after the optimization process is in the last figure.

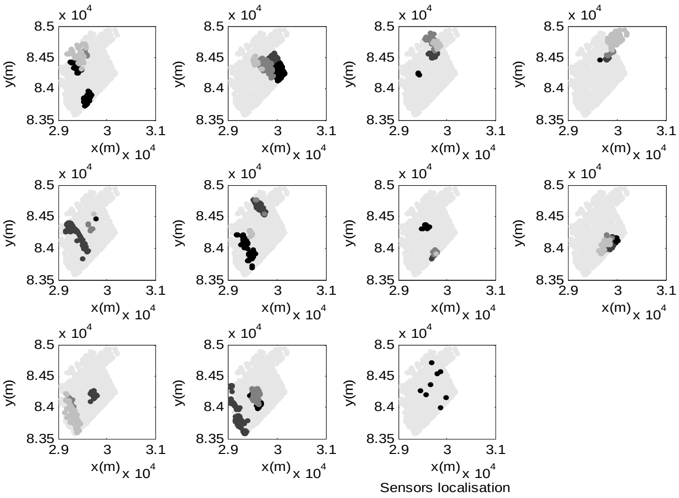

Fig. 6: Groups of nodes with the same leakage signature with 8 sensors and placement of sensors

In an ideal situation with a well calibrated network model a leakage should be searched in one of these regions instead of the whole sector.

## 5. LEAKAGE LOCALISATION

The localisation of leakages is based on the isolation techniques presented in *Section 2*. The binarised sensitivity matrix is used as a signature matrix for all leakages. In *Section 3*, the signatures were used to obtain an optimal distribution of the sensors and in Figure 6 the groups of nodes that generate the same signature are presented.

In the process of leak localization, the signature generated in case of having 8 sensors installed with a fault is compared with signature matrix. If the model were perfect, the leak should be localized with one measurement. Assuming that some parameters may be uncertain, the test has been done during 15 days of simulation (only the lowest consume hour is used each day) and then three options are used to assign the observed leakage signature to a group:

- Mean of the sensitivities
- Mean of binarised sensitivities
- The voting (all days the leak is assigned to a group and that with more assignation is the elected)

Results without uncertainty were not good using any of the three decision criteria. It was due to the changing boundary conditions that affected very much the sensitivity matrix. It is necessary to generate it each day when boundary conditions are known. When a new signature matrix for each day is generated the two first approaches are useless and the third one is tested. It provided perfect results without uncertainty, 100% localisation. It means that each day the group that was signalled suitable to have a leakage contained the node with leakage. Of course these groups were all different. Signature matrix is adapted to boundary conditions. Thus, there are different probabilities of having a leak in a node. This appears on Table 1. It shows the number of nodes with that have been signalled 0-15 times (each for each day). The shadowed line cell corresponds to the one that contains the node with leakage. In this case, it corresponds always to the 15. It has been done for the 39 groups (one leakage for each) that appeared in the first day.

The number in each cell corresponds to the nodes that have been signalled *n* times in the 15 days, being *n* the number of the row. Columns correspond to the 39 cases tested. In Figure 7, the nodes are presented in grey scale representing the times that have been signalled to be suitable of containing a leakage. The one that contained it appears in the black area.

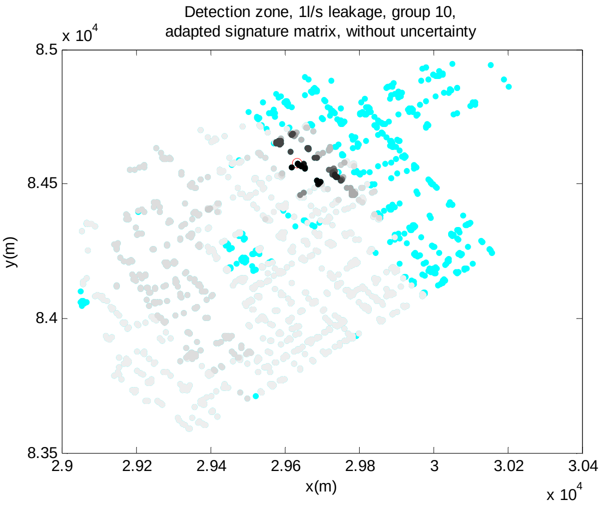

Fig. 7: Localisation of a leak in the correct zone with adapting signature matrix

Table 1: Results using voting criteria adapting signature matrix

| | | Leak | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| Number of detections | **0** | 529 | 316 | 503 | 316 | 986 | 798 | 782 | 884 | 1245 | 489 | 1253 | 1343 | 363 |
| | **1** | 629 | 505 | 639 | 519 | 438 | 615 | 491 | 518 | 325 | 761 | 131 | 64 | 778 |
| | **2** | 175 | 559 | 311 | 545 | 48 | 147 | 95 | 52 | 64 | 279 | 15 | 9 | 345 |
| | **3** | 126 | 88 | 12 | 88 | 18 | 37 | 76 | 5 | 3 | 9 | 29 | 12 | 59 |
| | **4** | 32 | 57 | 11 | 51 | 17 | 15 | 32 | 22 | 0 | 17 | 1 | 6 | 22 |
| | **5** | 19 | 39 | 21 | 37 | 30 | 0 | 63 | 12 | 0 | 15 | 0 | 7 | 33 |
| | **6** | 54 | 35 | 9 | 11 | 11 | 15 | 30 | 11 | 0 | 9 | 16 | 2 | 13 |
| | **7** | 31 | 8 | 5 | 39 | 7 | 0 | 14 | 5 | 0 | 4 | 117 | 6 | 10 |
| | **8** | 9 | 2 | 15 | 3 | 3 | 0 | 13 | 9 | 0 | 5 | 2 | 9 | 2 |
| | **9** | 1 | 1 | 31 | 0 | 12 | 0 | 2 | 6 | 1 | 5 | 11 | 94 | 4 |
| | **10** | 10 | 2 | 56 | 1 | 18 | 0 | 6 | 17 | 0 | 1 | 5 | 46 | 9 |
| | **11** | 3 | 1 | 15 | 2 | 10 | 2 | 7 | 55 | 0 | 26 | 11 | 7 | 0 |
| | **12** | 1 | 6 | 8 | 1 | 6 | 0 | 6 | 33 | 1 | 0 | 2 | 10 | 0 |
| | **13** | 9 | 18 | 2 | 9 | 0 | 1 | 15 | 10 | 0 | 5 | 4 | 2 | 0 |
| | **14** | 5 | 0 | 1 | 0 | 29 | 6 | 6 | 0 | 0 | 6 | 8 | 20 | 0 |
| | **15** | 7 | 3 | 1 | 18 | 7 | 4 | 2 | 1 | 1 | 9 | 35 | 3 | 2 |

| | | Leak | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** |
| Number of detections | **0** | 357 | 1311 | 1363 | 354 | 489 | 539 | 477 | 490 | 1396 | 1430 | 996 | 959 | 807 |
| | **1** | 869 | 66 | 44 | 853 | 552 | 575 | 778 | 512 | 122 | 92 | 372 | 347 | 533 |
| | **2** | 304 | 21 | 13 | 317 | 156 | 179 | 255 | 182 | 12 | 11 | 82 | 132 | 134 |
| | **3** | 50 | 83 | 5 | 57 | 170 | 59 | 65 | 47 | 55 | 52 | 100 | 90 | 53 |
| | **4** | 14 | 18 | 6 | 14 | 10 | 70 | 4 | 141 | 3 | 5 | 39 | 15 | 50 |
| | **5** | 12 | 119 | 9 | 11 | 7 | 53 | 5 | 4 | 7 | 8 | 2 | 23 | 1 |
| | **6** | 9 | 5 | 5 | 11 | 5 | 95 | 12 | 25 | 3 | 7 | 4 | 8 | 7 |
| | **7** | 9 | 2 | 35 | 7 | 25 | 18 | 33 | 31 | 4 | 1 | 2 | 19 | 1 |
| | **8** | 1 | 0 | 11 | 1 | 29 | 12 | 0 | 17 | 3 | 11 | 5 | 18 | 23 |
| | **9** | 0 | 0 | 23 | 0 | 53 | 9 | 2 | 25 | 11 | 3 | 1 | 11 | 9 |
| | **10** | 2 | 4 | 0 | 3 | 69 | 4 | 1 | 21 | 2 | 0 | 6 | 14 | 14 |
| | **11** | 4 | 8 | 9 | 4 | 11 | 8 | 4 | 46 | 2 | 5 | 15 | 1 | 0 |
| | **12** | 6 | 1 | 0 | 2 | 36 | 3 | 1 | 35 | 0 | 0 | 2 | 1 | 0 |
| | **13** | 2 | 1 | 17 | 4 | 9 | 8 | 0 | 23 | 9 | 11 | 8 | 1 | 6 |
| | **14** | 0 | 0 | 44 | 1 | 15 | 1 | 1 | 24 | 0 | 0 | 2 | 0 | 0 |
| | **15** | 1 | 1 | 56 | 1 | 4 | 7 | 2 | 17 | 11 | 4 | 4 | 1 | 2 |

| | | Leak | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **27** | **28** | **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** |
| Number of detections | **0** | 596 | 769 | 993 | 594 | 597 | 593 | 544 | 593 | 490 | 490 | 593 | 877 | 568 |
| | **1** | 477 | 585 | 293 | 471 | 597 | 593 | 429 | 593 | 512 | 518 | 595 | 357 | 610 |
| | **2** | 362 | 208 | 90 | 209 | 79 | 109 | 246 | 109 | 182 | 163 | 116 | 43 | 158 |
| | **3** | 109 | 45 | 97 | 55 | 115 | 76 | 127 | 76 | 47 | 5 | 81 | 105 | 48 |
| | **4** | 56 | 2 | 77 | 120 | 121 | 42 | 176 | 42 | 140 | 59 | 40 | 34 | 50 |
| | **5** | 9 | 2 | 34 | 19 | 43 | 16 | 28 | 18 | 5 | 43 | 34 | 11 | 30 |
| | **6** | 0 | 7 | 21 | 32 | 12 | 44 | 24 | 40 | 25 | 76 | 32 | 4 | 52 |
| | **7** | 1 | 5 | 3 | 35 | 33 | 19 | 16 | 26 | 35 | 74 | 13 | 7 | 35 |
| | **8** | 2 | 2 | 8 | 33 | 11 | 51 | 2 | 43 | 18 | 16 | 27 | 0 | 18 |
| | **9** | 6 | 1 | 10 | 14 | 9 | 9 | 2 | 16 | 22 | 101 | 23 | 2 | 8 |
| | **10** | 1 | 1 | 4 | 23 | 13 | 6 | 5 | 6 | 38 | 9 | 9 | 40 | 14 |
| | **11** | 6 | 5 | 2 | 5 | 2 | 4 | 23 | 3 | 25 | 79 | 27 | 139 | 8 |
| | **12** | 1 | 7 | 2 | 16 | 6 | 11 | 6 | 23 | 30 | 3 | 7 | 13 | 30 |
| | **13** | 1 | 0 | 5 | 8 | 0 | 40 | 4 | 22 | 38 | 2 | 27 | 1 | 10 |
| | **14** | 8 | 0 | 0 | 1 | 1 | 12 | 6 | 21 | 7 | 1 | 5 | 6 | 0 |
| | **15** | 5 | 1 | 1 | 5 | 1 | 15 | 2 | 9 | 26 | 1 | 11 | 1 | 1 |

Uncertainty was calculated using the monthly variation for a demand. It was of 18% of the total demand. Uncertainty was introduced as a coefficient multiplied to the demand of each node generated as a random number between 0.8 and 1.2. The global demand has been kept equal. The total demand affects greatly to the sensitivity.

Results are presented in Table 2 and Figure 8. In this case the leaky node is not always exactly in the most signalled group and the dark grey in the figure does not correspond to 15 but to 13 days. In Figure 8 the gray scale is lighter because there are less correct detections due to the uncertainty.

Table 2: Results using voting criteria

|  |  | Leak |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| Number of detections | **0** | 810 | 504 | 688 | 504 | 1027 | 1185 | 539 | 688 | 794 | 754 | 1223 | 1343 | 765 |
|  | **1** | 412 | 646 | 754 | 646 | 207 | 370 | 418 | 708 | 517 | 762 | 164 | 67 | 778 |
|  | **2** | 219 | 270 | 15 | 270 | 233 | 28 | 414 | 63 | 326 | 27 | 11 | 6 | 19 |
|  | **3** | 34 | 61 | 5 | 61 | 38 | 20 | 73 | 9 | 0 | 23 | 27 | 12 | 37 |
|  | **4** | 39 | 83 | 26 | 83 | 17 | 9 | 123 | 29 | 0 | 10 | 17 | 3 | 14 |
|  | **5** | 17 | 35 | 25 | 35 | 18 | 0 | 15 | 10 | 0 | 5 | 93 | 9 | 10 |
|  | **6** | 71 | 8 | 19 | 8 | 9 | 0 | 18 | 20 | 0 | 11 | 22 | 9 | 9 |
|  | **7** | 12 | 2 | 20 | 2 | 4 | 1 | 12 | 26 | 0 | 38 | 6 | 9 | 6 |
|  | **8** | 4 | 1 | 42 | 1 | 10 | 17 | 4 | 42 | 1 | 8 | 10 | 94 | 2 |
|  | **9** | 17 | 2 | 20 | 2 | 16 | 3 | 6 | 37 | 0 | 2 | 5 | 46 | 0 |
|  | **10** | 5 | 22 | 26 | 4 | 25 | 4 | 8 | 8 | 1 | 0 | 23 | 7 | 0 |
|  | **11** | 0 | 6 | 0 | 24 | 31 | 3 | 10 | 0 | 1 | 0 | 39 | 10 | 0 |
|  | **12** | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
|  | **13** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 0 |
|  | **14** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |
|  | **15** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  |  | Leak |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** |
| Number of detections | **0** | 645 | 1322 | 1363 | 623 | 637 | 528 | 1121 | 638 | 1272 | 1272 | 833 | 953 | 811 |
|  | **1** | 895 | 66 | 44 | 911 | 554 | 479 | 274 | 561 | 247 | 235 | 535 | 325 | 551 |
|  | **2** | 55 | 10 | 5 | 61 | 161 | 187 | 193 | 152 | 14 | 15 | 82 | 119 | 144 |
|  | **3** | 11 | 83 | 10 | 11 | 22 | 251 | 42 | 23 | 3 | 12 | 100 | 100 | 58 |
|  | **4** | 12 | 18 | 7 | 8 | 3 | 106 | 2 | 3 | 54 | 9 | 39 | 24 | 14 |
|  | **5** | 6 | 13 | 9 | 6 | 1 | 34 | 5 | 1 | 4 | 51 | 2 | 44 | 1 |
|  | **6** | 1 | 111 | 6 | 5 | 6 | 11 | 3 | 30 | 17 | 13 | 4 | 11 | 8 |
|  | **7** | 4 | 1 | 1 | 6 | 28 | 2 | 0 | 40 | 13 | 17 | 2 | 23 | 34 |
|  | **8** | 9 | 1 | 36 | 3 | 29 | 1 | 0 | 32 | 1 | 5 | 5 | 20 | 14 |
|  | **9** | 1 | 0 | 7 | 0 | 82 | 3 | 0 | 70 | 15 | 11 | 1 | 18 | 2 |
|  | **10** | 1 | 4 | 85 | 6 | 94 | 25 | 0 | 64 | 0 | 0 | 6 | 2 | 3 |
|  | **11** | 0 | 4 | 58 | 0 | 12 | 10 | 0 | 26 | 0 | 0 | 18 | 1 | 0 |
|  | **12** | 0 | 7 | 9 | 0 | 11 | 3 | 0 | 0 | 0 | 0 | 9 | 0 | 0 |
|  | **13** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 |
|  | **14** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | **15** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  |  | Leak |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **27** | **28** | **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** |
| Number of detections | **0** | 596 | 596 | 597 | 594 | 597 | 595 | 526 | 593 | 638 | 638 | 595 | 1165 | 570 |
|  | **1** | 477 | 477 | 601 | 478 | 597 | 619 | 464 | 613 | 501 | 511 | 620 | 10 | 636 |
|  | **2** | 362 | 367 | 100 | 270 | 85 | 94 | 197 | 99 | 16 | 21 | 106 | 101 | 116 |
|  | **3** | 103 | 153 | 118 | 68 | 145 | 85 | 120 | 87 | 80 | 6 | 98 | 106 | 88 |
|  | **4** | 29 | 15 | 118 | 50 | 80 | 52 | 192 | 29 | 70 | 0 | 28 | 34 | 54 |
|  | **5** | 37 | 3 | 42 | 34 | 42 | 17 | 68 | 32 | 67 | 101 | 19 | 17 | 50 |
|  | **6** | 5 | 7 | 23 | 45 | 39 | 18 | 13 | 21 | 40 | 151 | 65 | 4 | 47 |
|  | **7** | 3 | 6 | 9 | 30 | 19 | 64 | 8 | 59 | 54 | 23 | 24 | 2 | 12 |
|  | **8** | 7 | 1 | 14 | 24 | 15 | 15 | 8 | 21 | 37 | 100 | 9 | 3 | 41 |
|  | **9** | 5 | 1 | 8 | 10 | 7 | 5 | 3 | 7 | 44 | 9 | 19 | 38 | 10 |
|  | **10** | 8 | 8 | 4 | 10 | 6 | 20 | 2 | 3 | 93 | 73 | 35 | 144 | 16 |
|  | **11** | 7 | 6 | 4 | 10 | 4 | 34 | 16 | 39 | 0 | 4 | 20 | 9 | 0 |
|  | **12** | 1 | 0 | 2 | 12 | 3 | 20 | 14 | 15 | 0 | 3 | 2 | 7 | 0 |
|  | **13** | 0 | 0 | 0 | 5 | 1 | 2 | 9 | 20 | 0 | 0 | 0 | 0 | 0 |
|  | **14** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
|  | **15** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Increasing uncertainty interval, the proposed localization methodology produces poorer results. For a 50% uncertainty leaks were not well localized but they were localized in a neighbour zone.

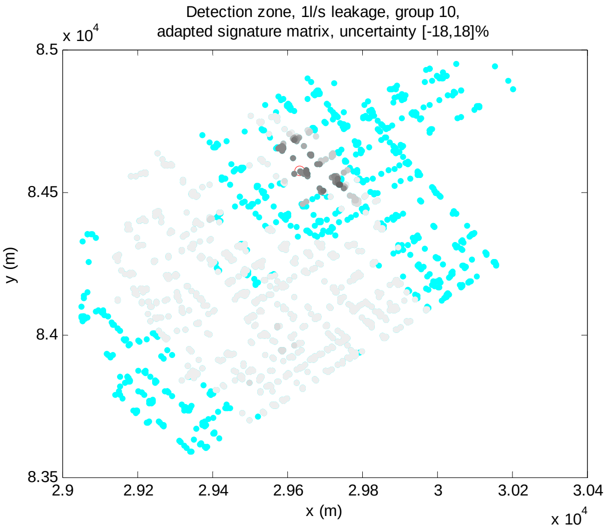

Fig. 8: Localisation of a leak in the correct zone

## 6. CONCLUSIONS

A leakage localization method based on the pressure measurements and sensitivity analysis of nodes in a network has been proposed. The leakage localization methodology is founded in standard model based fault diagnosis well established theory.

In order to maximise the isolability with a reasonable number of sensors an optimal sensor placement methodology based on genetic algorithms is also proposed. The objective function in the minimisation process was the size of the maximum group discriminated.

To assess the validity of the proposed approach, it has been applied to a real DMA of Barcelona network. Models and information were provided by the water company. For this sector (DMA) the sensor placement and the leakage detection and localization methodologies have been applied with successful results even in presence of demand uncertainty.

An issue in the process is to recalculate the sensitivity matrix for each boundary condition using the simulation model because of the high dependence of it to global consumption. This approach is being currently developed using linear parameter varying (LPV) models that consider the consumption as a scheduling variable (Vento, 2009).

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