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INVERSE KINEMATICS BY DISTANCE MATRIX COMPLETION

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ABSTRACT - This paper characterizes a family of serial robots whose inverse kinematics can be translated into a system of distance constraints that can be solved using a sequence of constructive operations taking as fixed reference either a triangle or a tetrahedron. The relevance of the obtained family of robots is established when it is shown to contain the best-known commercial serial robots.

KEYWORDS - Cayley-Menger determinants, position analysis of robots.

INTRODUCTION

The Theory of Distance Geometry [1] allows coordinate-free formulations for most position analysis problems. Using such formulations, the inverse kinematics of a serial robot can be performed by using a *distance matrix*, one whose entries are squared distances between pairs of points selected on the axes of the robot. While some of these distances are known (such as the distances between points on the same axis or between points on consecutive axes), many others are unknown. Then, finding all solutions to an inverse kinematics problem boils down to finding values for these unknown distances that permit completing the matrix into a "proper" Euclidean distance matrix [6]. If, by any means, the unknown distances are obtained, one can then easily assign coordinates to the selected points and trivially derive the possible configurations of the robot.

The determination of all values for the unknown distances is usually done via a *bound smoothing* process: a large range is initially assigned to the unknowns and their bounds are progressively reduced in an iterative manner, by applying triangular inequalities and other necessary conditions [5]. Finding all possible solutions for a given incomplete distance matrix can be extremely complex in general, as this problem is known to be NP-complete.

In this paper, we focus on a subclass of distance constraint solving problems where the values of all unknown distances can be derived following a constructive process in which the distance matrix is progressively completed by deducing the value of one unknown at a time. In order to identify all robots whose inverse kinematics can be solved in this way, we first characterize the family of distance matrices that encode all serial robots with six degrees of freedom (DoF). Then, we exhaustively search within this family for those matrices that can be completed in a constructive manner taking as fixed reference either a triangle or a tetrahedron. The result is the identification of a family of serial robots that includes the best-known industrial robots.

This paper is organized as follows. First, we describe how to translate an inverse kinematic problem into a distance constraint satisfaction problem. Then, we show how some incomplete distance matrices can be completed by using a sequence of two basic operations. Next, a comprehensive study of all serial robots whose associated distance matrix can be completed using this technique is presented. All these ideas are then applied to the resolution of the inverse kinematics of a Puma 560 manipulator. Finally, we summarize some points deserving further attention.

ROBOTS AND DISTANCE CONSTRAINTS

The position analysis of a mechanism can usually be translated into a set of distance constraints fixing the relative positions between n points $\mathbf{p}_1, \ldots, \mathbf{p}_n$, selected on its links. Such constraints are usually represented by means of a distance matrix $\mathbf{S}$, whose (i, j) entry is $s_{i,j} = \|\mathbf{p}_i - \mathbf{p}_j\|^2$, i.e., the square of the distance between $\mathbf{p}_i$ and $\mathbf{p}_j$. Some matrix entries are a priori known, and the goal is to solve for the remaining unknowns.

We next show how one can perform this translation for any serial manipulator, and how one can compute the coordinates of the selected points, with reference to an absolute frame, once *all* distances in $\mathbf{S}$ have been solved for.

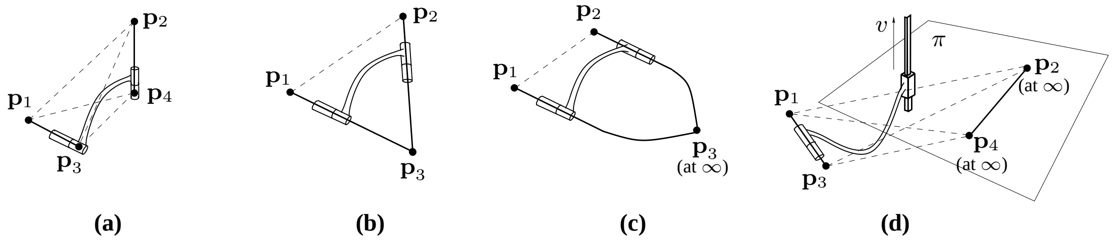


Figure 1: Modelling a link as a set of points with some fixed distances between them.

A serial robot is simply an open chain of seven rigid bodies (the links), pairwise articulated through prismatic or revolute joints. The inverse kinematics problem is to find all valid configurations of this chain that are compatible with a specified pose for the last link, relative to the first. Since the first and last links ar mutually fixed, the problem is equivalent to that of finding the valid configurations of a closed loop of six pairwise articulated links. The translation of each link into distance constraints depends on the type of joints it connects, either prismatic or revolute, and on whether the axes of these joints are skew or concurrent.

A link connecting two skew revolute axes can be modelled by taking two points on each of these axes, and by connecting them all with rigid bars to form a tetrahedron (Fig. 1-a). In this way, for example, a 6R linkage can be modelled as a ring of six pairwise-articulated tetrahedra, as indicated in Fig. 3.

If the two axes of the link are not skew but intersecting, we can economize points and simply model the link as a triangle of fixed distances (Fig. 1-b). Note that the case of parallel revolute axes can be seen as a specialization of the previous one, where the point of intersection is an improper point at infinity in the direction of the axes, instead of a common proper point (Fig. 1-c). This will cause no trouble in the analysis below, as a point at infinity can always be approximated by a proper point, sufficiently far away in some direction.

Similar transformations are applied to a link with one or two prismatic joints: since a translation along direction v can always be seen as a rotation about the line at infinity of any plane π orthogonal to v, we can model a prismatic joint as a revolute joint infinitely far away on this plane. Computationally, we will represent such joint by designating two points on π, placed sufficiently far away along different directions (points $\mathbf{p}_2$ and $\mathbf{p}_4$ in Fig. 1-d).

The procedure detailed above can be used to define an incomplete distance matrix from a serial robot. If, by any means we can complete $\mathbf{S}$, then we will need to assign coordinates to the selected points in order to obtain the actual configurations of the robot. The standard way to do so is by first computing the Gram matrix $\mathbf{G}$ associated with $\mathbf{S}$, whose entries are defined by $g_{ij} = \frac{1}{2}(s_{i,n}^2 + s_{j,n}^2 - s_{i,j}^2)$. The Cholesky factorization of $\mathbf{G}$ into $\mathbf{G} = \mathbf{X}\mathbf{X}^t$ yields the matrix $\mathbf{X}$, whose rows yield unique coordinates for $\mathbf{p}_1, \ldots, \mathbf{p}_n$, up to congruences and mirror transformations. For further details see [6]. Next, we need to obtain all angles between neighboring links from these coordinates.

COMPLETING DISTANCE MATRICES

First of all, let us define

$$D(i_1, \ldots, i_n; j_1, \ldots, j_n) = \begin{vmatrix} 0 & 1 & \ldots & 1 \\ 1 & s_{i_1,j_1} & \ldots & s_{i_1,j_n} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & s_{i_n,j_1} & \ldots & s_{i_n,j_n} \end{vmatrix}, \tag{1}$$

with $s_{i,j} = \|\mathbf{p}_i - \mathbf{p}_j\|^2$. This determinant is known as the *Cayley-Menger bi-determinant* of the point sequences $\mathbf{p}_{i_1}, \ldots, \mathbf{p}_{i_n}$, and $\mathbf{p}_{j_1}, \ldots, \mathbf{p}_{j_n}$. When the two point sequences are the same, it will be convenient to abbreviate $D(i_1, \ldots, i_n; i_1, \ldots, i_n)$ by $D(i_1, \ldots, i_n)$, which is simply called the *Cayley-Menger determinant* of the involved points. The square volume $V^2(\mathbf{p}_0, \ldots, \mathbf{p}_k)$ of a $k-$dimensional simplex defined by the $k+1$ points $\mathbf{p}_0, \ldots, \mathbf{p}_k$

can be expressed entirely in terms of distances as follows:

$$V^2(\mathbf{p}_0, \ldots, \mathbf{p}_k) = \frac{(-1)^{k+1}}{2^k (k!)^2} D(0, \ldots, k).$$

Next, we show how Cayley-Menger determinants play a central role in expressing unknown distances in terms of known ones.

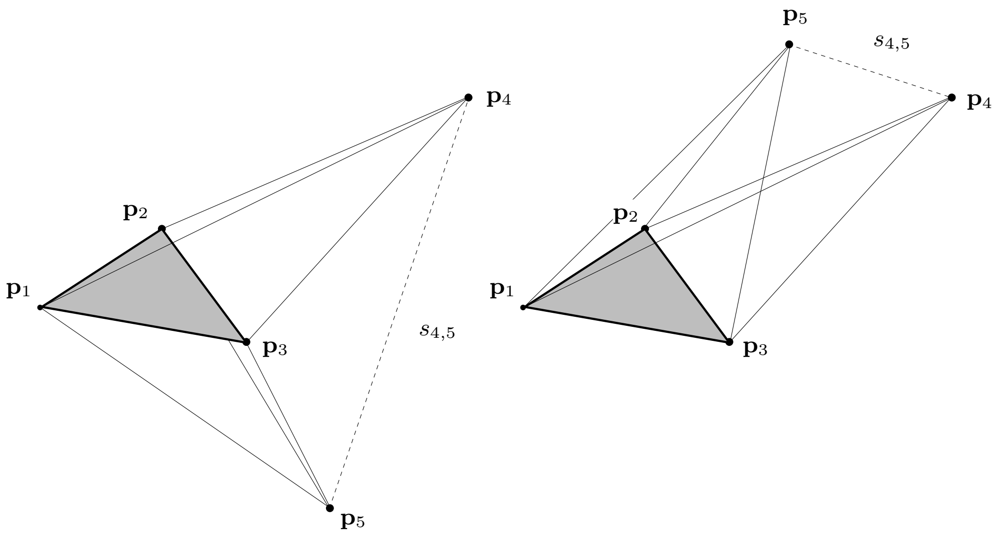


Figure 2: The computation of the distance between $\mathbf{p}_4$ and $\mathbf{p}_5$ from their distances to $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$ leads to two possible solutions that depend on the relative location of $\mathbf{p}_4$ and $\mathbf{p}_5$ with respect to the plane defined by points $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$.

Let us consider five points, $\mathbf{p}_0, \ldots, \mathbf{p}_4$, in $\mathbb{R}^3$, so that only the squared distance between $\mathbf{p}_3$ and $\mathbf{p}_4$, i.e. s_{34}, is unknown. This configuration of points can be seen as two pyramids sharing the same triangular base so that the distance between their apexes is unknown. Clearly, there are two solutions to this problem (Fig. 2). These five points define a simplex in $\mathbb{R}^4$ but, since it is embedded in $\mathbb{R}^3$, its volume is null. Hence, $D(0,1,2,3,4) = 0$. This defines a quadratic equation in s_{34} that can be simplified by applying Jacobi's theorem to the following partition of $D(0,1,2,3,4)$

$$\left| \begin{array}{cccc|cc} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & s_{01} & s_{02} & s_{03} & s_{04} \\ 1 & s_{10} & 0 & s_{12} & s_{13} & s_{14} \\ 1 & s_{20} & s_{21} & 0 & s_{23} & s_{24} \\ \hline 1 & s_{30} & s_{31} & s_{32} & 0 & s_{34} \\ 1 & s_{40} & s_{41} & s_{42} & s_{43} & 0 \end{array} \right|,$$

concluding that $D(0,1,2,3,4) = 0$ yields

$$\frac{D(0,1,2,3)D(0,1,2,4) - D^2(0,1,2,3;0,1,2,4)}{D(0,1,2)} = 0.$$

Assuming that the triangular base defined by $\mathbf{p}_0$, $\mathbf{p}_1$, and $\mathbf{p}_3$ is not degenerate, then

$$D(0,1,2,3;0,1,2,4) = \pm\sqrt{D(0,1,2,3)D(0,1,2,4)}. \quad (2)$$

Since the left hand side of this equation is linear in $s_{34} = \|\mathbf{p}_4 - \mathbf{p}_3\|^2$, there exist two possible solutions for s_{34} that correspond to the two possible signs for the square root in the right hand side.

Now, let us consider six points, $\mathbf{p}_0, \ldots, \mathbf{p}_5$, so that only the distance between $\mathbf{p}_5$ and $\mathbf{p}_4$ is unknown. This configuration is similar to the one in Fig. 2 where the role of the triangle has been substituted by a tetrahedron. These six points define a simplex in $\mathbb{R}^5$ but, since it is embedded in $\mathbb{R}^3$, its volume is null. Hence, $D(0,1,2,3,4,5) = 0$. Again, this defines a quadratic equation in s_{45}. Nevertheless, proceeding as above, this equation can be simplified by applying Jacobi's theorem to the following partition of D(0, 1, 2, 3, 4, 5)

$$\left|\begin{array}{ccccc|cc} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & s_{01} & s_{02} & s_{03} & s_{04} & s_{05} \\ 1 & s_{10} & 0 & s_{12} & s_{13} & s_{14} & s_{15} \\ 1 & s_{20} & s_{21} & 0 & s_{23} & s_{24} & s_{25} \\ 1 & s_{30} & s_{31} & s_{32} & 0 & s_{34} & s_{35} \\ \hline 1 & s_{40} & s_{41} & s_{42} & s_{43} & 0 & s_{45} \\ 1 & s_{50} & s_{51} & s_{52} & s_{53} & s_{54} & 0 \end{array}\right|,$$

concluding that $D(0,1,2,3,4,5) = 0$ yields

$$\frac{D(0,1,2,3,4)D(0,1,2,3,5)D^2(0,1,2,3,4;0,1,2,3,5)}{D(0,1,2,3)} = 0.$$

Now, note that $D(0,1,2,3,4) = 0$ and $D(0,1,2,3,5) = 0$ because they correspond to the volumes of simplices in $\mathbb{R}^4$. Thus, assuming that the tetrahedron defined by $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ is not degenerate,

$$D(0,1,2,3,4;0,1,2,3,5) = 0, \quad (3)$$

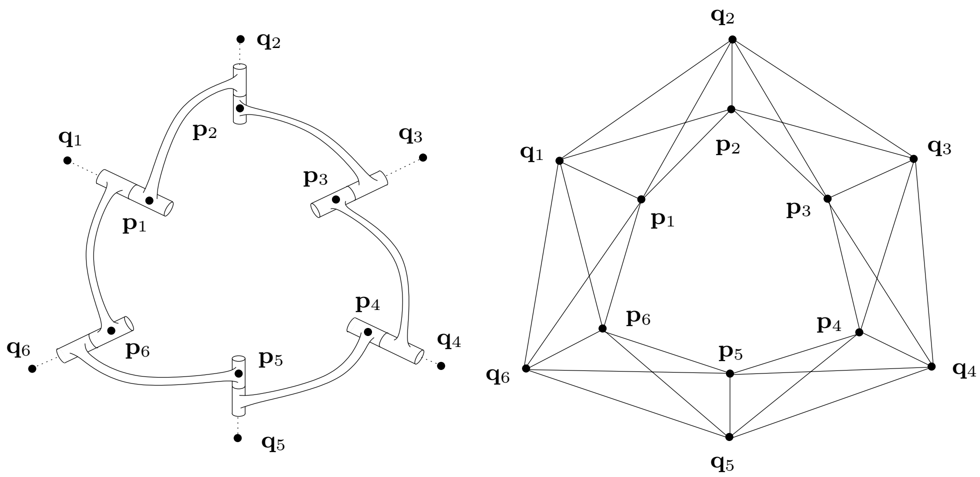
which is linear in s_{45}. As a consequence, there is only one solution for s_{45} compatible with all other distances.
In what follows, we will say that a distance matrix can be completed if its unknown distances can be determined by iteratively finding reference tetrahedra and triangles and using equations (2) and (3), respectively, to obtain the unknown distances. To reduce the number of possible matrix completions to a minimum, one should look for reference triangles only when no reference tetrahedra are available, because reference triangles lead to two possible solutions.

APPLICATION TO SPATIAL SERIAL ROBOTS

A 6 DoF serial robot whose end effector is fixed relative to its base can be seen as a closed chain of six rigid bodies pairwise articulated, that is, as a 6R closed mechanism (Fig. 3-a). Since translations can be seen as rotations centered at infinity, this representation is general enough for our purposes. By taking two points on each axis, this 6R mechanism can be translated into a set of distance constrains between 12 points, as explained above, which correspond to the edge lengths of six pairwise articulated tetrahedra (Fig. 3-b). If two consecutive axes meet at a point (possibly at infinity if they are parallel), the tetrahedron can be substituted by a triangle. However, note that no more than three axes can meet at a point if we want the corresponding serial robot to have 6 DoF.
In Fig. 3-b, we have two cycles of points, $\{\mathbf{p}_1, \ldots, \mathbf{p}_6\}$ and $\{\mathbf{q}_1, \ldots, \mathbf{q}_6\}$, where the couples $(\mathbf{p}_i, \mathbf{q}_i)$ define the rotation axes. Since we allow that two or more consecutive axes meet at a point, consecutive points in these cycles can actually coincide and, thus, in general, the two cycles will be $\{\mathbf{p}_1, \ldots, \mathbf{p}_m\}$ and $\{\mathbf{q}_1, \ldots, \mathbf{q}_n\}$ with $m, n \leq 6$. Then, the adjacency matrix S representing all interconnections has the form:

$$S = \begin{pmatrix} C_m & A \\ A^T & C_n \end{pmatrix}, \quad (4)$$

where C_m and C_n are the adjacency matrices for the points in the cycles $\{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$ and $\{\mathbf{q}_1, \ldots, \mathbf{q}_m\}$, respectively (i.e., they are cyclic tridiagonal matrices), and A is the matrix encoding the connections between points lying in different cycles. Now, all candidates for matrix A can be readily enumerated by realizing that the connections they encode should only correspond to a sequence of tetrahedra and triangles. The number of options can be further reduced taking into account that at least one tetrahedron must be included in the sequence because the robot end-effector can be placed arbitrarily with respect to its base. The result of this enumeration leads to 243 candidates. Discarding those that have more than three co-punctual axes and those that cannot be completed



(a) (b)

Figure 3: A general 6R mechanism (a), and its representation in terms of distance constraints (b).

as described in the previous section, we finally obtain 8 non-isomorphic configurations that correspond to the 8 serial robots shown in Fig. 4. Each robot is represented here as a bipartite graph made out of two vertex sets, each corresponding to one of the vertex cycles $\{\mathbf{p}_1, \ldots, \mathbf{p}_m\}$ and $\{\mathbf{q}_1, \ldots, \mathbf{q}_n\}$. Edges between the vertices represent the robot's joint axes.

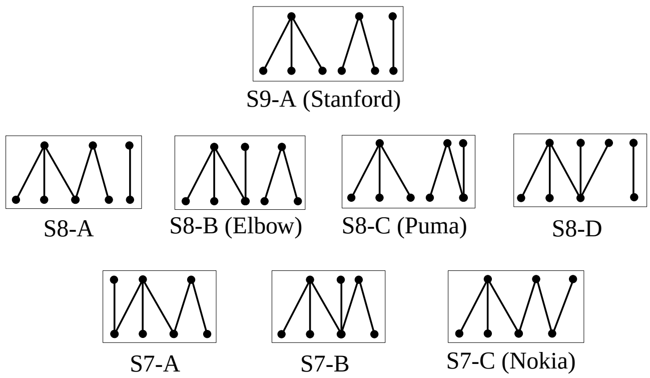


Figure 4: Schematic representation of all serial robots whose associated distance matrix can be completed using the two operations presented in this paper.

A close look to this family of serial robots reveals that all of its members have three consecutive co-punctual axes. Thus, all serial robots that can be solved using the presented technique are decoupled, that is, their inverse kinematics can be decomposed into rotational and translational components.

We note that the just characterized family of serial robots, although small, includes most popular 6 DoF serial manipulators such as the Stanford, Cincinnati Milacron, Puma and Nokia manipulators, as shown in Fig. 5.

AN EXAMPLE

The presented methodology has been implemented in MATLAB. As an example, we here follow in detail how it can be used to solve the inverse kinematics of a PUMA 560 manipulator.

First, each link of the robot must be modelled as a set of distance constraints between a set of points. Fig. 5-b shows this robot in its home position, and the points that should be selected to derive such constraints. Coordinates

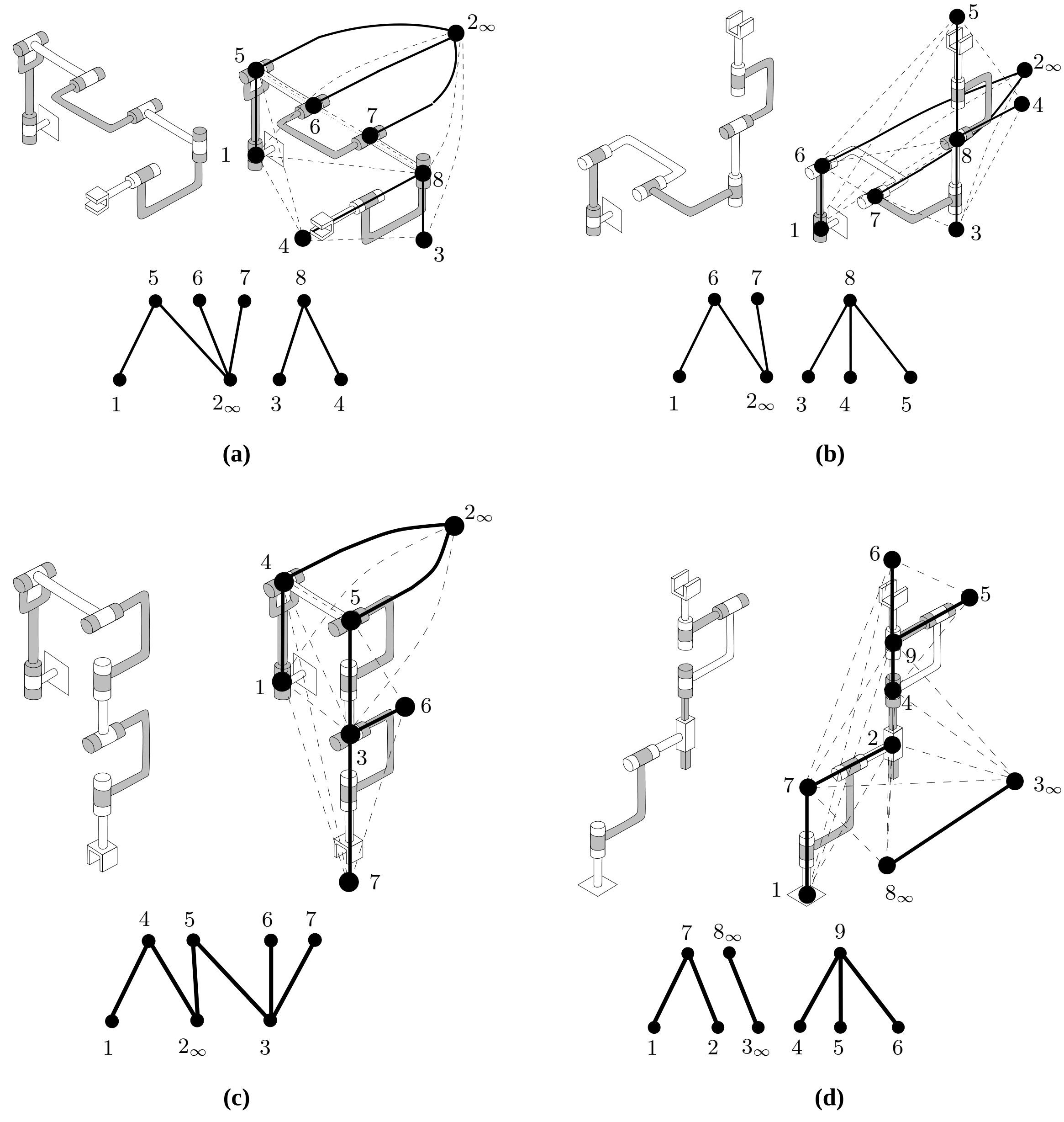


Figure 5: (a) Elbow manipulator or Cincinnati Milacron (S8-B). (b) Puma manipulator (S8-C). (c) Nokia manipulator (S7-C). (d) Stanford manipulator (S9-A). In (a), (b), and (c), parallelism between axes is exploited to simplify its analysis, but note that, even if this artifact is not introduced, the distance matrices associated with (a) and (b) can still be completed. Robot (d) includes a translational degree of freedom that, according to our formalism, is represented by a rotation axis at infinity. (Figure partially adapted from [3].)

i	α_i (deg.)	a_i (m)	θ_i (deg.)	d_i (m)
1	90	0	θ_1	0
2	0	0.4318	θ_2	0
3	-90	0.0203	θ_3	0.15005
4	90	0	θ_4	0.4318
5	-90	0	θ_5	0
6	0	0	θ_6	0

Table 1: Denavit-Hartenberg parameters of the PUMA 560 robot.

for all points can be easily obtained in terms of the Denavit-Hartenberg parameters of this robot, shown in Table 1. According to the process explained above, link 1 can be modelled by the fixed distances between three points, 1, 6, and 2, as it connects two intersecting axes. Similarly, link 2 is modelled by the fixed distances between points 6, 2, and 7, but point 2 must be placed at infinity, as the axes of this link are parallel. Link 3 has two skew axes and, thus, it is modelled by the fixed distances of the tetrahedron defined by 7, 2, 3, and 8. Links 4 and 5 connect concurrent axes and are thus represented by the triangles (3, 8, 4), and (4, 8, 5), respectively. These distances yield the non-bold numeric entries in the following distance matrix:

$$\mathbf{S} = \begin{pmatrix} 0 & 101.05 & s_{1,3} & s_{1,4} & \mathbf{1.314} & 1 & s_{1,7} & \mathbf{1.877} \\ 101.05 & 0 & 97.077 & s_{2,4} & s_{2,5} & 100.05 & 100.05 & 97.264 \\ s_{1,3} & 97.077 & 0 & 1.186 & s_{3,5} & s_{3,6} & 0.023 & 0.186 \\ s_{1,4} & s_{2,4} & 1.186 & 0 & 2 & s_{4,6} & s_{4,7} & 1 \\ \mathbf{1.314} & s_{2,5} & s_{3,5} & 2 & 0 & \mathbf{0.596} & s_{5,7} & 1 \\ 1 & 100.05 & s_{3,6} & s_{4,6} & \mathbf{0.596} & 0 & 0.186 & \mathbf{0.318} \\ s_{1,7} & 100.05 & 0.023 & s_{4,7} & s_{5,7} & 0.186 & 0 & 0.209 \\ \mathbf{1.877} & 97.264 & 0.186 & 1 & 1 & \mathbf{0.318} & 0.209 & 0 \end{pmatrix}.$$

The usual formulation of the inverse kinematics further considers a 0th and a 6th link, respectively modelling the ‘world’ and the ‘hand’, and a homogeneous transformation positioning the hand with respect to the world. Note that, in doing so, these two elements become mutually rigid, and it is thus possible to consider them as a *single* link constraining the relative position between the first and last revolute axes. In our setting, this is equivalent to fixing the distances of the tetrahedron (1, 6, 5, 8). Note that these distances, while fixed, depend on the specific pose of the end effector relative to the world, or, in other words, on the specific inverse kinematics problem to be solved. They correspond to the numeric bold entries in $\mathbf{S}$, computed in this example for a pose of the hand given by the joint angles $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (229.25°, 339.86°, 14.68°, 102.84°, 243.81°, 211.03°)$.
Now, by taking the sequence of tetrahedra and triangles shown in the first column of the following table as references, the distances in the second column can be obtained thus completing $\mathbf{S}$.

tetrahedron/triangle		squared distance
$(2,6,8)$	$\rightarrow$	$s_{1,7}$
$(1,6,8)$	$\rightarrow$	$s_{2,5}$
$(2,7,8)$	$\rightarrow$	$s_{3,6}$
$(1,2,6,8)$	$\rightarrow$	$s_{5,7}$
$(2,6,7,8)$	$\rightarrow$	$s_{1,3}$
$(1,2,6,7)$	$\rightarrow$	$s_{3,5}$
$(3,5,8)$	$\rightarrow$	$s_{4,6}$
$(3,5,6,8)$	$\rightarrow$	$s_{4,7}$
$(3,5,6,7)$	$\rightarrow$	$s_{1,4}$
$(1,3,5,6)$	$\rightarrow$	$s_{2,4}$

Since there are 4 steps in this sequence in which a triangle is involved, a total of $2^4 = 16$ possible completions exist for $\mathbf{S}$. Them all correspond to consistent Euclidean matrices; that is, their associated Gram matrices are positive semidefinite of rank 3. Each such matrix yields two specular coordinatizations of the points $1, \ldots, 8$. If among these coordinatizations we discard those where the orientations of tetrahedra $(1, 6, 5, 8)$ and $(2, 3, 7, 8)$ are different from that in the actual robot, we finally obtain the following 8 solutions for the joint angles $\theta_1, \ldots, \theta_6$.

sol. #	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
1	11.75	200.37	171.57	133.73	124.20	28.30
2	11.75	200.37	171.57	313.73	235.80	208.30
3	228.62	339.63	14.67	283.19	115.57	30.58
4	228.62	339.63	14.67	103.19	244.43	210.58
5	11.75	97.66	14.67	221.40	239.70	77.82
6	11.75	97.66	14.67	41.40	120.30	257.83
7	228.62	82.34	171.57	58.262	264.26	106.19
8	228.62	82.34	171.57	238.26	95.74	286.19

The reader can check that the fourth such solution is very close to the pose of the hand specified above, the small error being due to the fact that point 2 is not sufficiently far away from the robot (here it is placed at a distance $d_\infty = 10$m from the line defined by points 6 and 7). The following table shows how, by solving the same problem at increasing values of d_∞, this solution tends to the exact one.

d_∞	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
1	222.76	337.40	14.88	106.64	250.01	206.41
10	228.62	339.63	14.67	103.19	244.43	210.58
100	229.18	339.84	14.68	102.88	243.87	210.98
1000	229.24	339.86	14.68	102.84	243.82	211.02
10000	229.25	339.86	14.68	102.84	243.81	211.03
∞ (exact)	229.25	339.86	14.68	102.84	243.81	211.03

CONCLUSIONS

We have presented an explicit enumeration of all six DoF serial robots whose inverse kinematics can be solved by completing a distance matrix. This completion is performed by applying a sequence of two simple operations. The method has been implemented in MATLAB, and the resolution of the inverse kinematics of a PUMA 560 robot has been reported as an example. This method, if implemented in a symbolic algebra package, would yield closed-form formulas. This is a point that deserves further efforts as it would permit to analyze the computational efficiency of the resulting formulas, and their numerical stability, in front of other well-known formulations obtained for particular architectures.

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