ON THE NUMERICAL CLASSIFICATION OF THE SINGULARITIES OF ROBOT MANIPULATORS

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August 2012

SINGULARITIES

COMPUTATION CLASSIFICATION?

C-SPACE
INPUT SINGULARITIES
OUTPUT

$$\boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0}$$
$$\boldsymbol{L} \cdot \boldsymbol{m} = \mathbf{0}$$

input output

passive

~~FIKP~~

~~IIKP~~

$$\left.\begin{array}{c}\boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0} \\ \boldsymbol{L}_I^{\mathsf{T}}\boldsymbol{\xi} = \mathbf{0} \\ \|\boldsymbol{\xi}\|^2 = 1\end{array}\right\}$$

$$\left.\begin{array}{c}\boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0} \\ \boldsymbol{L}_O^{\mathsf{T}}\boldsymbol{\xi} = \mathbf{0} \\ \|\boldsymbol{\xi}\|^2 = 1\end{array}\right\}$$

REDUNDANT INPUT

IMPOSSIBLE INPUT

REDUNDANT OUTPUT

IMPOSSIBLE OUTPUT

REDUNDANT PASSIVE MOTION

INCREASED INSTANTANEOUS MOTION

SINGULARITIES

6 TYPES

21 CLASSES

SINGULARITIES

REDUNDANT INPUT

IMPOSSIBLE INPUT

REDUNDANT OUTPUT

IMPOSSIBLE OUTPUT

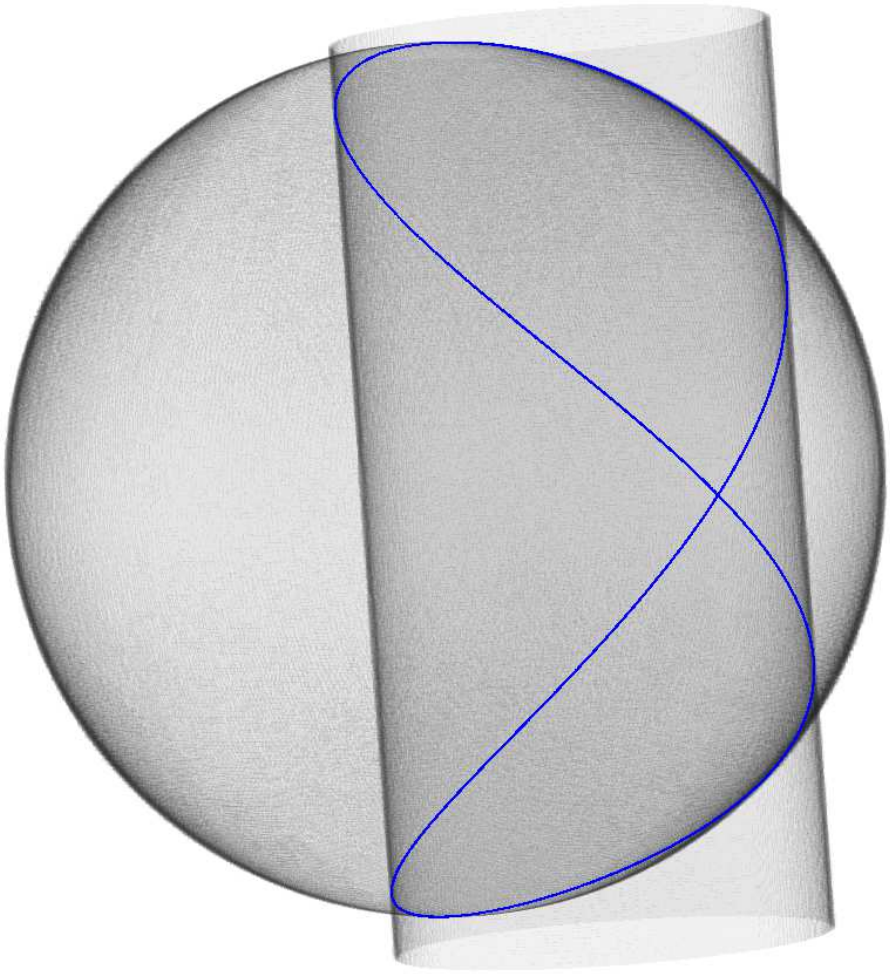
REDUNDANT PASSIVE MOTION

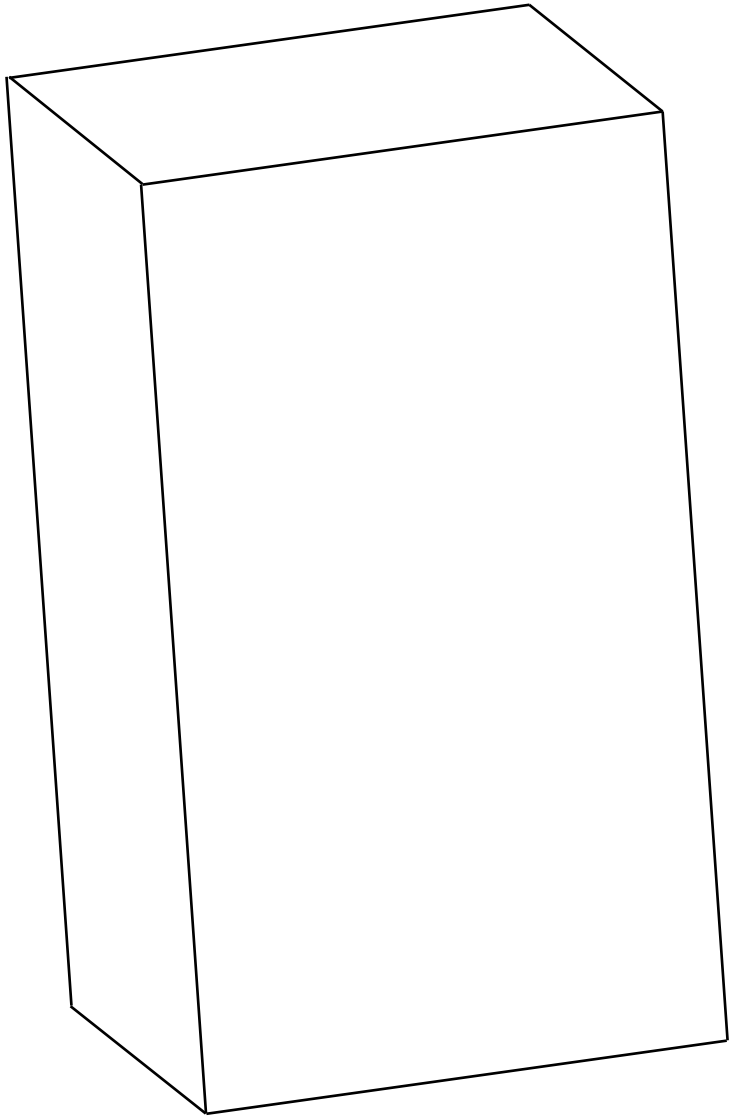
INCREASED INSTANTANEOUS MOTION

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
RPM			Y	Y			Y
RI *and* RPM			Y		Y		Y
RO *and* RPM			Y			Y	Y
RI *and* RO *and* RPM			Y	Y	Y	Y	Y

NUMERICAL COMPUTATION ⟶ CLASSIFICATION

NUMERICAL COMPUTATION





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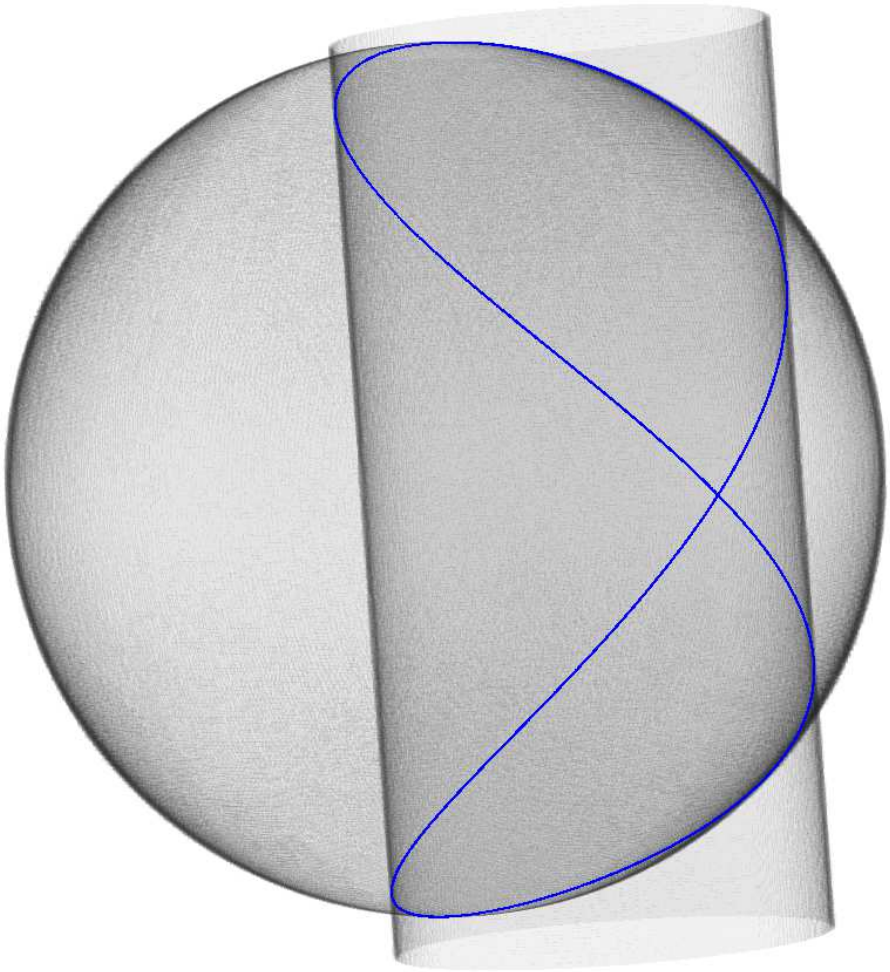

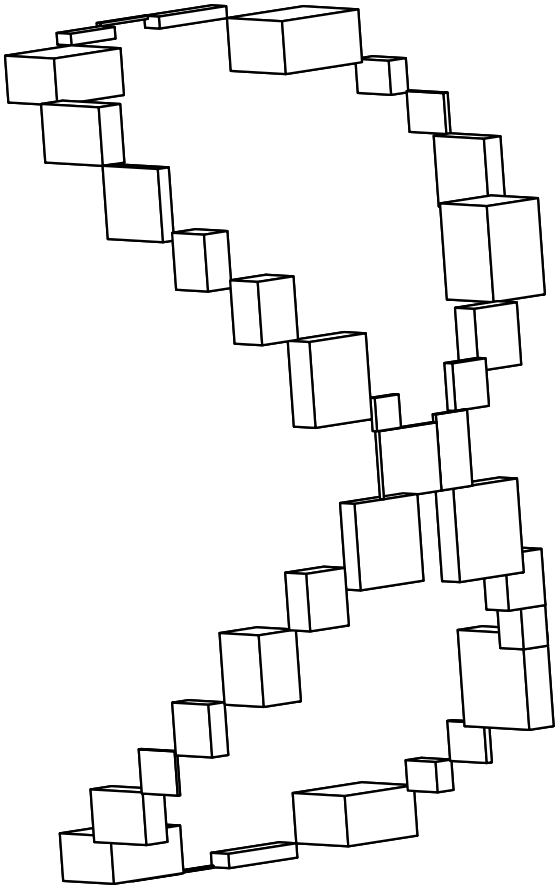

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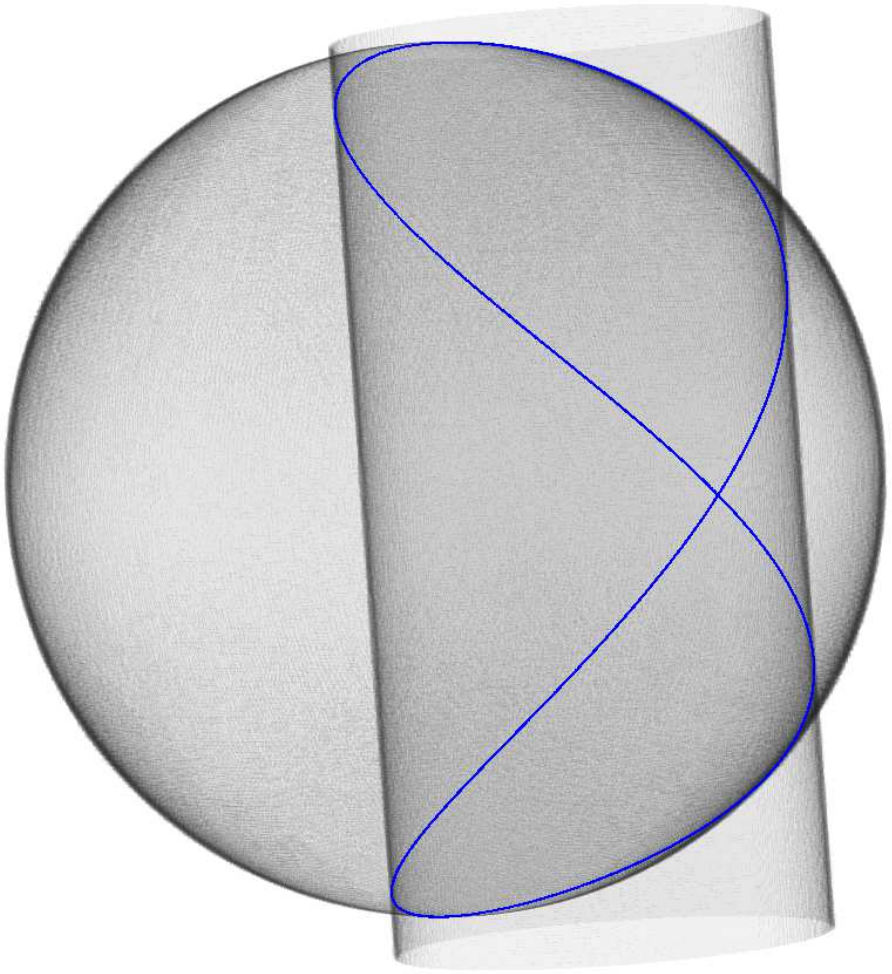

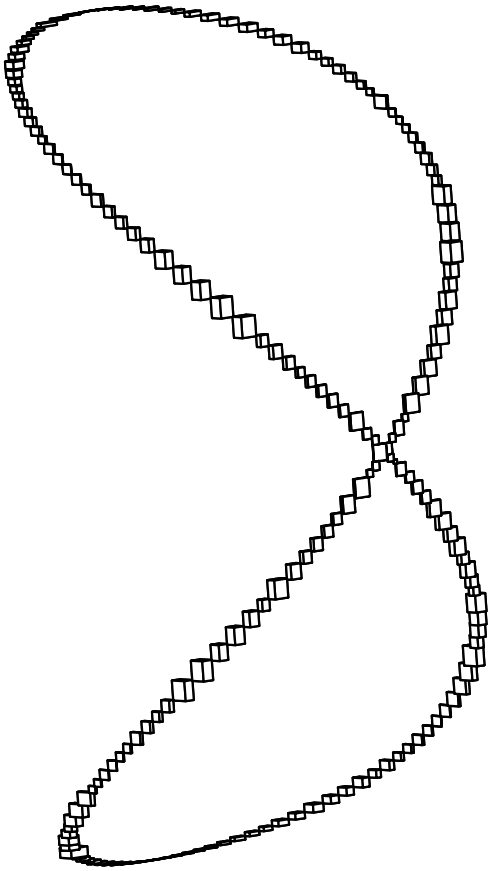

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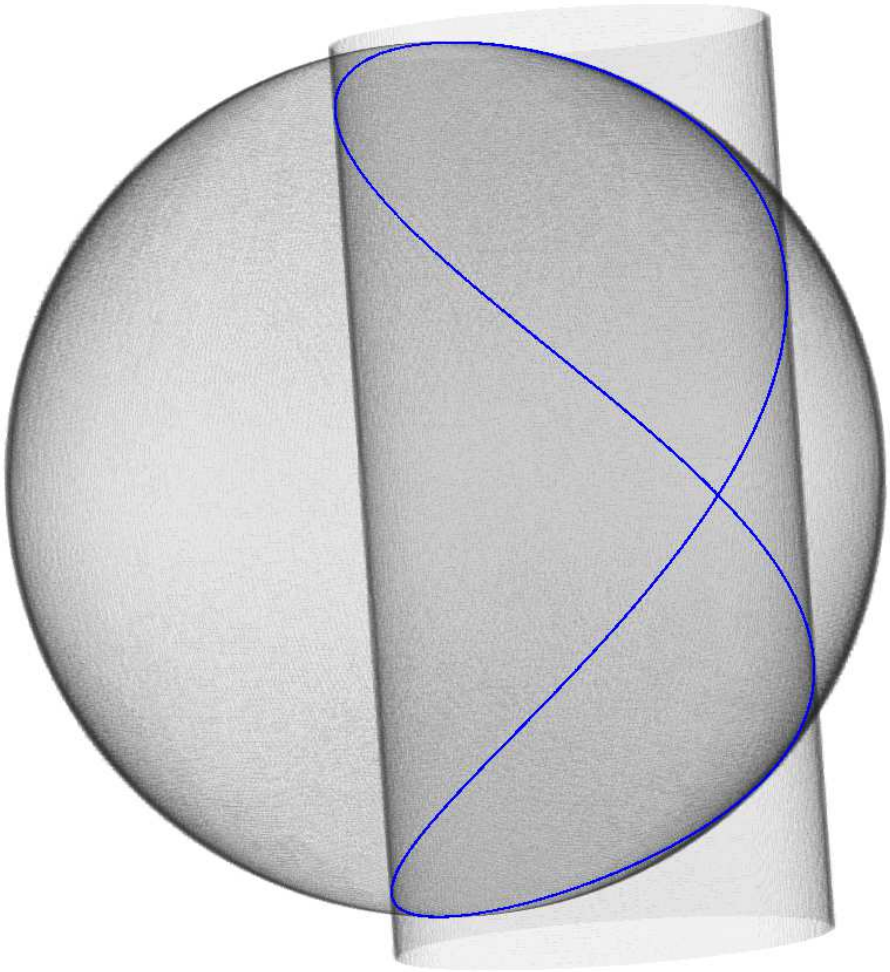
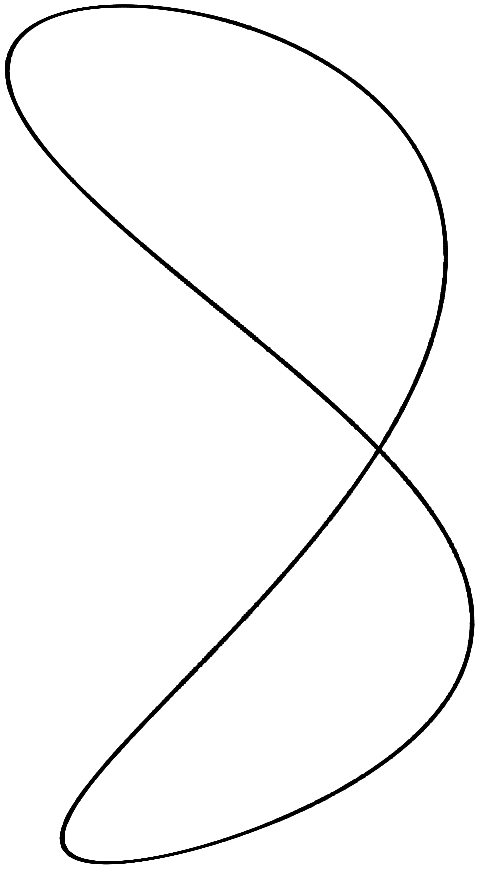
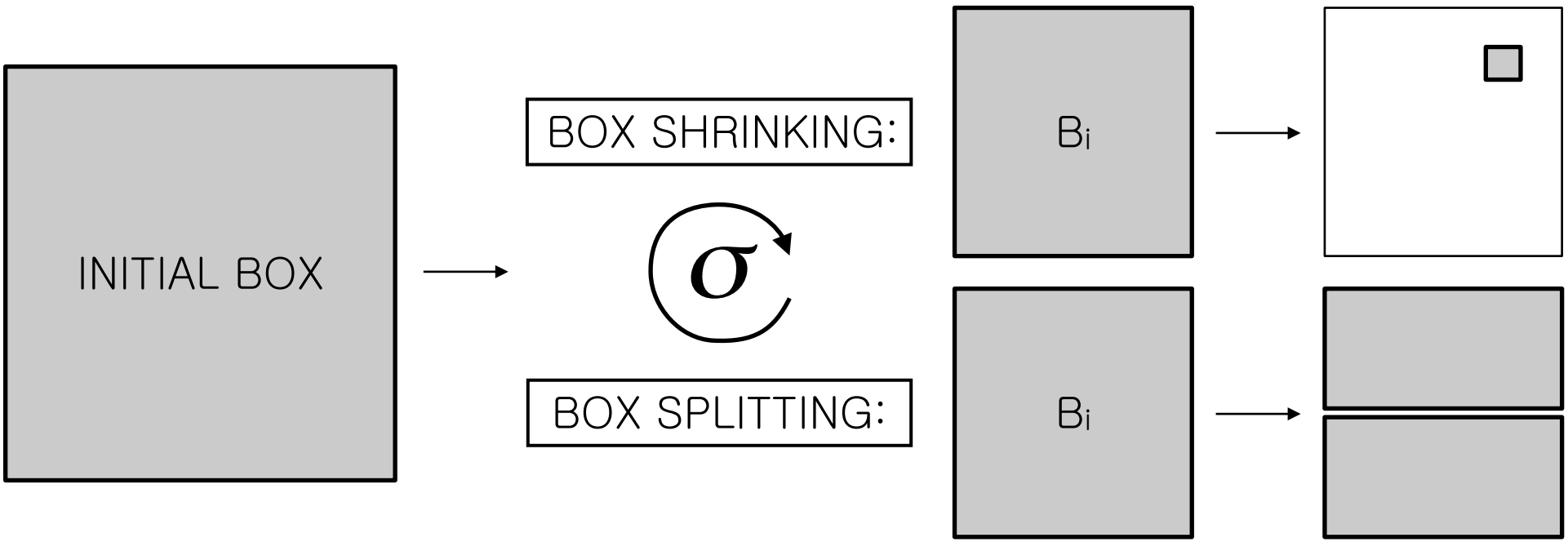


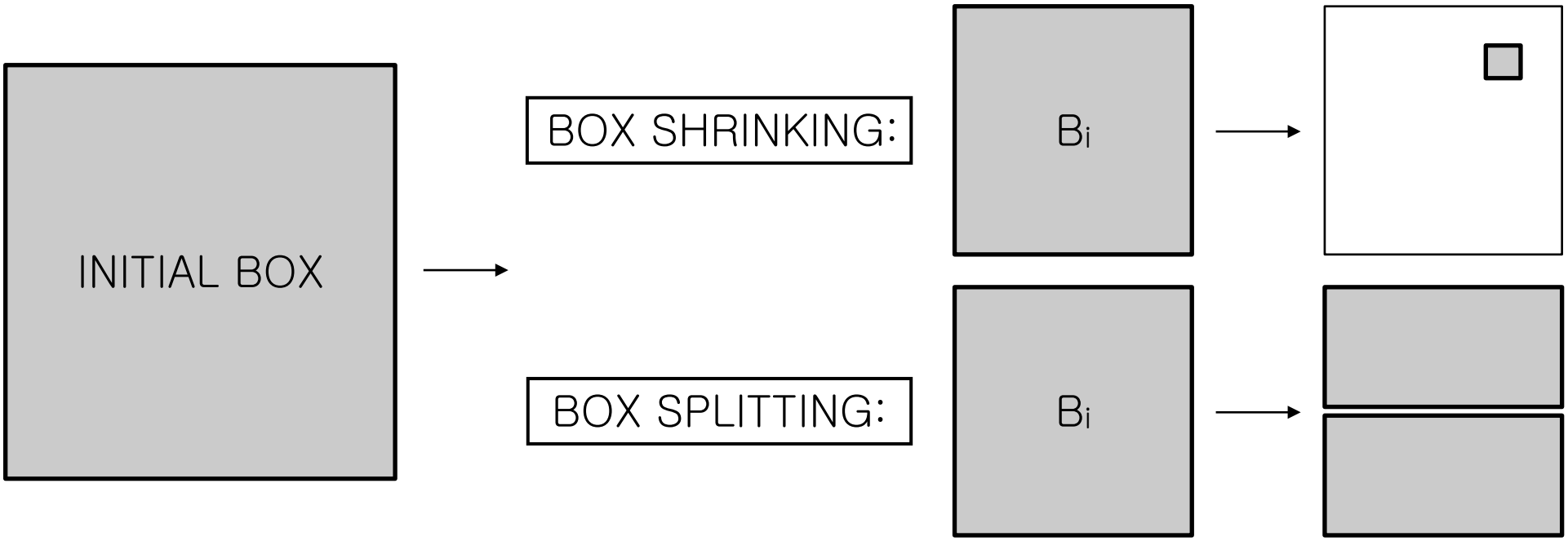


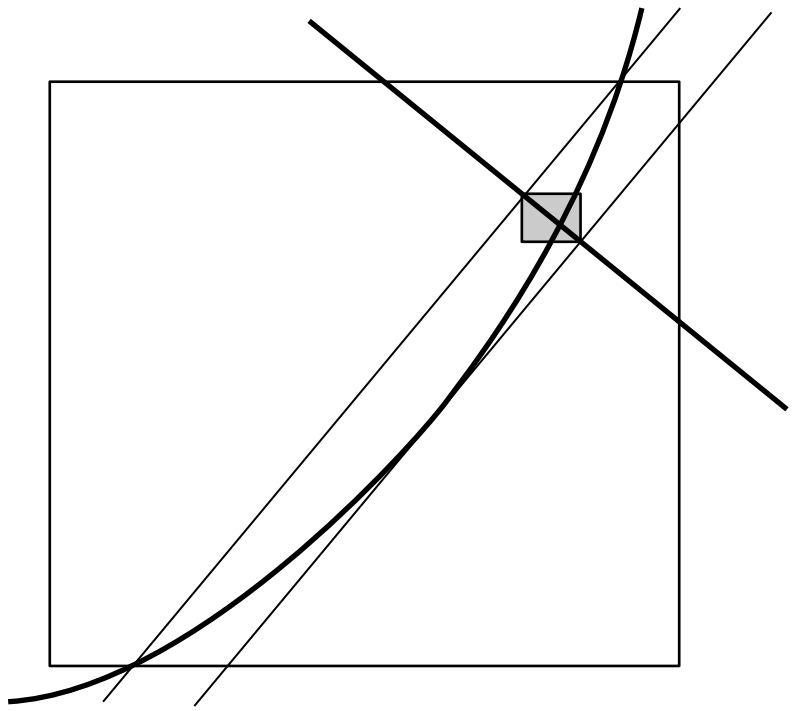
NUMERICAL COMPUTATION

$$\boldsymbol{F}(\boldsymbol{y}) = \boldsymbol{0}$$

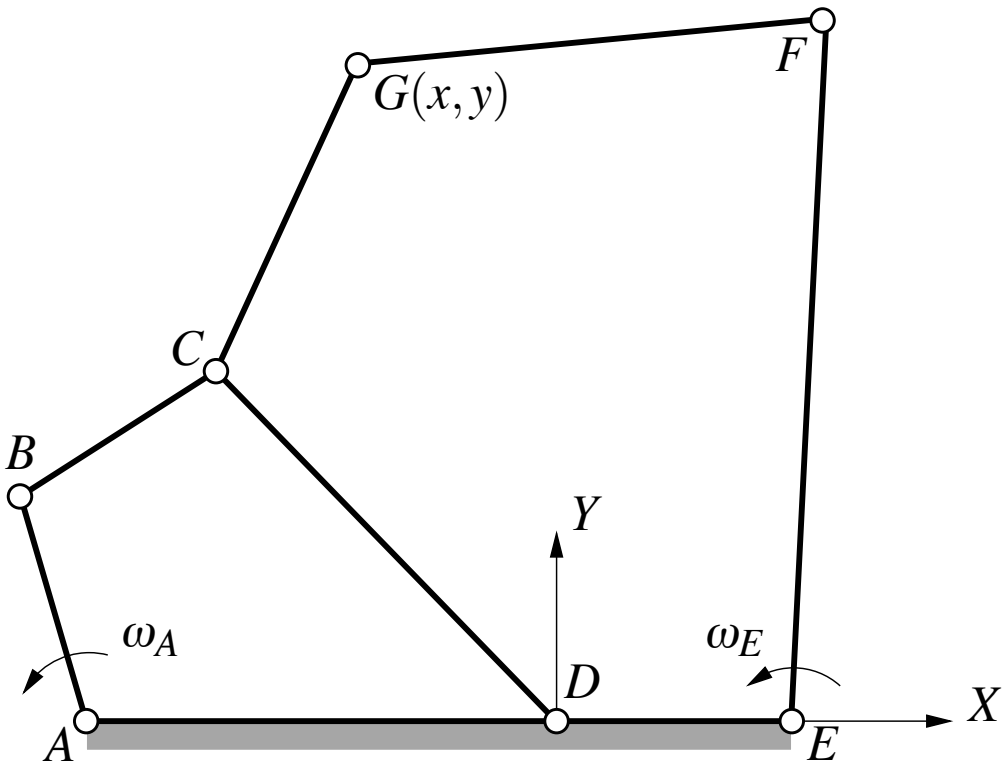


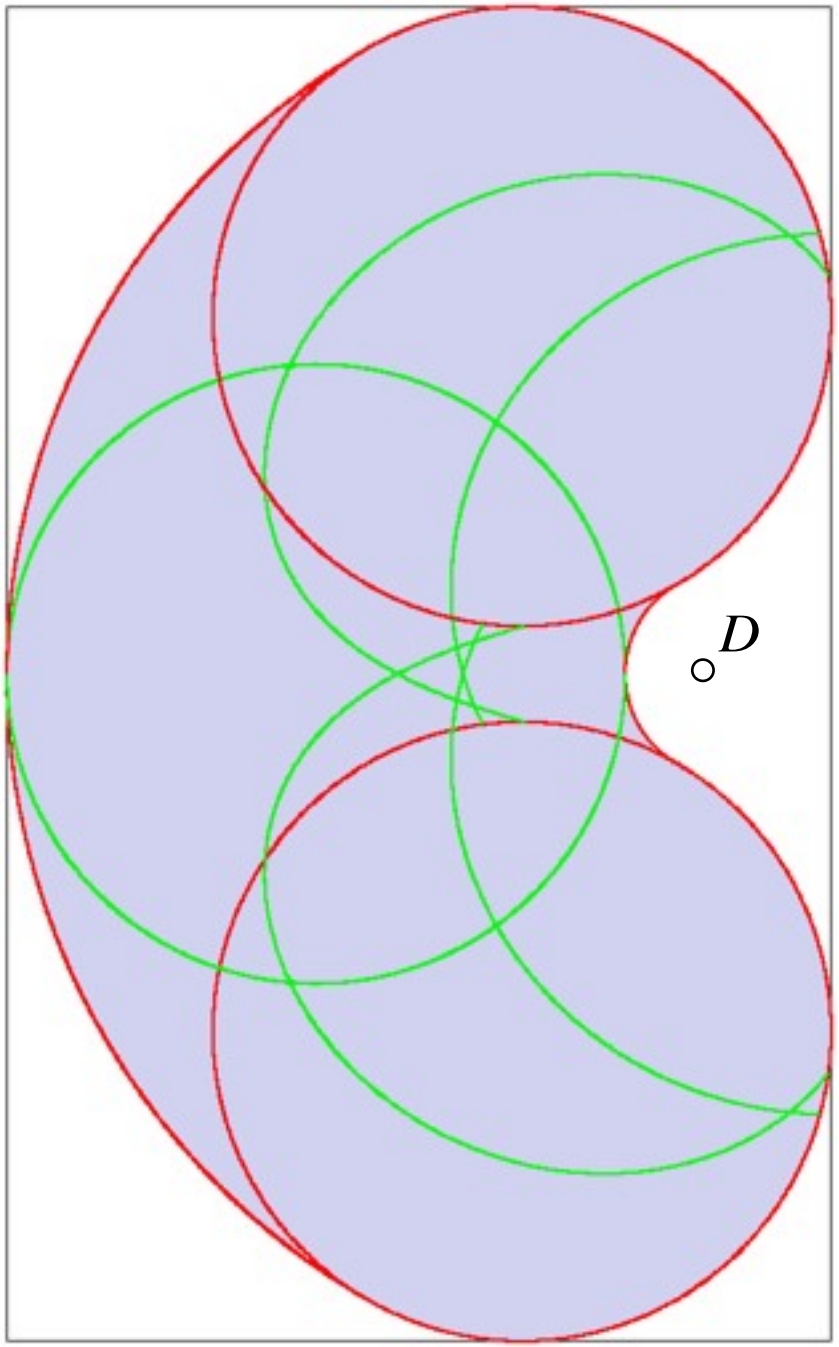
NUMERICAL COMPUTATION



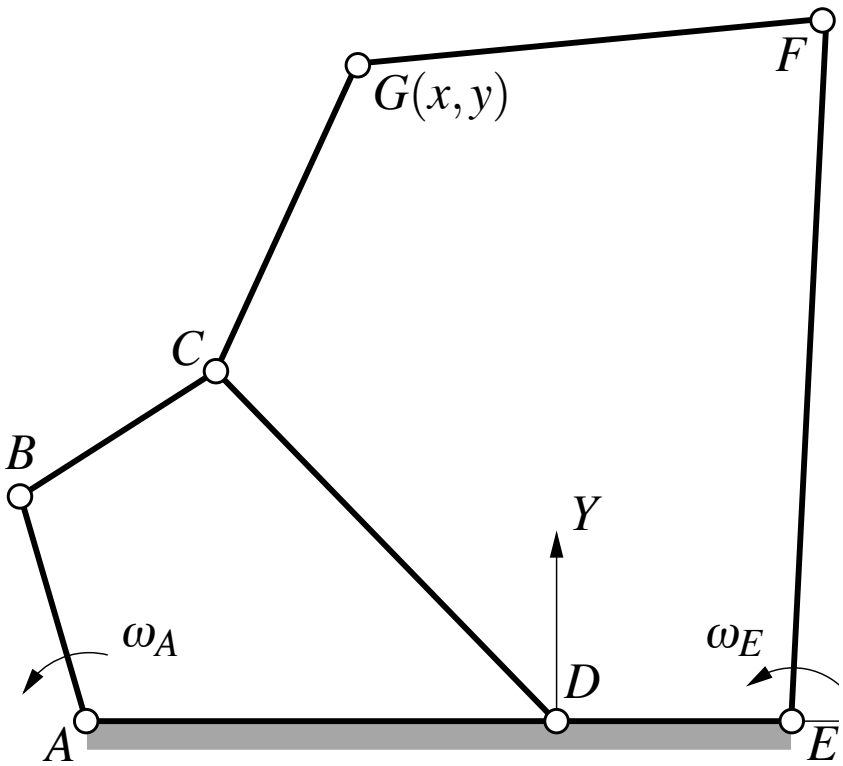


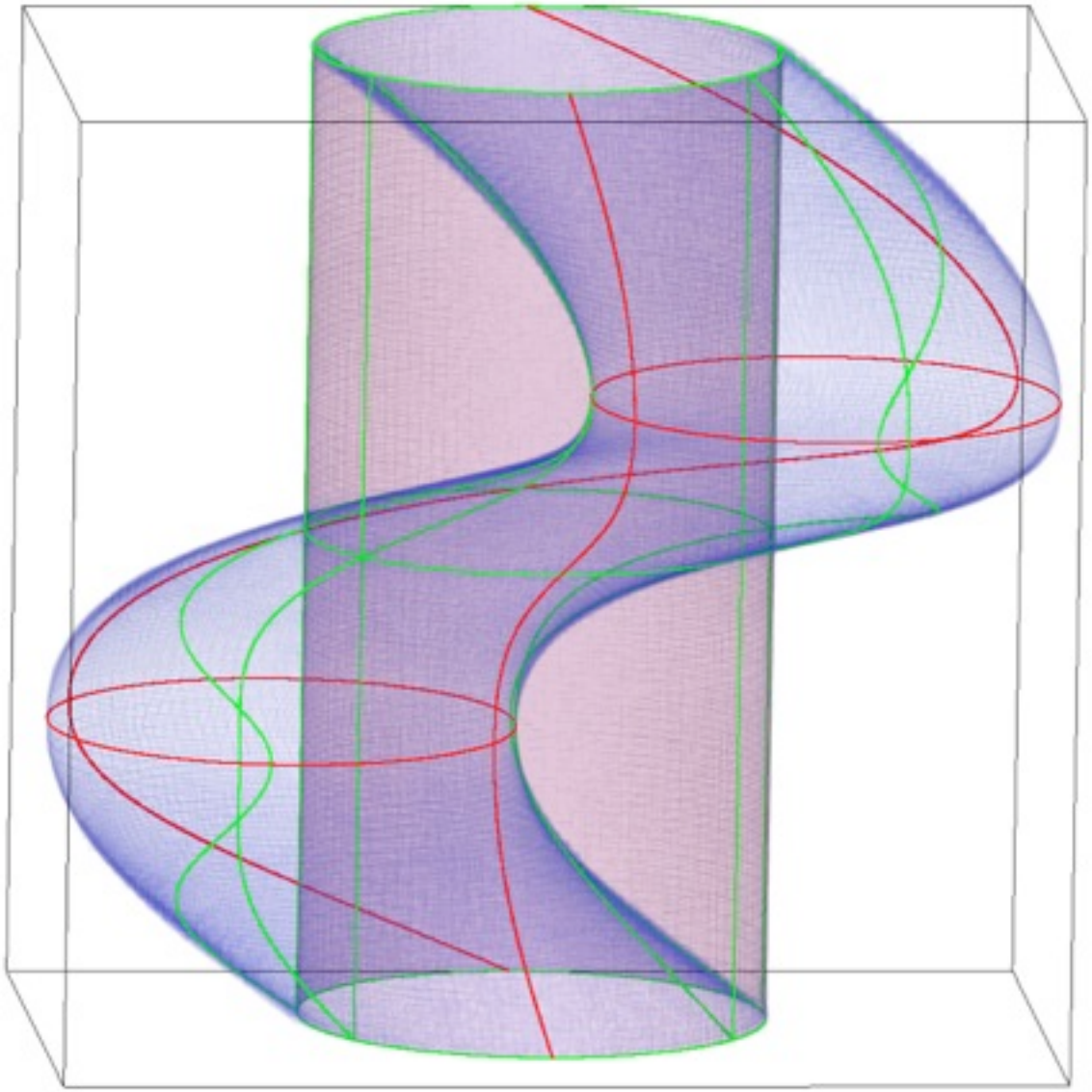
NUMERICAL COMPUTATION





NUMERICAL COMPUTATION





CLASSIFICATION

CONDITIONS OF SINGULARITY

(i) $\boldsymbol{q} \in \{RI\} \iff rank\ \boldsymbol{L}_O < rank\ \boldsymbol{L}_p + n,$

(ii) $\boldsymbol{q} \in \{RO\} \iff rank\ \boldsymbol{L}_I < rank\ \boldsymbol{L}_p + n,$

(iii) $\boldsymbol{q} \in \{RPM\} \iff rank\ \boldsymbol{L}_p < N - n,$

(iv) $\boldsymbol{q} \in \{II\} \iff rank\ \boldsymbol{L}_I < rank\ \boldsymbol{L},$

(v) $\boldsymbol{q} \in \{IO\} \iff rank\ \boldsymbol{L}_O < rank\ \boldsymbol{L},$

(vi) $\boldsymbol{q} \in \{IIM\} \iff rank\ \boldsymbol{L} < N,$

(vii) $\boldsymbol{q} \in \{RI\}$ or $\boldsymbol{q} \in \{RPM\} \iff \boldsymbol{q} \in \{IO\}$ or $\boldsymbol{q} \in \{IIM\} \iff \boldsymbol{L}_O$ is singular,

(viii) $\boldsymbol{q} \in \{RO\}$ or $\boldsymbol{q} \in \{RPM\} \iff \boldsymbol{q} \in \{II\}$ or $\boldsymbol{q} \in \{IIM\} \iff \boldsymbol{L}_I$ is singular

CLASSIFICATION

CONDITIONS OF SINGULARITY

(i) $\boldsymbol{q} \in \{RI\} \iff rank\ \boldsymbol{L}_O < rank\ \boldsymbol{L}_p + n,$

(ii) $\boldsymbol{q} \in \{RO\} \iff rank\ \boldsymbol{L}_I < rank\ \boldsymbol{L}_p + n,$

(iii) $\boldsymbol{q} \in \{RPM\} \iff rank\ \boldsymbol{L}_p < N - n,$

(iv) $\boldsymbol{q} \in \{II\} \iff rank\ \boldsymbol{L}_I < rank\ \boldsymbol{L},$

(v) $\boldsymbol{q} \in \{IO\} \iff rank\ \boldsymbol{L}_O < rank\ \boldsymbol{L},$

(vi) $\boldsymbol{q} \in \{IIM\} \iff rank\ \boldsymbol{L} < N,$

(vii) $\boldsymbol{q} \in \{RI\}$ or $\boldsymbol{q} \in \{RPM\} \iff \boldsymbol{q} \in \{IO\}$ or $\boldsymbol{q} \in \{IIM\} \iff \boldsymbol{L}_O$ is singular,

(viii) $\boldsymbol{q} \in \{RO\}$ or $\boldsymbol{q} \in \{RPM\} \iff \boldsymbol{q} \in \{II\}$ or $\boldsymbol{q} \in \{IIM\} \iff \boldsymbol{L}_I$ is singular

$$\text{(iii)} \longrightarrow \left.\begin{array}{c} \boldsymbol{\Phi}(\boldsymbol{q}) = \boldsymbol{0} \\ \boldsymbol{L}_P \boldsymbol{\xi}_P = \boldsymbol{0} \\ \|\boldsymbol{\xi}_P\|^2 = 1 \end{array}\right\} \quad \boxed{\text{T1}}\ \text{RPM}$$

CLASSIFICATION

T1	RPM
T2	IIM
T3	RI+RPM
T4	RO+RPM
T5	II+IIM
T6	IO+IIM
T7	II,RI,IO,RO
T8	II,RI,IO,RO

CLASSIFICATION

- T1 RPM
- T2 IIM
- T3 RI+RPM
- T4 RO+RPM
- T5 II+IIM
- T6 IO+IIM
- T7 II,RI,IO,RO
- T8 II,RI,IO,RO

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
~~RPM~~			~~Y~~	~~Y~~			~~Y~~
~~RI *and* RPM~~			~~Y~~		~~Y~~		~~Y~~
~~RO *and* RPM~~			~~Y~~			~~Y~~	~~Y~~
~~RI *and* RO *and* RPM~~			~~Y~~	~~Y~~	~~Y~~	~~Y~~	~~Y~~

CLASSIFICATION

T1 RPM

T2 IIM

T3 RI+RPM

T4 RO+RPM

T5 II+IIM

T6 IO+IIM

T7 II,RI,IO,RO

T8 II,RI,IO,RO

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
RPM			Y	Y			Y
RI *and* RPM			Y		Y		Y
RO *and* RPM			Y			Y	Y
RI *and* RO *and* RPM			Y	Y	Y	Y	Y

CLASSIFICATION

T1 RPM

T2 IIM

T3 RI+RPM

T4 RO+RPM

T5 II+IIM

T6 IO+IIM

T7 II,RI,IO,RO

T8 II,RI,IO,RO

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
~~RPM~~			~~Y~~	~~Y~~			~~Y~~
~~RI *and* RPM~~			~~Y~~		~~Y~~		~~Y~~
~~RO *and* RPM~~			~~Y~~			~~Y~~	~~Y~~
~~RI *and* RO *and* RPM~~			~~Y~~	~~Y~~	~~Y~~	~~Y~~	~~Y~~

CLASSIFICATION

T1 RPM

T2 IIM

T3 RI+RPM

T4 RO+RPM

T5 II+IIM

T6 IO+IIM

T7 II,RI,IO,RO

T8 II,RI,IO,RO

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
RPM			Y	Y			Y
~~RI *and* RPM~~			~~Y~~		~~Y~~		~~Y~~
~~RO *and* RPM~~			~~Y~~			~~Y~~	~~Y~~
~~RI *and* RO *and* RPM~~			~~Y~~	~~Y~~	~~Y~~	~~Y~~	~~Y~~

CLASSIFICATION

T1 RPM

T2 IIM

T3 RI+RPM

T4 RO+RPM

T5 II+IIM

T6 IO+IIM

T7 II,RI,IO,RO

T8 II,RI,IO,RO

T1+T2

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
RPM			Y	$\boxed{Y}$			Y
~~RI *and* RPM~~			~~Y~~		~~Y~~		~~Y~~
~~RO *and* RPM~~			~~Y~~			~~Y~~	~~Y~~
~~RI *and* RO *and* RPM~~			~~Y~~	~~Y~~	~~Y~~	~~Y~~	~~Y~~

CLASSIFICATION

T1 RPM

T2 IIM

T3 RI+RPM

T4 RO+RPM

T5 II+IIM

T6 IO+IIM

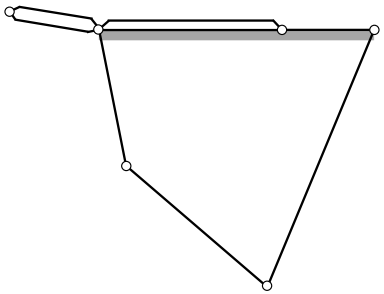
T7 II,RI,IO,RO

T8 II,RI,IO,RO

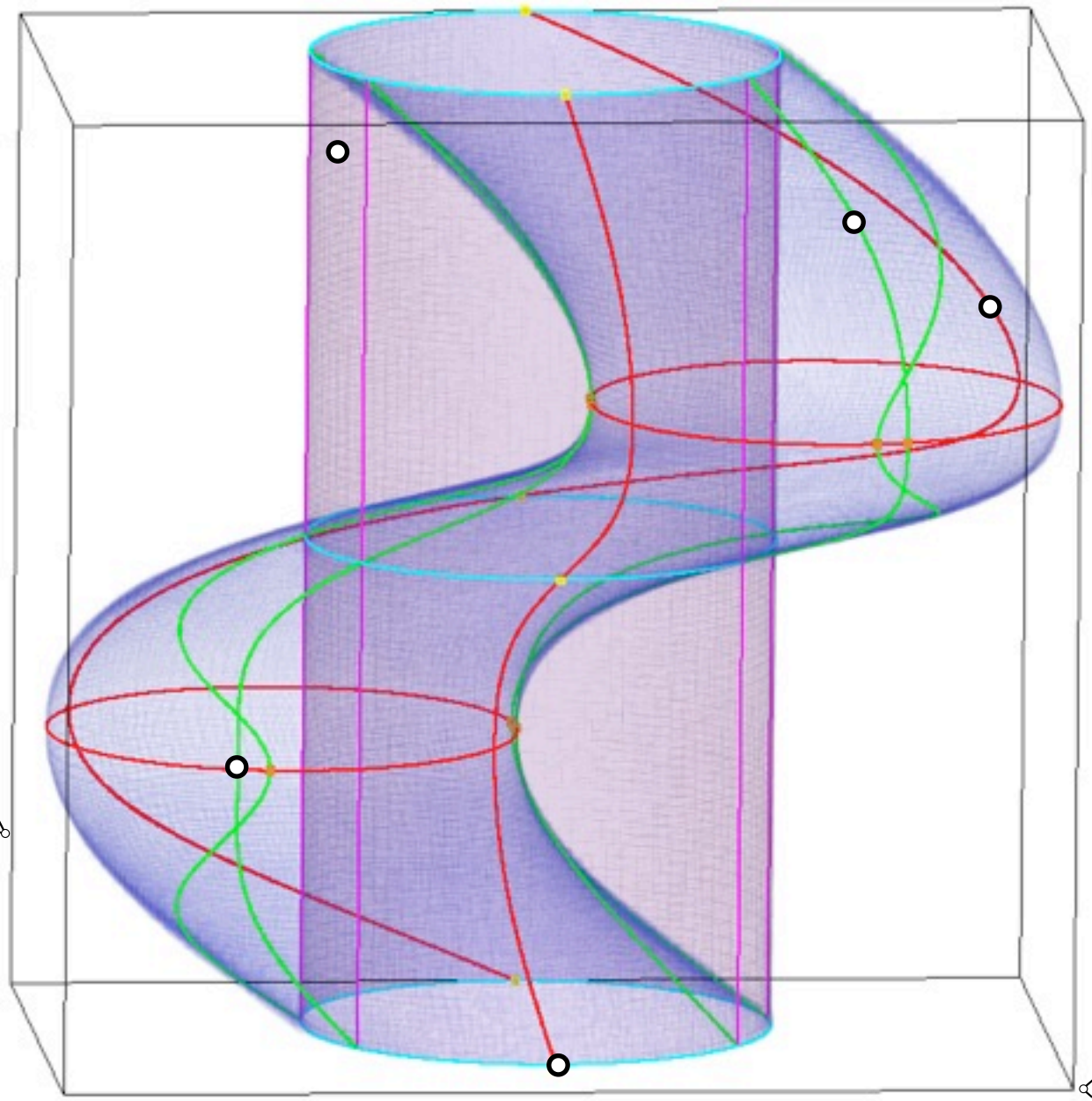
T1+T5+T6

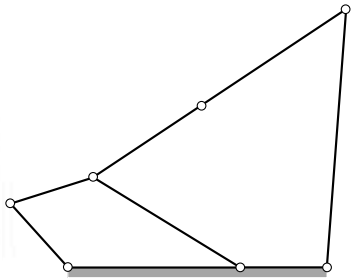
	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
RPM			Y	Y			Y
~~RI *and* RPM~~			~~Y~~		~~Y~~		~~Y~~
~~RO *and* RPM~~			~~Y~~			~~Y~~	~~Y~~
~~RI *and* RO *and* RPM~~			~~Y~~	~~Y~~	~~Y~~	~~Y~~	~~Y~~

CLASSIFICATION

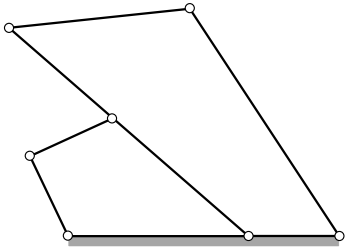


{RI,IO}

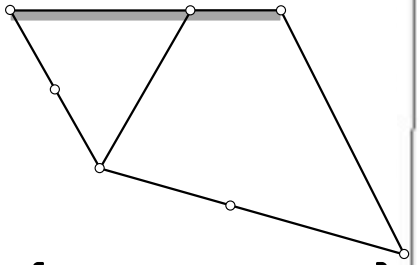




{RO,II}

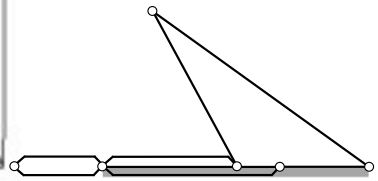


{RI,IO}



{RI,RO,II,IO}

128000 BOXES



{RI,RPM,IO,IIM}

CONCLUSION

CLASSIFICATION OF THE SINGULARITY SET

COMPUTATION

$$\boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0}$$

$$\boldsymbol{L} \cdot \boldsymbol{m} = \mathbf{0}$$

~~FIKP~~

~~IIKP~~

$$\boldsymbol{F}(\boldsymbol{y}) = \mathbf{0}$$

COLLECTION OF BOXES

COMPUTATION

	IO	II	IO *and* II	IIM	IO *and* IIM	II *and* IIM	IO *and* II *and* IIM
RI	Y						
RO		Y					
RI *and* RO			Y	Y	Y	Y	Y
RPM			Y	Y			Y
RI *and* RPM			Y		Y		Y
~~RO *and* RPM~~			~~Y~~			~~Y~~	~~Y~~
~~RI *and* RO *and* RPM~~			~~Y~~	~~Y~~	~~Y~~	~~Y~~	~~Y~~

SEQUENCE OF TESTS

NUMERICAL METHOD BASED ON LINEAR RELAXATIONS